Optimizing the Crisp Representation of the Fuzzy Description Logic $\mathcal{SROIQ}$

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Outline

1. Motivation
2. Fuzzy $\mathcal{SROIQ}$
3. A Crisp Representation for Fuzzy $\mathcal{SROIQ}$
4. Implementation
5. Conclusions and Future Work

Crisp Representations for Fuzzy DLs

- Classical ontologies are not appropriate for imprecise and vague knowledge. A solution are **fuzzy Description Logics** (DLs).
- Fuzzy DLs require that new languages need to be used, and hence to adapt the available resources.
  - Specially difficult with reasoners: significant gap between the design of a decision procedure and a practical implementation.
- Alternative: To **represent fuzzy DLs using crisp DLs** and to reduce reasoning within fuzzy DLs to reasoning within crisp ones.
- Advantages:
  - No need to agree a new standard fuzzy language.
  - Use of standard languages and reuse of available resources.
  - Use of existing crisp reasoners.
    - This will support early reasoning in future fuzzy languages.
- An immediate practical application of fuzzy ontologies is feasible because it relies on existing valid languages and tools.

Our contributions

- A crisp **representation of fuzzy $\mathcal{SROIQ}$**.
  - G. Stoilos et al. proposed fuzzy $\mathcal{SROIQ}$ but only provided reasoning for a fragment of it, missing:
    - Qualified cardinality restrictions e.g. $\geq 2 \mathit{hasSon}.\mathit{Male}$,
    - Negated local reflexivity concepts e.g. $\neg \exists \mathit{likes}.\mathit{Self}$,
    - Negative role assertions e.g. $(\mathit{fernando}, \mathit{juan}) : \neg \mathit{hasFriend}$,
- **Fuzzy** Concept and Role Inclusion Axioms (**GCIs and RIAs**) can be true to some degree using Gödel implication in the semantics.
  - First work supporting reasoning with fuzzy RIAs.
- The reduction is **optimized** in several ways:
  - We reduce the number of new crisp atomic elements (which are needed to represent the elements in the fuzzy KB).
  - We reduce the new axioms needed to preserve their semantics.
  - We show how to optimize some important GCIs.
- Implementation of **DELOREAN**, the first reasoner supporting a fuzzy extension of $\mathcal{SHOIN}$ (and hence OWL DL).

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Complex fuzzy concept and roles

Constructor	**Syntax**	**Semantics**
(top concept)	$\top$	1
(bottom concept)	$\bot$	0
(atomic concept)	A	$A^{\mathcal{I}}(a)$
(concept conjunction)	$C \sqcap D$	$C^{\mathcal{I}}(a) \otimes D^{\mathcal{I}}(a)$
(concept disjunction)	$C \sqcup D$	$C^{\mathcal{I}}(a) \oplus D^{\mathcal{I}}(a)$
(concept negation)	$\neg C$	$\ominus C^{\mathcal{I}}(a)$
(universal quantification)	$\forall R.C$	$\inf_{b \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(a, b) \rightarrow C^{\mathcal{I}}(b)\}$
(existential quantification)	$\exists R.C$	$\sup_{b \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(a, b) \otimes C^{\mathcal{I}}(b)\}$
(fuzzy nominals)	$\bigcup_{i=1}^{m} \{\alpha_i / o_i\}$	$\sup_{i \mid a \in \{o_i^{\mathcal{I}}\}} \alpha_i$
(at-least restriction)	$\geq n\ S.C$	$\sup_{b_1, \ldots, b_m \in \Delta^{\mathcal{I}}} [(\otimes_{i=1}^{m} \{S^{\mathcal{I}}(a, b_i) \otimes C^{\mathcal{I}}(b_i)\}) \bigotimes (\otimes_{j<k} \{b_j \neq b_k\})]$
(at-most restriction)	$\leq n\ S.C$	$\inf_{b_1, \ldots, b_{n+1} \in \Delta^{\mathcal{I}}} [(\otimes_{i=1}^{n+1} \{S^{\mathcal{I}}(a, b_i) \otimes C^{\mathcal{I}}(b_i)\}) \rightarrow (\oplus_{j<k} \{b_j = b_k\})]$
(local reflexivity)	$\exists S.Self$	$S^{\mathcal{I}}(a, a)$
(atomic role)	R_A	$R_A^{\mathcal{I}}(a, b)$
(inverse role)	R^-	$R^{\mathcal{I}}(b, a)$
(universal role)	U	1

Fuzzy axioms in the ABox, TBox and RBox

Axiom	Syntax	Semantics
(concept ass.)	$\langle a : C \bowtie \alpha \rangle$	$C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie \alpha$
(role ass.)	$\langle (a, b) : R \bowtie \alpha \rangle$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie \alpha$
(inequality ass.)	$\langle a \neq b \rangle$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$
(equality ass.)	$\langle a = b \rangle$	$a^{\mathcal{I}} = b^{\mathcal{I}}$
(GCI)	$\langle C \sqsubseteq D \rhd \alpha \rangle$	$\inf_{a \in \Delta^{\mathcal{I}}} \{C^{\mathcal{I}}(a) \rightarrow D^{\mathcal{I}}(a)\} \rhd \alpha$
(RIA)	$\langle R_1 R_2 \ldots R_n \sqsubseteq R' \rhd \alpha \rangle$	$\sup_{b_1 \ldots b_{n+1} \in \Delta^{\mathcal{I}}} \bigotimes [R_1^{\mathcal{I}}(b_1, b_2), \ldots, R_n^{\mathcal{I}}(b_n, b_{n+1})] \rightarrow R^{\mathcal{I}}(b_1, b_{n+1}) \rhd \alpha$
(transitive)	$trans(R)$	$\forall a, b \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a, b) \geq \sup_{c \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(a, c) \otimes R^{\mathcal{I}}(c, b)$
(disjoint)	$dis(S_1, S_2)$	$\forall a, b \in \Delta^{\mathcal{I}}, S_1^{\mathcal{I}}(a, b) \otimes S_2^{\mathcal{I}}(a, b) = 0$
(reflexive)	$ref(R)$	$\forall a \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a, a) = 1$
(irreflexive)	$irr(S)$	$\forall a \in \Delta^{\mathcal{I}}, S^{\mathcal{I}}(a, a) = 0$
(symmetric)	$sym(R)$	$\forall a, b \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a, b) = R^{\mathcal{I}}(b, a)$
(asymmetric)	$asy(S)$	$\forall a, b \in \Delta^{\mathcal{I}}$, if $S^{\mathcal{I}}(a, b) > 0$ then $S^{\mathcal{I}}(b, a) = 0$

Fuzzy operators

- We consider $f_{KD}\mathcal{SROIQ}$:
  - **Minimum t-norm**, $\alpha \otimes \beta = \min\{\alpha, \beta\}$
  - **Maximum t-conorm**, $\alpha \oplus \beta = \max\{\alpha, \beta\}$
  - **Łukasiewicz negation**, $\ominus\alpha = 1 - \alpha$
  - **KD implication** except in GCIs and RIAs: $\alpha \rightarrow \beta = \max\{1 - \alpha, \beta\}$
  - **Gödel implication** in GCIs and RIAs: $\alpha \rightarrow \beta = \begin{cases} 1 & \alpha \leq \beta \\ \beta & \alpha > \beta \end{cases}$
- These fuzzy operators make possible the reduction to a crisp KB (other fuzzy operators are not suitable in principle).

Fuzzy GCIs and RIAs

- The most common semantics for GCIs and RIAs, based on **Zadeh's set inclusion**, forces them to be either true or false:

$$C \sqsubseteq D \text{ iff } \forall x \in \Delta^{\mathcal{I}}, C^{\mathcal{I}}(x) \leq D^{\mathcal{I}}(x)$$

- The use of **Kleene-Dienes implication** in the semantics of GCIs and RIAs brings about two counter-intuitive effects:
  - In general concepts (and roles) do not fully subsume themselves.
  - $\langle C \sqsubseteq D \geq 1 \rangle$ force some fuzzy concepts and roles to be interpreted as crisp.
- **Gödel implication**:
  - Solves these problems,
  - It is suitable for a classical representation.
  - For GCIs of the form $\langle C \sqsubseteq D \geq 1 \rangle$, it is equivalent to consider Zadeh's set inclusion.

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Idea of the reduction

1. Compute the **set of degrees of truth** which must be considered.
   - Degrees x in the fuzzy KB and their complementaries $1 - x$.
2. For each fuzzy atomic concept and role, add **new crisp elements** (α-cut and strict α-cut.)
3. Add **new axioms** to preserve their semantics.
4. Reduce **fuzzy axioms** using the new crisp elements.

- The size of the crisp KB is **quadratic**.
  - The size is linear under a fixed set of degrees.
- The reduction **preserves reasoning**.
  - Consistency of the fuzzy KB and the crisp KB are equivalent.
- The reduction can be **reused** when adding new axioms.
  - If the new axiom do not introduce new vocabulary nor degrees.

Example of reduction

h is a hotel at a German-speaking country with at-least degree 0.5.

$KB = \{ \langle h : Hotel \sqcap \forall isIn.\{(ge, 1), (au, 1), (sw, 0.67)\} > 0.5\rangle \}$

1. **Degrees of truth** to be considered:
   $\{0, 0.5, 1\}$
2. New **crisp elements**:
   $Hotel_{>0}, Hotel_{\geq 0.5}, Hotel_{>0.5}, Hotel_{\geq 1}, isIn_{>0}, isIn_{\geq 0.5}, isIn_{>0.5}, isIn_{\geq 1}$
3. New **axioms**:
   $Hotel_{\geq 0.25} \sqsubseteq Hotel_{>0}, Hotel_{>0.25} \sqsubseteq Hotel_{\geq 0.25}, \ldots$
   $IsIn_{\geq 0.25} \sqsubseteq IsIn_{>0}, IsIn_{>0.25} \sqsubseteq IsIn_{\geq 0.25}, \ldots$
4. **Reduction** of every axiom in the KB:
   $h : \rho(Hotel, > 0.5) \sqcap \forall \rho(isIn, \geq 0.5).\rho(\{(ge, 1), (au, 1), (sw, 0.67)\},$
   $> 0.5)$ = $h : Hotel_{>0.5} \sqcap \forall isIn_{\geq 0.5}.\{ge, au, sw\}$

Example: Reduction of a fuzzy GCI

- Consider the GCI $\langle C \sqsubseteq D \geq \alpha \rangle$.
- If it is satisfied, $\inf_{a \in \Delta^{\mathcal{I}}} C^{\mathcal{I}}(a) \Rightarrow D^{\mathcal{I}}(a) \geq \alpha$.
- An arbitrary a must satisfy that $C^{\mathcal{I}}(a) \Rightarrow D^{\mathcal{I}}(a) \geq \alpha$.
- From the semantics of Gödel implication, this is true if:
  - $C^{\mathcal{I}}(a) \leq D^{\mathcal{I}}(a)$, or
  - $D^{\mathcal{I}}(a) \geq \alpha$.
- Roughly, for very γ such that $\gamma < \alpha$, $C^{\mathcal{I}}(a) \rhd \gamma$ implies $D^{\mathcal{I}}(a) \rhd \gamma$.

$$\rho(C, \rhd\gamma) \sqsubseteq \rho(D, \rhd\gamma)$$

- Additionally, $C^{\mathcal{I}}(a) \geq \alpha$ implies $D^{\mathcal{I}}(a) \geq \alpha$.

$$\rho(C, \geq \alpha) \sqsubseteq \rho(D, \geq \alpha)$$

Optimizing the number of new elements and axioms

- Roughly, for each $\alpha, \beta \in N^{fK}$ we create:
  - Two new crisp atomic concepts $A_{\geq \alpha}, A_{> \beta}$.
  - Two new crisp atomic roles $R_{\geq \alpha}, R_{> \beta}$.
- **Previous works use two more atomic concepts** $A_{\leq \beta}, A_{< \alpha}$, but:
  - We use $\neg A_{> \gamma_k}$ rather than $A_{\leq \gamma_k}$.
  - We use $\neg A_{\geq \gamma_k}$ instead of $A_{< \gamma_k}$.
- We also need some new axioms to preserve their semantics:

$$A_{\geq \gamma_{i+1}} \sqsubseteq A_{> \gamma_i} \quad A_{> \gamma_j} \sqsubseteq A_{\geq \gamma_j}$$
$$R_{\geq \gamma_{i+1}} \sqsubseteq R_{> \gamma_i} \quad R_{> \gamma_i} \sqsubseteq R_{\geq \gamma_i}$$

- **Previous works also use some additional axioms**, which now are superfluous (they follow immediately from the semantics):

$$A_{< \gamma_k} \sqsubseteq A_{\leq \gamma_k} \quad A_{\leq \gamma_i} \sqsubseteq A_{< \gamma_{i+1}}$$
$$A_{\geq \gamma_k} \sqcap A_{< \gamma_k} \sqsubseteq \bot \quad A_{> \gamma_i} \sqcap A_{\leq \gamma_i} \sqsubseteq \bot$$
$$\top \sqsubseteq A_{\geq \gamma_k} \sqcup A_{< \gamma_k} \quad \top \sqsubseteq A_{> \gamma_i} \sqcup A_{\leq \gamma_i}$$

Optimizing some GCIs

- $\langle C \sqsubseteq \top \bowtie \gamma \rangle$ and $\langle \bot \sqsubseteq D \bowtie \gamma \rangle$ are tautologies.
  - They are unnecessary in the resulting KB.
- $\kappa(\top \sqsubseteq D \bowtie \gamma) = \top \sqsubseteq \rho(D, \bowtie \gamma)$.
  - It appears in role range axioms, $range(R) = C$ iff $\top \sqsubseteq \forall R.C \geq 1$.
- $\kappa(C \sqsubseteq \bot \bowtie \gamma) = \rho(C, > 0) \sqsubseteq \bot$.
  - It appears in disjointness, $disjoint(C, D) = C$ iff $C \sqcap D \sqsubseteq \bot \geq 1$.
- If the resulting TBox contains $A \sqsubseteq B$, $A \sqsubseteq C$ and $B \sqsubseteq C$, then $A \sqsubseteq C$ is unnecessary.
  - Example: $\kappa(C \sqsubseteq \{1/o_1, 0.5/o_2\}) = \{C_{>0} \sqsubseteq \{o_1, o_2\}, C_{\geq 0.5} \sqsubseteq \{o_1, o_2\}, C_{>0.5} \sqsubseteq \{o_1\}, C_{\geq 1} \sqsubseteq \{o_1\}\}$ can be optimized to: $\{C_{>0} \sqsubseteq \{o_1, o_2\}, C_{\geq 0.5} \sqsubseteq \{o_1\}\}$.

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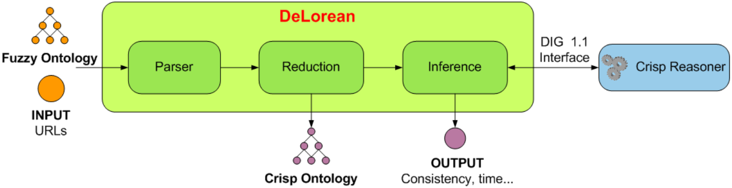
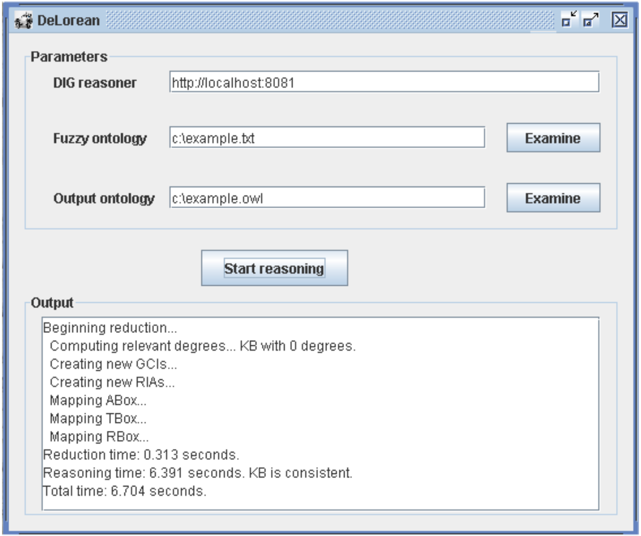
Implementation: DELOREAN

- **DELOREAN** = DEscription LOgic REasoner with vAgueNess.
- Using Java, Jena API, JavaCC and **DIG 1.1** interface.
- Architecture:



  - The **Parser** reads an input file with a fuzzy ontology.
  - **Reduction** module implements the reduction, builds a Jena model, and saves it as an OWL file with an equivalent crisp ontology.
  - **Inference** module perform a consistency test, using any crisp reasoner through the DIG interface.
  - **User interface** communicates with the user.
- Currently the logic supported is $f_{KD}\mathcal{SHOIN}$ **(OWL DL)**, since DIG interface does not yet support full $\mathcal{SROIQ}$.

DELOREAN User Interface



Experimentation

- Experiments have shown that the **results of the reasoning tasks** over the crisp ontology were the expected.
- We extended the axioms of *Koala*, a small $\mathcal{ALCON}(\mathcal{D})$ ontology, with random degrees and used PELLET reasoner through DIG.
- Time of a classification test over the resulting crisp ontology:

Number of degrees	crisp	3	5	7	11
Reduction time	-	1.18	6.28	23.5	148.25
Reasoning time	0.56	0.98	1.343	2.88	6.47

- The **reduction time is currently high**, so the implementation should be optimized. Anyway, the reduction can be reused and hence needs to be computed just once (possibly off-line).
- The **reasoning time is reasonable** at least for small ontologies and using a limited number of degrees of truth.

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Conclusions and Future Work

- **Conclusions**:
  - We have shown how to reduce fuzzy $\mathcal{SROIQ}$ into $\mathcal{SROIQ}$.
  - Crisp representations can be optimized in several ways.
  - Restricting the number of truth degrees is important to control the complexity of the reduction.
  - DELOREAN is the first reasoner supporting fuzzy $\mathcal{SHOIN}$ (and hence fuzzy OWL DL).
- **Future work**:
  - Compare DELOREAN with other fuzzy DL reasoners.
  - Extend the reasoner to fuzzy $\mathcal{SROIQ}$ (and hence OWL 1.1) as soon as DIG 2.0 interface is available.
  - To allow the definition of crisp concepts and roles.
  - To allow the use of two implications in the semantics of GCIs and RIAs: Gödel and Kleene-Dienes.

Questions?

Thank you very much for your attention