Representing Uncertain Concepts in Rough Description Logics via Contextual Indiscernibility Relations

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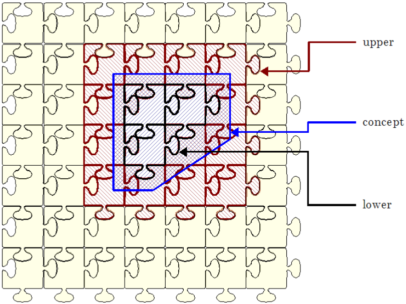
Contents

Introduction & Motivation

- Modeling *uncertain concepts* in description logics (DLs) is generally done via numerical approaches
  - probabilistic approach
  - possibilistic approach
- **Drawback:** uncertainty is introduced into the model ⇒
  - increase of conceptual and/or computational complexity
  - new reasoners may be necessary
- *To overcome such a drawback* we **focus on:** modelling uncertain concepts via *Rough Description Logics* (RDLs) [Schlobach et al. @ IJCAI 2007]

Rough Description Logics: Basics...

- RDLs allow for modeling an *underspecified (uncertain) concept* C by means of *crisp approximations of* C
- **RDLs extend classical DLs with two modal-like operators**
  - the *lower approximation* of $C \Rightarrow$ sub-concept of C
  - the *upper approximation* of a $C \Rightarrow$ super-concept of C



...Rough Description Logics: Basics...

The concept approximations are based on capturing uncertainty as an *indiscernibility relation* R *among individuals*

- **the upper approximation** describes which elements are **possibly** elements of C
  - defined as *the set of individuals that are* **indiscernible** from *at least an individual* that is known to *belong to* C
    $$\overline{C} := \{a \mid \exists b : R(a,b) \wedge b \in C\}$$
- **the lower approximation** describes which elements are **definitely** elements of C
  - defined as *the set of individuals* that are **indiscernible** from *all individuals* that are known to *belong to* C
    $$\underline{C} := \{a \mid \forall b : R(a,b) \rightarrow b \in C\}$$

RDL concept description: Example

Task: model the uncertain concept *Addressee* $\Leftrightarrow$ potential addressees for the advertising campaign of a new product (Porsche)

- $\underline{Addressee}$ = people with an income above 1 million dollars
- $\overline{Addressee}$ = people with an income above 100 000 dollars

RDL can be simulated in standard DL $\Leftrightarrow$ A RDL concept can be easily translated in a DL concept without adding expressive power

Defining Conceptual Indescernibility Relations

Problem: *RDLs do not specify any indiscernibility relation* $R \Rightarrow$ we introduce two general ways for defining it

1. **Context-based Indescernibility relation**
   - the indiscernibility of the individuals is defined w.r.t. a specific context described by concepts (in the ontology)
2. **Similarity-based Indescernibility relation**
   - it allows for relaxing the discernibility of individuals using a tolerance threshold

Defining a Context

Definition (context)

A *context* is a finite set of *relevant features* in the form of DL concepts (of the knowledge base) $\mathsf{C} = \{F_1, \ldots, F_m\}$

Example (Advertising Campaign cont'd)

A possible context C for the advertising campaign is:

$$\mathsf{C} = \{\mathit{SalaryAboveMillion}, \mathit{HouseOwner}, \mathit{Manager}\},$$

where *SalaryAboveMillion*, *HouseOwner*, and *Manager* are DL concepts.

Context-based Indescernibility Relation

Definition (indiscernibility relation)

Let $\mathsf{C} = \{F_1, \ldots, F_m\}$ be a context, let $\mathcal{I}$ be a DL interpretation. *The indiscernibility relation* R_{C} *induced by* C is defined as:

$$R_{\mathsf{C}} = \Big\{(a,b) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}) \mid \forall i \in \{1, \ldots, m\} : \ \pi_{F_i}(a) = \pi_{F_i}(b)\Big\}$$

where $\forall i \in \{1, \ldots, m\}$ the *projection function* $\pi_{F_i} : \Delta^{\mathcal{I}} \mapsto \{0, \frac{1}{2}, 1\}$ is defined as ***[Fanizzi et al. 2007]***:

$$\forall a \in \Delta^{\mathcal{I}} : \quad \pi_{F_i}(a) = \begin{cases} 1 & \mathcal{I} \models F_i(a); \\ 0 & \mathcal{I} \models \neg F_i(a); \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

Contextual upper and lower approximations

- Any indiscernibility relation R splits $\Delta^{\mathcal{I}}$ in a partition of equivalence classes $[a]_{\mathsf{C}}$ (*elementary sets*), for a generic individual a.
- Each $[a]_{\mathsf{C}}$ naturally induces a concept C_a
  - a possible desciption for C_a is $\mathsf{C}_a \equiv F_1 \sqcap F_2 \sqcap \neg F_3 \sqcap \cdots \sqcap F_m$

Definition (Contextual Approximations)

Let $\mathsf{C} = \{F_1, \ldots, F_m\}$ be a context, let D be a DL concept, and let $\mathcal{I}$ be an interpretation. The *contextual upper and lower approximations* of D relative to the context C, are defined as:

- $\overline{D}^{\mathsf{C}} = \{a \in \Delta^{\mathcal{I}} \mid \mathsf{C}_a \sqcap D \not\models \bot\}$,
- $\underline{D}_{\mathsf{C}} = \{a \in \Delta^{\mathcal{I}} \mid \mathsf{C}_a \sqsubseteq D\}$.

Contextual Approximations: Properties

Given a context $\mathsf{C} = \{F_1, \ldots, F_m\}$ and two concepts D and E, *the following properties holds for the approximation operators*:

1. $\underline{\bot}_{\mathsf{C}} = \overline{\bot}^{\mathsf{C}} = \bot$,
2. $\underline{\top}_{\mathsf{C}} = \overline{\top}^{\mathsf{C}} = \top$,
3. $\underline{D \sqcup E}_{\mathsf{C}} \sqsupseteq \underline{D}_{\mathsf{C}} \sqcup \underline{E}_{\mathsf{C}}$,
4. $\overline{D \sqcup E}^{\mathsf{C}} = \overline{D}^{\mathsf{C}} \sqcup \overline{E}^{\mathsf{C}}$,
5. $\underline{D \sqcap E}_{\mathsf{C}} = \underline{D}_{\mathsf{C}} \sqcap \underline{E}_{\mathsf{C}}$,
6. $\overline{D \sqcap E}^{\mathsf{C}} \sqsubseteq \overline{D}^{\mathsf{C}} \sqcap \overline{E}^{\mathsf{C}}$,
7. $\underline{\neg D}_{\mathsf{C}} = \neg \overline{D}^{\mathsf{C}}$,
8. $\overline{\neg D}^{\mathsf{C}} = \neg \underline{D}_{\mathsf{C}}$,
9. $\underline{\underline{D}_{\mathsf{C}}}_{\mathsf{C}} = \underline{D}_{\mathsf{C}}$,
10. $\overline{\overline{D}^{\mathsf{C}}}^{\mathsf{C}} = \overline{D}^{\mathsf{C}}$.

Rough Membership Function

- A (rough) concept description may include boundary individuals which cannot be ascribed to a concept with absolute certainty
- As uncertainty is related to the membership to a set, **a (rough) membership function can be defined**
  - *it can be considered a numerical measure of the uncertainty*

Definition (Rough Membership Function)

Let $\mathsf{C} = \{F_1, \ldots, F_m\}$ be a context and let $\mathcal{I}$ be the canonical interpretation. *The* C*-rough membership function of an individual* a *to a concept* D is defined by:

$$\mu_{\mathsf{C}}(a, D) = \frac{|(\mathsf{C}_a \sqcap D)^{\mathcal{I}}|}{|(\mathsf{C}_a)^{\mathcal{I}}|},$$

Similarity based Indescernibility Relation

Goal: relaxing individual indescernibility by a similarity measure

Definition (Contextual Similarity Measure)

Let $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ be a knowledge base. Given a context $\mathsf{C} = \{F_1, F_2, \ldots, F_m\}$, a family of similarity functions $s_p^{\mathsf{C}} : \mathrm{Ind}(\mathcal{A}) \times \mathrm{Ind}(\mathcal{A}) \mapsto [0,1]$ is defined as $(\forall a, b \in \mathrm{Ind}(\mathcal{A}))$:

$$s_p^{\mathsf{C}}(a,b) := \sqrt[p]{\sum_{i=1}^{m} \left| \frac{\sigma_i(a,b)}{m} \right|^p},$$

where $p > 0$ and the *basic similarity function* σ_i is:

$$\forall a, b \in \mathrm{Ind}(\mathcal{A}): \qquad \sigma_i(a,b) = 1 - |\pi_i(a) - \pi_i(b)|.$$

Similarity Measure: Discussion

- **UNDERLYING IDEA**: *on a semantic level, similar individuals should behave similarly w.r.t. the same concepts*
    - Similarity is determined relative to a context (set of concepts) acting as *discriminating feature set*
- Two individuals are *maximally similar* w.r.t. a concept F_i if they exhibit the same behavior, i.e., both are instances of the concept or of its negation
- The *minimal similarity* holds when individuals belong to opposite concepts
- *Due to the OWA*, a reasoner cannot assess the concept-membership, hence, since both possibilities are open, *an intermediate value is assigned to reflect such uncertainty*
- **Optimal discriminating feature set could be learned**

Concept Approximation by Tolerance

- Given a *tolerance threshold* $\theta \in [0, 1]$, the upper and lower approximation of a concept D can be defined as:
$$\overline{D} := \{a \mid \exists b : s_p^{\mathsf{C}}(a, b) \geq \theta \wedge b \in D\}$$
$$\underline{D} := \{a \mid \forall b : s_p^{\mathsf{C}}(a, b) \geq \theta \rightarrow b \in D\}$$
- The equivalence classes $[a]_{\mathsf{C}}$ for an individual a consist of the individuals within a certain degree of similarity θ w.r.t. a: $[a]_{\mathsf{C}} = \nu_\theta(a)$ where $\nu_\theta(a) = \{b \in \Delta^{\mathcal{I}} \mid s_p^{\mathsf{C}}(a, b) \geq \theta\}$ *the set* $\nu_\theta(a)$ *is called the neighborhood of* a
- **since these approximations depend on the threshold** $\Rightarrow$ *we have a numerical way to control the degree of indiscernibility*

Conclusions & Future Works

Conclusions

- A notion of context have been introduced
- Two Indescernability functions have been proposed
  - Context-based indescernibility function
  - Similarity-based indescernibility function
- They are used for modelling uncertain concept via Rough Description Logics

Future Works

- Implement Indescernibility functions
- Exploit RDLs for matchmaking problems

The End

That's all!

Questions?