

Extending Fuzzy Description Logics with a Possibilistic Layer

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Abstract. Classical ontologies are not suitable to represent imprecise nor uncertain pieces of information. As a solution we will combine fuzzy Description Logics with a possibilistic layer. Then, we will show how to perform reasoning by relying on classical existing reasoners.

Description Logics (DLs for short) are a family of logics for representing structured knowledge which have proved to be very useful as ontology languages. Nevertheless, it has been widely pointed out that classical ontologies are not appropriate to deal with imprecise, vague and uncertain knowledge, which is inherent to several real-world domains and Semantic Web tasks (e.g. the integration or merging of ontologies). Fuzzy and possibilistic logics have proved to be suitable formalisms to handle imprecise/vague and uncertain knowledge respectively. Fuzzy and possibilistic logics are orthogonal, the former handling degrees of truth and the latter handling degrees of certainty.

There exist several fuzzy and possibilistic extensions of DLs in the literature (see [1] for an overview). These extensions are appropriate to handle either vagueness or uncertainty, but handling both of them has not received such attention. An exception is [2], where every fuzzy set is represented using two crisp sets (its support and core) and then axioms are extended with necessity degrees. Although for some applications this representation may be enough (and the own authors suggest to consider more α -cuts), there is a loss of information which we will overcome here. Another related work combines fuzzy vagueness and probabilistic uncertainty with description logic programs [3].

We propose to build a layer to deal with uncertain knowledge on top of a fuzzy Knowledge Base (KB) defined as in [4], by annotating the axioms with possibility and necessity degrees, and to reduce it to a possibilistic layer over a crisp ontology. Interestingly, this makes possible to perform reasoning tasks relying on existing classical reasoners e.g. Pellet (<http://pellet.owldl.com>).

Syntax. A *possibilistic fuzzy knowledge base* pfK is a fuzzy KB where each fuzzy axiom τ (see [4] for details) is equipped with a possibility or necessity degree, $(\tau, P \alpha)$ or $(\tau, N \alpha)$ respectively with $\alpha \in (0, 1]$. If no degree is specified, $N 1$ is assumed. Necessity degrees express to what extent a formula is necessary true, whereas possibility degrees express to what extent a formula is possible.

Semantics. Let \mathcal{I} be the set of all (fuzzy) interpretations. A *possibilistic interpretation* is a mapping $\pi : \mathcal{I} \rightarrow [0, 1]$ such that $\pi(\mathcal{I}) = 1$ for some $\mathcal{I} \in \mathcal{I}$. The intuition here is that $\pi(\mathcal{I})$ represents the degree to which the world \mathcal{I} is possible.

\mathcal{I} is impossible if $\pi(\mathcal{I}) = 0$ and fully possible if $\pi(\mathcal{I}) = 1$. The possibility of an axiom τ is defined as $Poss(\tau) = \sup\{\pi(\mathcal{I}) \mid \mathcal{I} \in \mathcal{J}, \mathcal{I} \models \tau\}$ (where $\sup \emptyset = 0$), and the necessity is defined as $Nec(\tau) = 1 - Poss(\neg\tau)$. A possibilistic interpretation π satisfies a possibilistic axiom $(\tau, \Pi\gamma)$, denoted $\pi \models (\tau, \Pi\gamma)$, iff $Poss(\tau) \geq \gamma$ and a possibilistic axiom $(\tau, N\gamma)$, denoted $\pi \models (\tau, N\gamma)$, iff $Nec(\tau) \geq \gamma$.

Reasoning. B. Hollunder showed that reasoning within a possibilistic DL can be reduced to reasoning within a classical DL [5]. We will reduce here our possibilistic fuzzy DL to a possibilistic DL. A fuzzy KB fK can be reduced to a crisp KB $\mathcal{K}(fK)$ and every axiom $\tau \in fK$ is reduced to $\mathcal{K}(\tau)$, which can be an axiom or a set of axioms [4]. Adding degrees of certainty to fK formulae is equivalent to adding degrees of certainty to their reductions, as long as we also consider axioms preserving the semantics of the whole process (which are assumed to be necessarily true and do not have any degree of certainty associated). For every axiom $(\tau, \Pi\gamma) \in pfK$, $Poss(\tau) \geq \gamma$ iff $Poss(\mathcal{K}(\tau)) \geq \gamma$. Similarly, $(\tau, N\gamma) \in pfK$, $Nec(\tau) \geq \gamma$ iff $Nec(\mathcal{K}(\tau)) \geq \gamma$.

Example 1. The axiom $(\langle tom : High \geq 0.5 \rangle, N 0.2)$ means that it is possible with degree 0.2 that *tom* can be considered a *High* person with (at least) degree 0.5. It is reduced into $(\langle tom : High_{\geq 0.5} \rangle, N 0.2)$, meaning that it is possible with degree 0.2 that *tom* belongs to the crisp set $High_{\geq 0.5}$. The final crisp KB would also need some additional axioms (consequence of the reduction of the fuzzy KB): $High_{\geq 0.5} \sqsubseteq High_{>0}$, $High_{>0.5} \sqsubseteq High_{\geq 0.5}$ and $High_{\geq 1} \sqsubseteq High_{>0.5}$. \square

Final remarks. [4] reduces a fuzzy KB to a crisp KB and reasoning is performed by computing a consistency test on the crisp KB. Our case is more difficult and needs to perform several entailment tests. Moreover, how to represent the possibilistic DL using a classical DL remains an open issue.

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