Finite Fuzzy Description Logics: A Crisp Representation for Finite Fuzzy $\mathcal{ALCH}$

Fernando Bobillo

fbobillo@unizar.es

Department of Computer Science and Systems Engineering
University of Zaragoza, Spain

Joint research with U. Straccia (ISTI-CNR, Pisa, Italy)

URSW 2010
Shanghai (China)
November 2010

Universidad Zaragoza

Outline

Universidad Zaragoza

Outline

Introduction

- Classical ontology languages are not appropriate to deal with vagueness or imprecision in the knowledge.
  - Solution: **Fuzzy Description Logics** (DLs).
- Different **fuzzy logics** (families of fuzzy operators), e.g., Gödel.
- They lead to fuzzy DLs with **different properties**.
  - Clearly, different applications may need different fuzzy logics.
- In fuzzy DLs, some fuzzy operators have undesired properties.
  - In Zadeh fuzzy logic concepts and roles do not fully subsume themselves.
  - Łukasiewicz logic may not be suitable for combining information as the conjunction easily collapses to zero.
- The **study of new fuzzy operators** is an interesting topic.

Introduction

- Assuming a **finite set of degrees of truth** is useful in fuzzy DLs.
  - Instead of dealing with degrees of truth in $[0, 1]$, we will assume a finite (totally ordered) set of linguistic terms or labels.
  - In the Zadeh case it is interesting for computational reasons.
  - In Gödel logic, it is necessary to show that the logic verifies the Witnessed Model Property.
  - In Łukasiewicz logic, it is necessary to obtain a crisp representation.
- A question that immediately arise is **whether this assumption is possible when different fuzzy logics are considered**.
- We take the previous research on finite fuzzy logics and **generalize the previous research on finite fuzzy DLs**.
- **Benefits of this approach**:
  - Since experts' knowledge is usually expressed using a set of linguistic terms, the process of knowledge acquisition is easier.
  - New fuzzy operators in the setting of fuzzy DLs.

Universidad Zaragoza

Outline

Universidad Zaragoza

Finite chain of degrees of truth

- A **finite chain** of degrees of truth is a totally ordered set

$$\mathcal{N} = \{0 = \gamma_0 < \gamma_1 < \cdots < \gamma_p = 1\}$$

  where $p \geq 1$.

- Example: $\{\texttt{false}, \texttt{closeToFalse}, \texttt{neutral}, \texttt{closeToTrue}, \texttt{true}\}$.
- $\mathcal{N}$ can be understood as a set of **linguistic terms or labels**.
- For our purposes all finite chains with the same number of elements are equivalent.
  - This makes possible to **abstract from the numerical interpretations** of these labels.
- In practice, it is usually sufficient to use a small p.

Finite fuzzy logics

- The intersection, union, complement and implication are performed by a **t-norm** function $\otimes$, a **t-conorm** function $\oplus$, a **negation** function $\ominus$, and an **implication** function $\Rightarrow$, respectively.
- T-norms, t-conorms, negations and implications can be restricted to finite chains. Popular **examples**:

Family	$\gamma_i \otimes \gamma_j$	$\gamma_i \oplus \gamma_j$	$\ominus \gamma_i$	$\gamma_i \Rightarrow \gamma_j$
Zadeh	$\min\{\gamma_i, \gamma_j\}$	$\max\{\gamma_i, \gamma_j\}$	γ_{p-i}	$\max\{\gamma_{p-i}, \gamma_j\}$
Gödel	$\min\{\gamma_i, \gamma_j\}$	$\max\{\gamma_i, \gamma_j\}$	$\begin{cases} \gamma_p, & \gamma_i = 0 \\ \gamma_0, & \gamma_i > 0 \end{cases}$	$\begin{cases} \gamma_p, & \gamma_i \leq \gamma_j \\ \gamma_j, & \gamma_i > \gamma_j \end{cases}$
Łukasiewicz	$\gamma_{\max\{i+j-p,0\}}$	$\gamma_{\min\{i+j,p\}}$	γ_{p-i}	$\gamma_{\min\{p-i+j,p\}}$

Universidad Zaragoza

Finite smooth t-norms

- **Smoothness** is a discrete counterpart of continuity in $[0, 1]$.
  - A function $f : \mathcal{N} \to \mathcal{N}$ is *smooth* iff it satisfies for all $i \in \mathcal{N}^+$ $f(\gamma_i) = \gamma_j$ implies that $f(\gamma_{i-1}) = \gamma_k$ with $j - 1 \leq k \leq j + 1$.
- A **t-norm** on $\mathcal{N}$ is a function $\otimes : \mathcal{N}^2 \to \mathcal{N}$ satisfying commutativity, associativity, monotonicity, and some boundary conditions.

Proposition

*There is one and only one Archimedean smooth t-norm on $\mathcal{N}$ given by $\gamma_i \otimes \gamma_j = \gamma_{\max\{0, i+j-p\}}$. Moreover, given any subset J of $\mathcal{N}$ containing γ_0, γ_p, there is one and only one **smooth t-norm** $\otimes^J$ on $\mathcal{N}$ that has J as the set of idempotent elements. In fact, if J is the set $J = \{0 = \gamma_{i_0} < \gamma_{i_1} < \cdots < \gamma_{i_{m-1}} < \gamma_{i_m} = 1\}$ such a t-norm is given by:*

$$\gamma_i \otimes^J \gamma_j = \begin{cases} \gamma_{\max\{i_k, i+j-i_{k+1}\}} & \textit{if } \gamma_i, \gamma_j \in [i_k, i_{k+1}] \textit{ for some } 0 \leq k \leq m-1 \\ \gamma_{\min\{i,j\}} & \textit{otherwise} \end{cases}$$

Finite smooth t-norms

- The **Archimedean smooth t-norm** happens with $J = \{\gamma_0, \gamma_p\}$.
- The **minimum t-norm** happens with $J = \mathcal{N}$.

Example

- $\mathcal{N} = \{\gamma_0, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5\}$.
- $J = \{\gamma_0, \gamma_3, \gamma_5\}$.
- Then, $\otimes^J$ is defined as:

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
γ_0	γ_0	γ_0	γ_0	γ_0	γ_0	γ_0
γ_1	γ_0	γ_0	γ_0	γ_1	γ_1	γ_1
γ_2	γ_0	γ_0	γ_1	γ_2	γ_2	γ_2
γ_3	γ_0	γ_1	γ_2	γ_3	γ_3	γ_3
γ_4	γ_0	γ_1	γ_2	γ_3	γ_3	γ_4
γ_5	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5

Finite strong negations and smooth t-conorms

- A **strong negation** verifies $\ominus(\ominus\gamma) = \gamma, \forall \gamma \in \mathcal{N}$.
- There is only one strong negation on $\mathcal{N}$: $\ominus\gamma_i = \gamma_{p-i}$.
- Given a smooth t-norm $\otimes$ and the strong negation $\ominus$, we can define the **dual t-conorm** $\oplus_{\otimes}$ as: $\gamma_i \oplus_{\otimes} \gamma_j = \ominus((\ominus\gamma_i) \otimes (\ominus\gamma_j))$.

Proposition

There is one and only one Archimedean smooth t-conorm on $\mathcal{N}$ given by $\gamma_i \oplus \gamma_j = \gamma_{\min\{p,i+j\}}$. Moreover, given any subset J of $\mathcal{N}$ containing γ_0, γ_p, there is one and only one ***smooth t-conorm*** *$\oplus^J$ on $\mathcal{N}$ that has J as the set of idempotent elements. In fact, if J is the set $J = \{0 = \gamma_{i_0} < \gamma_{i_1} < \cdots < \gamma_{i_{m-1}} < \gamma_{i_m} = 1\}$ such a t-conorm is given by:*

$$\gamma_i \oplus^J \gamma_j = \begin{cases} \gamma_{\min\{i_{k+1}, i+j-i_k\}} & \textit{if } \gamma_i, \gamma_j \in [i_k, i_{k+1}] \textit{ for some } 0 \leq k \leq m-1 \\ \gamma_{\max\{i,j\}} & \textit{otherwise} \end{cases}$$

Finite S-implications

- A binary operator $\Rightarrow: \mathcal{N}^2 \rightarrow \mathcal{N}$ is said to be an **implication**, if it is non-increasing in the first place, non-decreasing in the second place, and satisfies some boundary conditions.
- Given a smooth t-norm $\otimes$ and the strong negation $\ominus$, an **S-implication** satisfies: $\gamma_i \Rightarrow_{s\otimes} \gamma_j = \ominus(\gamma_i \otimes (\ominus\gamma_j)) = (\ominus\gamma_i) \oplus \gamma_j$.

Proposition

*Let $\otimes^J : \mathcal{N}^2 \rightarrow \mathcal{N}$ be a smooth t-norm with $J = \{0 = \gamma_{i_0} < \gamma_{i_1} < \dots < \gamma_{i_{m-1}} < \gamma_{i_m} = 1\}$. Then, the **S-implication** $\Rightarrow_{s\otimes}$ is given by:*

$$\gamma_i \Rightarrow_{s\otimes} \gamma_j = \begin{cases} \gamma_{\min\{p-i_k, i_{k+1}+j-i\}} & \textit{if } \exists \gamma_{i_k} \in J \textit{ s.t. } \gamma_{i_k} \leq \gamma_i, \gamma_{p-j} \leq \gamma_{i_{k+1}} \\ \gamma_{\max\{p-i,j\}} & \textit{otherwise} \end{cases}$$

- Kleene-Dienes implication: with the minimum t-norm.
- Łukasiewicz implication: with the Archimedean smooth t-norm.

Universidad Zaragoza

Finite R-implications

- Given a smooth t-norm $\otimes$, an **R-implication** $\Rightarrow_{r\otimes}$ is defined as:

$$\gamma_i \Rightarrow_{r\otimes} \gamma_j = \max\{\gamma_k \in \mathcal{N} | (\gamma_i \otimes \gamma_k) \leq \gamma_j\}, \forall \gamma_i, \gamma_j \in \mathcal{N}$$

Proposition

Let $\otimes^J : \mathcal{N}^2 \rightarrow \mathcal{N}$ be a smooth t-norm with $J = \{0 = \gamma_{i_0} < \gamma_{i_1} < \ldots < \gamma_{i_{m-1}} < \gamma_{i_m} = 1\}$. Then, the implication $\Rightarrow_{r\otimes}$ is given by:

$$\gamma_i \Rightarrow_{r\otimes} \gamma_j = \begin{cases} \gamma_p & \textit{if } \gamma_i \leq \gamma_j \\ \gamma_{i_{k+1}+j-i} & \textit{if } \exists \gamma_{i_k} \in J \textit{ such that } \gamma_{i_k} \leq \gamma_j < \gamma_i \leq \gamma_{i_{k+1}} \\ \gamma_j & \textit{otherwise} \end{cases}$$

- Gödel implication: with the minimum t-norm.
- Łukasiewicz implication: with the Archimedean smooth t-norm.

Universidad Zaragoza

Finite R-implications

Example

Given the previous smooth t-norm, $\Rightarrow_{r\otimes}$ is defined as:

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
γ_0	γ_5	γ_5	γ_5	γ_5	γ_5	γ_5
γ_1	γ_2	γ_5	γ_5	γ_5	γ_5	γ_5
γ_2	γ_1	γ_2	γ_5	γ_5	γ_5	γ_5
γ_3	γ_0	γ_1	γ_2	γ_5	γ_5	γ_5
γ_4	γ_0	γ_1	γ_2	γ_4	γ_5	γ_5
γ_5	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5

- S-implications are smooth iff the t-norm is.
- R-implications may **not be smooth**.

Finite QL-implications and D-implications

- **QL-implication**: Implication verifying $\gamma_i \Rightarrow \gamma_j = (\ominus\gamma_i) \oplus (\gamma_i \otimes \gamma_j)$

Proposition

Let $\otimes^J : \mathcal{N} \times^J \mathcal{N} \to \mathcal{N}$ *be a smooth t-norm with* $J = \{0 = \gamma_{i_0} < \gamma_{i_1} < \ldots < \gamma_{i_{m-1}} < \gamma_{i_m} = 1\}$*. Then,* $\Rightarrow_{ql\otimes}$ *is given by:*

$$\gamma_i \Rightarrow_{ql\otimes} \gamma_j = \begin{cases} \gamma_{\max\{p-i+i_k, p+j-i_{k+1}\}} & \textit{if } \gamma_i, \gamma_j \in [i_k, i_{k+1}], 0 \leq k \leq m-1 \\ \gamma_{p-i+j} & \textit{if } \gamma_j \leq i_k \leq \gamma_i \textit{ for some } i_k \in J \\ \gamma_p & \textit{otherwise} \end{cases}$$

- The Łukasiewicz implication: with the minimum t-norm.
- Kleene-Dienes implication: with the Archimedean t-norm.
- **D-implication**: Implication satisfying $\gamma_i \Rightarrow \gamma_j = ((\ominus\gamma_i) \otimes (\ominus\gamma_j)) \oplus \gamma_j$, for all $\gamma_i, \gamma_j \in \mathcal{N}$.
- QL-implications and D-implications on $\mathcal{N}$ actually coincide.
  - $\Rightarrow_{ql\otimes^J} \equiv \Rightarrow_{d\otimes^{\bar{J}}}$, where $\bar{J} = \{\gamma_{p-x} | \gamma_x \in J\}$.

Universidad Zaragoza

Outline

Universidad Zaragoza

Main features of the logic

- **Concepts** denote fuzzy sets of individuals.
- **Roles** denote fuzzy binary relations.
- **Degrees of truth** are taking from a finite chain $\mathcal{N}$.
- **Axioms** have a degree of truth associated.
- The **fuzzy connectives** used are:
  - A smooth t-norm $\otimes$ on $\mathcal{N}$,
  - The strong negation $\ominus$ on $\mathcal{N}$,
  - The dual t-conorm $\oplus$,
  - The implications $\Rightarrow_{s\otimes}, \Rightarrow_{r\otimes}, \Rightarrow_{ql\otimes}$.

Universidad Zaragoza

Syntax and semantics

Element	Syntax	Semantics
Concepts	$\top$	γ_p
	$\bot$	γ_0
	A	$A^{\mathcal{I}}(x)$
	$C \sqcap D$	$C^{\mathcal{I}}(x) \otimes D^{\mathcal{I}}(x)$
	$C \sqcup D$	$C^{\mathcal{I}}(x) \oplus D^{\mathcal{I}}(x)$
	$\neg C$	$\ominus C^{\mathcal{I}}(x)$
	$\forall_s R.C$	$\inf_{y \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(x,y) \Rightarrow_s C^{\mathcal{I}}(y)\}$
	$\forall_r R.C$	$\inf_{y \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(x,y) \Rightarrow_r C^{\mathcal{I}}(y)\}$
	$\forall_{ql} R.C$	$\inf_{y \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(x,y) \Rightarrow_{ql} C^{\mathcal{I}}(y)\}$
	$\exists R.C$	$\sup_{y \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(x,y) \otimes C^{\mathcal{I}}(y)\}$
Roles	R	$R^{\mathcal{I}}(x,y)$
ABox axioms	$\langle a : C \bowtie \gamma \rangle$	$C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie \gamma$
	$\langle (a,b) : R \bowtie \gamma \rangle$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie \gamma$
TBox axioms	$\langle C \sqsubseteq_s D \rhd \gamma \rangle$	$\inf_{x \in \Delta^{\mathcal{I}}} \{C^{\mathcal{I}}(x) \Rightarrow_s D^{\mathcal{I}}(x)\} \rhd \gamma$
	$\langle C \sqsubseteq_r D \rhd \gamma \rangle$	$\inf_{x \in \Delta^{\mathcal{I}}} \{C^{\mathcal{I}}(x) \Rightarrow_r D^{\mathcal{I}}(x)\} \rhd \gamma$
	$\langle C \sqsubseteq_{ql} D \rhd \gamma \rangle$	$\inf_{x \in \Delta^{\mathcal{I}}} \{C^{\mathcal{I}}(x) \Rightarrow_{ql} D^{\mathcal{I}}(x)\} \rhd \gamma$
RBox axioms	$\langle R_1 \sqsubseteq_s R_2 \rhd \gamma \rangle$	$\inf_{x,y \in \Delta^{\mathcal{I}}} \{R_1^{\mathcal{I}}(x) \Rightarrow_s R_2^{\mathcal{I}}(x)\} \rhd \gamma$
	$\langle R_1 \sqsubseteq_r R_2 \rhd \gamma \rangle$	$\inf_{x,y \in \Delta^{\mathcal{I}}} \{R_1^{\mathcal{I}}(x) \Rightarrow_r R_2^{\mathcal{I}}(x)\} \rhd \gamma$
	$\langle R_1 \sqsubseteq_{ql} R_2 \rhd \gamma \rangle$	$\inf_{x,y \in \Delta^{\mathcal{I}}} \{R_1^{\mathcal{I}}(x) \Rightarrow_{ql} R_2^{\mathcal{I}}(x)\} \rhd \gamma$

Universidad Zaragoza

Reasoning tasks

- **Fuzzy KB satisfiability**. A fuzzy interpretation $\mathcal{I}$ *satisfies* (is a model of) a fuzzy KB $\mathcal{K} = \langle \mathcal{A}, \mathcal{T}, \mathcal{R} \rangle$ iff it satisfies each element in $\mathcal{A}$, $\mathcal{T}$ and $\mathcal{R}$.
- **Concept satisfiability**. C is α-satisfiable w.r.t. a fuzzy KB $\mathcal{K}$ iff $\mathcal{K} \cup \{\langle a : C \geq \alpha \rangle\}$ is satisfiable, where a is a new individual.
- **Entailment**.
  - $\mathcal{K} \models \langle a : C \bowtie \alpha \rangle$ iff $\mathcal{K} \cup \{\langle a : C \neg\bowtie \alpha \rangle\}$ is unsatisfiable.
  - $\mathcal{K} \models \langle (a, b) : R \geq \alpha \rangle$ iff $\mathcal{K} \cup \{\langle b : B \geq \gamma_p \rangle\} \models \langle a : \exists R.B \geq \alpha \rangle$, where B is a new concept.
- **Greatest lower bound**. The greatest lower bound of a concept or role assertion τ is defined as the $\sup\{\alpha : \mathcal{K} \models \langle \tau \geq \alpha \rangle\}$.
  - It can be computed performing at most $\log |\mathcal{N}|$ entailment tests.
- **Concept subsumption**. D subsumes C with degree α w.r.t. a fuzzy KB $\mathcal{K}$ iff $\mathcal{K} \models \langle C \sqsubseteq_X D \bowtie \alpha \rangle$, where $X \in \{s, r, ql\}$.
  - Subsumption can be reduced to fuzzy KB satisfiability.

Universidad Zaragoza

Logical properties

Proposition

Finite fuzzy $\mathcal{ALCH}$ interpretations coincide with crisp interpretations if we restrict the degrees to $\{\gamma_0 = 0, \gamma_p = 1\}$.

1. Concept simplification
2. Involutive negation
3. Excluded middle and contradiction
4. Idempotence of conjunction/disjunction
5. De Morgan laws
6. Inter-definability of concepts
7. Inter-definability of axioms
8. Contrapositive symmetry
9. Modus ponens
10. Self-subsumption

Universidad Zaragoza

Outline

Universidad Zaragoza

Idea of the reduction

- The reduction is **similar to the case of Łukasiewicz** fuzzy DLs.
- Use crisp concepts and roles (**α-cuts** of the fuzzy ones).
- To keep the semantics of the α-cuts, **some axioms must be introduced** (GCIs and role hierarchies).
- Every axiom of the fuzzy ontology is represented, independently from other axioms, using these basic crisp elements.
- We need to build **disjunctions or conjunctions over all possible degrees** of truth.

Universidad Zaragoza

Reduction of the t-norm

Example

- Let us compute the α-cut $\rho(A_1 \sqcap A_2, \geq \alpha)$, if $J = \{\gamma_0, \gamma_3, \gamma_5\}$ and $\otimes$:

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
γ_0	γ_0	γ_0	γ_0	γ_0	γ_0	γ_0
γ_1	γ_0	γ_0	γ_0	γ_1	γ_1	γ_1
γ_2	γ_0	γ_0	γ_1	γ_2	γ_2	γ_2
γ_3	γ_0	γ_1	γ_2	γ_3	γ_3	γ_3
γ_4	γ_0	γ_1	γ_2	γ_3	γ_3	γ_4
γ_5	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5

- If $\alpha = \gamma_2$, there are two possibilities:
  - $A_1^{\mathcal{I}}(x) \geq \gamma_2$ and $A_2^{\mathcal{I}}(x) \geq \gamma_3$, or
  - $A_1^{\mathcal{I}}(x) \geq \gamma_3$ and $A_2^{\mathcal{I}}(x) \geq \gamma_2$.
- Hence, $\rho(A_1 \sqcap A_2, \geq \gamma_2) = \Big(\rho(A_1, \geq \gamma_2) \sqcap \rho(A_2, \geq \gamma_3)\Big) \sqcup \Big(\rho(A_1, \geq \gamma_3) \sqcap \rho(A_2, \geq \gamma_2)\Big)$.
- If $\alpha \notin J$: similar as in finite Łukasiewicz.

Reduction of the t-norm

Example

- Let us compute the α-cut $\rho(A_1 \sqcap A_2, \geq \alpha)$, if $J = \{\gamma_0, \gamma_3, \gamma_5\}$ and $\otimes$:

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
γ_0	γ_0	γ_0	γ_0	γ_0	γ_0	γ_0
γ_1	γ_0	γ_0	γ_0	γ_1	γ_1	γ_1
γ_2	γ_0	γ_0	γ_1	γ_2	γ_2	γ_2
γ_3	γ_0	γ_1	γ_2	γ_3	γ_3	γ_3
γ_4	γ_0	γ_1	γ_2	γ_3	γ_3	γ_4
γ_5	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5

- If $\alpha = \gamma_3$, there is only one possibility:
  - $A_1^{\mathcal{I}}(a^{\mathcal{I}}) \geq \gamma_3$ and $A_2^{\mathcal{I}}(a^{\mathcal{I}}) \geq \gamma_3$.
- Hence, $\rho(A_1 \sqcap A_2, \geq \gamma_3) = \rho(A_1, \geq \gamma_3) \sqcap \rho(A_2, \geq \gamma_3)$.
- If $\alpha \in J$: as in Zadeh and Gödel.

Universidad Zaragoza

Reduction of R-implication

- R-implications are, in general, non smooth. Hence, we consider **optimal pairs** of elements γ_x, γ_y such that $\gamma_x \Rightarrow_r \gamma_y \geq \alpha$.

Definition

Let $\gamma_x, \gamma_y \in \mathcal{N}^+$. (γ_x, γ_y) is a $(\Rightarrow_{\geq \alpha})$**-optimal** pair iff:

- $\gamma_x \Rightarrow \gamma_y \geq \alpha$.
- There are no $\gamma'_x, \gamma'_y \in \mathcal{N}^+$ such that $\gamma'_x \Rightarrow \gamma'_y \geq \alpha$, and such that either $\gamma'_x < \gamma_x$ or $\gamma'_y < \gamma_y$.

Example

The $(\Rightarrow_{\geq \gamma_3})$-optimal pairs are (γ_3, γ_3), (γ_2, γ_2), and (γ_1, γ_1) for $\Rightarrow$:

	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5
γ_0	γ_5	γ_5	γ_5	γ_5	γ_5	γ_5
γ_1	γ_2	γ_5	γ_5	γ_5	γ_5	γ_5
γ_2	γ_1	γ_2	γ_5	γ_5	γ_5	γ_5
γ_3	γ_0	γ_1	γ_2	γ_5	γ_5	γ_5
γ_4	γ_0	γ_1	γ_2	γ_4	γ_5	γ_5
γ_5	γ_0	γ_1	γ_2	γ_3	γ_4	γ_5

Properties of the reduction

- **Correctness**

Theorem

- *The satisfiability problem in finite fuzzy $\mathcal{ALCH}$ is decidable.*
- *A finite fuzzy $\mathcal{ALCH}$ fuzzy KB $\mathcal{K}$ is satisfiable iff crisp($\mathcal{K}$) is.*

- **Complexity**
  - The size of crisp($\mathcal{K}$) is $\mathcal{O}(|\mathcal{K}| \cdot |\mathcal{N}|^k)$.
    - k is the maximal depth of the concepts appearing in $\mathcal{K}$.
  - In the particular case of Zadeh, the size of crisp($\mathcal{K}$) is $\mathcal{O}(|\mathcal{K}| \cdot |\mathcal{N}|)$.
  - For other fuzzy operators the case is more complex, as we need to build disjunctions or conjunctions over all possible degrees of truth.
- **Modularity**
  - The reduction of an ontology can be reused when adding new axioms if they do not introduce new atomic concepts/roles.
  - It is enough to add the reduction of the new axioms.

Outline

Universidad Zaragoza

Conclusions and future work

- We provide a general framework for **fuzzy DLs with a finite chain** of degrees of truth $\mathcal{N}$.
  - Very useful in practice, since expert knowledge is usually expressed using linguistic terms and avoiding numerical interpretations.
- Starting from a smooth finite t-norm on $\mathcal{N}$, we define the syntax and semantics of **finite fuzzy $\mathcal{ALCH}$** with 3 different implications.
  - We study some logical properties to help in the choice.
- The **decidability** of the logic has been shown by presenting a reasoning preserving **reduction to the crisp case**.
  - The **complexity** of the crisp representation is higher than in finite Zadeh fuzzy DLs, because it is necessary to build disjunctions or conjunctions over all possible degrees of truth.
- **Future work**: More expressive fuzzy DLs, towards OWL 2.

Universidad Zaragoza

Thank you very much for your attention