Tractability of the Crisp Representations of Tractable Fuzzy Description Logics

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Introduction

- Classical ontology languages are not appropriate to deal with vagueness or imprecision in the knowledge.
  - Solution: **Fuzzy Description Logics** (DLs).
- An important line of research is the computation of an **equivalent crisp representation** of a fuzzy ontology.
- This way, it is possible to reason with the obtained crisp ontology, making it possible to reuse classical ontology languages, DL reasoners, and other resources.
- It is possible to reason with very expressive fuzzy DLs, and with **different fuzzy logics**:
  - **Zadeh**
  - **Gödel**
  - **Łukasiewicz**
- **Our goal** is to study some property (tractability) of the crisp representations of fuzzy ontologies.

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Tractable DLs

- **Expressive power compromised** for the efficiency of reasoning.
- The standard language OWL 2 has 3 fragments (**profiles**):
  - **OWL 2 EL**
  - **OWL 2 QL**
  - **OWL 2 RL**
- **Complexity**:
  - **OWL 2 EL, OWL 2 RL**: polynomial time w.r.t. the ontology size.
  - **OWL 2 QL**: LOGSPACE w.r.t. the size of the ABox.

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Tractable DLs

- **Relation** of some **OWL 2** constructors and its **profiles**:

OWL 2	OWL 2 EL	OWL 2 QL	OWL 2 RL
Class	✓	✓	✓
ObjectIntersectionOf	✓	restricted	✓
ObjectUnionOf			restricted
ObjectComplementOf		restricted	restricted
ObjectAllValuesFrom			restricted
ObjectSomeValuesFrom	✓	restricted	restricted
DataAllValuesFrom			restricted
DataSomeValuesFrom	✓	✓	restricted
...			
ObjectProperty	✓	✓	✓
DatatypeProperty	✓	✓	✓
...			
ClassAssertion	✓	✓	✓
ObjectPropertyAssertion	✓	✓	✓
SubClassOf	✓	✓	✓
SubObjectPropertyOf	✓	✓	✓
SubDataPropertyOf	✓	✓	✓
...			

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Definition

A fuzzy DL language $\mathcal{X}$ is **closed under reduction** iff the crisp representation of a fuzzy ontology in $\mathcal{X}$ is in the (crisp) DL language $\mathcal{X}$.

- Sometimes, fuzzy DL languages are closed under reduction.
- The objective of this paper is to determine in a precise way **when this property holds**, focusing on **tractable fuzzy DLs**.

The case of Zadeh fuzzy logic

- Zadeh logic makes it possible to obtain **smaller crisp representations** than with Gödel and Łukasiewicz logics.
- Example:
  - From $\langle a : C \sqcap D \geq 0.6 \rangle$ we can deduce both $\langle a : C \geq 0.6 \rangle$ and $\langle a : D \geq 0.6 \rangle$.
- **In Łukasiewicz logic**, this is not possible, and we have to build a disjunction over all the possibilities.
  - From $\langle a : C \sqcap D \geq 0.6 \rangle$, deduce $\langle a : C \geq 1 \rangle$ and $\langle a : D \geq 0.6 \rangle$,
    or $\langle a : C \geq 0.9 \rangle$ and $\langle a : D \geq 0.7 \rangle$,
    or $\langle a : C \geq 0.8 \rangle$ and $\langle a : D \geq 0.8 \rangle$,
    or $\langle a : C \geq 0.7 \rangle$ and $\langle a : D \geq 0.9 \rangle$,
    or $\langle a : C \geq 0.6 \rangle$ and $\langle a : D \geq 1 \rangle$.
- **In Gödel implication**, we have a similar problem.

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The case of Zadeh fuzzy logic

Property

In Zadeh fuzzy logic, a fuzzy DL language $\mathcal{X}$ is closed under reduction iff it includes GCIs and role hierarchies.

- This result applies to **OWL 2 EL, OWL 2 QL, and OWL 2 RL**.

Example

- Let us assume the language $\mathcal{ALC}$.
- Since $\mathcal{ALC}$ does not contain role hierarchies, the property fails.
- Hence, fuzzy $\mathcal{ALC}$ is not closed under reduction.
- This is intuitive, because the crisp representations contains role hierarchies ($R_{\geq \alpha} \sqsubseteq R_{\geq \beta}$), which are not part of $\mathcal{ALC}$.

The case of Gödel fuzzy logic

Property

In Gödel fuzzy logic, a fuzzy DL language $\mathcal{X}$ is closed under reduction iff it verifies each of the following conditions:

- *$\mathcal{X}$ includes GCIs.*
- *$\mathcal{X}$ includes role hierarchies.*
- *If $\mathcal{X}$ includes universal restrictions, then it also include conjunction.*

- This result applies to **OWL 2 EL, OWL 2 QL, and OWL 2 RL**.

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The case of Łukasiewicz fuzzy logic

Property

In Łukasiewicz fuzzy logic, a fuzzy DL language $\mathcal{X}$ is **not** *closed under reduction if it verifies some of the following conditions:*

- *$\mathcal{X}$ does not include GCIs.*
- *$\mathcal{X}$ does not include role hierarchies.*
- *$\mathcal{X}$ includes one and only one of disjunction and conjunction.*
- *$\mathcal{X}$ includes existential restrictions, but not disjunction.*
- *$\mathcal{X}$ includes universal restrictions, but not conjunction.*

- This result applies to **OWL 2 EL, OWL 2 QL, and OWL 2 RL**.
  - OWL 2 EL / OWL 2 QL support conjunction but not disjunction.
  - OWL 2 RL allows intersection as a superclass expression, but it does not allow disjunction there.
- We only have **a partial result**.
  - We only know a crisp representation for $Ł_n$ $\mathcal{ALCHOI}$.

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Size of the crisp representations

- **Zadeh and Gödel OWL 2 QL**:
  - Crisp representations are in crisp OWL 2 QL.
  - A crisp ontology with an **ABox with the same size** of the fuzzy one.
    - The complexity of reasoning depends on the number of assertions.
  - TBox and RBox are larger than the original fuzzy ones.
- **Zadeh and Gödel OWL 2 EL / OWL 2 RL**:
  - Crisp representations are in crisp OWL 2 EL / RL.
  - **TBox and RBox are larger** than the original fuzzy ones.
    - Reasoning depends on the size of the ontology.
- **Gödel OWL 2 RL makes concept expressions larger** than Zadeh, because of universal restrictions.
  - Gödel OWL 2 EL / QL do not, since there are not universal restrictions.
- It is specially important to use **optimized crisp representations** (e.g., do not consider domain/range axioms as GCIs).

Thank you very much for your attention

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