Tractability of the Crisp Representations of Tractable Fuzzy Description Logics

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Introduction

- Classical ontology languages are not appropriate to deal with vagueness or imprecision in the knowledge.
  - Solution: **Fuzzy Description Logics** (DLs).

- An important line of research is the computation of an **equivalent crisp representation** of a fuzzy ontology.

- This way, it is possible to reason with the obtained crisp ontology, making it possible to reuse classical ontology languages, DL reasoners, and other resources.

- It is possible to reason with very expressive fuzzy DLs, and with **different fuzzy logics**:
  - Zadeh
  - Gödel
  - Łukasiewicz

- **Our goal** is to study some property (tractability) of the crisp representations of fuzzy ontologies.
Tractable DLs

Expressive power compromised for the efficiency of reasoning.

The standard language OWL 2 has 3 fragments (profiles):

- OWL 2 EL
- OWL 2 QL
- OWL 2 RL

Complexity:

- OWL 2 EL, OWL 2 RL: polynomial time w.r.t. the ontology size.
- OWL 2 QL: LOGSPACE w.r.t. the size of the ABox.
### Relation of some OWL 2 constructors and its profiles:

<table>
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<th>OWL 2</th>
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<th>OWL 2 QL</th>
<th>OWL 2 RL</th>
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</table>
Motivation

Definition

A fuzzy DL language $\mathcal{X}$ is **closed under reduction** iff the crisp representation of a fuzzy ontology in $\mathcal{X}$ is in the (crisp) DL language $\mathcal{X}$.

- Sometimes, fuzzy DL languages are closed under reduction.
- The objective of this paper is to determine in a precise way **when this property holds**, focusing on **tractable fuzzy DLs**.
The case of Zadeh fuzzy logic

- Zadeh logic makes it possible to obtain **smaller crisp representations** than with Gödel and Łukasiewicz logics.

**Example:**
- From \( \langle a : C \sqcap D \geq 0.6 \rangle \) we can deduce both \( \langle a : C \geq 0.6 \rangle \) and \( \langle a : D \geq 0.6 \rangle \).

**In Łukasiewicz logic**, this is not possible, and we have to build a disjunction over all the possibilities.
- From \( \langle a : C \sqcap D \geq 0.6 \rangle \), deduce \( \langle a : C \geq 1 \rangle \) and \( \langle a : D \geq 0.6 \rangle \),
  - or \( \langle a : C \geq 0.9 \rangle \) and \( \langle a : D \geq 0.7 \rangle \),
  - or \( \langle a : C \geq 0.8 \rangle \) and \( \langle a : D \geq 0.8 \rangle \),
  - or \( \langle a : C \geq 0.7 \rangle \) and \( \langle a : D \geq 0.9 \rangle \),
  - or \( \langle a : C \geq 0.6 \rangle \) and \( \langle a : D \geq 1 \rangle \).

**In Gödel implication**, we have a similar problem.
The case of Zadeh fuzzy logic

**Property**

In Zadeh fuzzy logic, a fuzzy DL language $\mathcal{X}$ is closed under reduction iff it includes GCIs and role hierarchies.

- This result applies to OWL 2 EL, OWL 2 QL, and OWL 2 RL.

**Example**

- Let us assume the language $\mathcal{ALC}$.
- Since $\mathcal{ALC}$ does not contain role hierarchies, the property fails.
- Hence, fuzzy $\mathcal{ALC}$ is not closed under reduction.
- This is intuitive, because the crisp representations contains role hierarchies ($R_{\geq\alpha} \sqsubseteq R_{\geq\beta}$), which are not part of $\mathcal{ALC}$.
The case of Gödel fuzzy logic

Property

In Gödel fuzzy logic, a fuzzy DL language $\mathcal{X}$ is closed under reduction iff it verifies each of the following conditions:

- $\mathcal{X}$ includes GCIs.
- $\mathcal{X}$ includes role hierarchies.
- If $\mathcal{X}$ includes universal restrictions, then it also include conjunction.

This result applies to OWL 2 EL, OWL 2 QL, and OWL 2 RL.
In Łukasiewicz fuzzy logic, a fuzzy DL language $\mathcal{X}$ is not closed under reduction if it verifies some of the following conditions:

- $\mathcal{X}$ does not include GCIs.
- $\mathcal{X}$ does not include role hierarchies.
- $\mathcal{X}$ includes one and only one of disjunction and conjunction.
- $\mathcal{X}$ includes existential restrictions, but not disjunction.
- $\mathcal{X}$ includes universal restrictions, but not conjunction.

This result applies to OWL 2 EL, OWL 2 QL, and OWL 2 RL.

- OWL 2 EL / OWL 2 QL support conjunction but not disjunction.
- OWL 2 RL allows intersection as a superclass expression, but it does not allow disjunction there.

We only have a partial result.

- We only know a crisp representation for $L_n$ ALCHOI.
Size of the crisp representations

- **Zadeh and Gödel OWL 2 QL:**
  - Crisp representations are in crisp OWL 2 QL.
  - A crisp ontology with an **ABox with the same size** of the fuzzy one.
  - The complexity of reasoning depends on the number of assertions.
  - TBox and RBox are larger than the original fuzzy ones.

- **Zadeh and Gödel OWL 2 EL / OWL 2 RL:**
  - Crisp representations are in crisp OWL 2 EL / RL.
  - **TBox and RBox are larger** than the original fuzzy ones.
  - Reasoning depends on the size of the ontology.

- **Gödel OWL 2 RL makes concept expressions larger** than Zadeh, because of universal restrictions.
  - Gödel OWL 2 EL / QL do not, since there are not universal restrictions.

- It is specially important to use **optimized crisp representations** (e.g., do not consider domain/range axioms as GCIs).
Comments?

Thank you very much for your attention