Claudia d’Amato  Volha Bryl,  Luciano Serafini

November 11, 2012

8th International Workshop on Uncertainty
Reasoning for the Semantic Web

11th ISWC, Boston (MA), USA.
Heterogeneous resources of knowledge about the same domain:

**Non or simply structured data** (e.g., sensor data, signals, DB tracks, texts, bag of words, etc) containing (alpha)numeric feature based data

**Structured knowledge** (e.g., ontologies = T-box + A-box) describing the entity types, the relations and the objects of a particular domain.
Heterogeneous Data

Heterogeneous resources of knowledge about the same domain:

**Non or simply structured data** (e.g., sensor data, signals, DB tracks, texts, bag of words, etc) containing (alpha)numeric feature based data

For instance a database of a company containing anagraphic data, salaries, evaluations, performances, and other relevant information about the employees.

**Structured knowledge** (e.g., ontologies = T-box + A-box) describing the entity types, the relations and the objects of a particular domain.

For instance the ontology for the organizational structure of a company describing, the roles, the activities, the responsibilities, etc. and the instantiation to the set of the company employees.
Different type of data support different type of inference

**Non or simply structured data** enables the discovering of new knowledge as regularity patterns of data via **inductive reasoning**.

**Structured knowledge** allows to discover new knowledge via inferencing of classes or properties of objects based on logical **deductive reasoning**.
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Non or simply structured data + Structured knowledge will enable the combination of knowledge induced from data and knowledge encoded in an ontology in a new form of mixed reasoning, that we call data drive inference.

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**Usually older employees earn more than their younger colleagues**

**If a person is a project leader than he or she coordinates the work of all the people allocated to the project**

**Since project coordinators receive always the highest salary within a project team, the oldest person of project team, is most probably the project coordinator**
Abstract objectives

1. To define a reference framework capable to represent quantitative data and logical knowledge in an integrated way
2. Extend machine learning algorithms to support the induction of new knowledge from quantitative data integrated with logical knowledge
3. Extend the logical reasoning algorithms to support logical inference in presence of knowledge induced from logical data integrated with logical knowledge
objectives

Concrete contributions (EKA 2012)

1. define the notion of ontology integrated with a dataset, called grounded ontologies
2. introduce semantically enriched association rules to represent knowledge induced from grounded ontologies
3. define a simple algorithm for learning semantically enriched association rules from grounded ontologies.
Concrete contributions (EKAW 2012)

1. define the notion of ontology integrated with a dataset, called grounded ontologies
2. introduce semantically enriched association rules to represent knowledge induced from grounded ontologies
3. define a simple algorithm for learning semantically enriched association rules from grounded ontologies.

Concrete contributions (URSW 2012)

4. define a new reasoning task, called the most plausible model that computes the most common model of an ontology w.r.t., a set of semantically enriched association rules
5. propose a first approximated tableaux algorithm to compute the most plausible model
Dataset \( D \) is a non-empty set of objects

- \( f_1, \ldots, f_n \) are \( n \) feature functions defined on every element of \( D \), with \( f_i : D \to D_i \).
Dataset $D$ is a non-empty set of objects.
- $f_1, \ldots, f_n$ are $n$ feature functions defined on every element of $D$, with $f_i : D \rightarrow D_i$.

Knowledge base $\mathcal{K}$ on an alphabet $\Sigma$, composed of three disjoint sets of symbols, $\Sigma_C, \Sigma_R$ and $\Sigma_I$, is a set $\mathcal{K}$ of DL inclusion axioms and DL assertions.
Grounded Ontology

- **Dataset** $\mathbf{D}$ is a non-empty set of objects
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- **A grounding** is a total function $g : \mathbf{D} \to \Sigma_I$.
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- for every $d \in D$ there is a constant $a \in \Sigma_I$ with $g(d) = a$.
- intuitively $g(d) = a$ represents the fact that data $d$ is an observation about the object $a$ of the knowledge base;
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- A **grounding** is a total function \( g : D \to \Sigma_I \)
  - for every \( d \in D \) there is a constant \( a \in \Sigma_I \) with \( g(d) = a \).
  - intuitively \( g(d) = a \) represents the fact that data \( d \) is an observation about the object \( a \) of the knowledge base;
  - there can be more than one observation for \( a \) in \( D \). This implies that \( g(d) = a \) and \( g(d') = a \) for \( d \neq d' \)
## Grounded Ontology - Example

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⊤ ⊑ Person ⊔ Prj

Person ⊑ ∃leads.Praj ⊔ ∃worksFor.Praj
Prj ⊑ (= 1)leads⁻ Prj ⊑ ∃worksFor⁻

dom(worksFor) ⊑ Person  dom(leads) ⊑ Person

leads(Alice, P)  worksFor(Bob, P)  worksFor(Chris, P)
leads(Bob, Q)  worksFor(Chris, Q)  worksFor(Dan, Q)
leads(Alice, R)  worksFor(Dan, R)

allDifferent(Alice, Bob, Chris, Dan)
⊤(Eva),  ∀leads⊥(Chris)  ∀leads⊥(Dan)
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\[
\top \sqsubseteq \text{Person} \sqcup \text{Prj} \\
\text{Person} \sqsubseteq \exists \text{leads} . \text{Prj} \sqcup \exists \text{worksFor} . \text{Prj} \\
\text{Prj} \sqsubseteq (\equiv 1) \text{leads}^{-} \quad \text{Prj} \sqsubseteq \exists \text{worksFor}^{-} \\
\text{dom(worksFor)} \sqsubseteq \text{Person} \quad \text{dom(leads)} \sqsubseteq \text{Person} \\
\text{leads}(\text{Alice}, P) \quad \text{worksFor}(\text{Bob}, P) \quad \text{worksFor}(\text{Chris}, P) \\
\text{leads}(\text{Bob}, Q) \quad \text{worksFor}(\text{Chris}, Q) \quad \text{worksFor}(\text{Dan}, Q) \\
\text{leads}(\text{Alice}, R) \quad \text{worksFor}(\text{Dan}, R) \\
\text{allDifferent}(\text{Alice}, \text{Bob}, \text{Chris}, \text{Dan}) \\
\top(\text{Eva}), \quad \forall \text{leads} \perp (\text{Chris}) \quad \forall \text{leads} \perp (\text{Dan})
\]

\[\begin{array}{l}
\text{g} \\
001 \quad \text{Alice} \\
002 \quad \text{Alice} \\
003 \quad \text{Alice} \\
004 \quad \text{Bob} \\
005 \quad \text{Bob} \\
006 \quad \text{Chris} \\
007 \quad \text{Dan} \\
008 \quad \text{Dan} \\
009 \quad \text{Eva}
\end{array}\]
Grounding allow to join data with knowledge

Let \((\mathcal{K}, \mathbf{D}, g)\) be a grounded ontology, the semantic enrichment of \(\mathbf{D}\) w.r.t., \(\mathcal{K}\), denoted by \(\mathbf{D}^{\mathcal{K}}\) is dataset containing the same data as \(\mathbf{D}\) with the following additional semantic attributes:
Let $(\mathcal{K}, \mathcal{D}, g)$ be a grounded ontology, the semantic enrichment of $\mathcal{D}$ w.r.t., $\mathcal{K}$, denoted by $\mathcal{D}^{+\mathcal{K}}$ is dataset containing the same data as $\mathcal{D}$ with the following additional semantic attributes:

**Conceptual attributes** for every primitive concept $C \in \Sigma_c$ the attribute $f_C$ is defined as follows:

$$f_C(d) = \begin{cases} 
1 & \text{if } \mathcal{K} \models C(g(d)) \\
0 & \text{if } \mathcal{K} \models \neg C(g(d)) \\
unknown & \text{otherwise}
\end{cases}$$
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\text{unknown} & \text{otherwise}
\end{cases}
\]

Relational attributes for every relation \(R \in \Sigma_r\) the attribute \(f_R\) is defined as follows:

\[
f_R(d) = \begin{cases} 
1 & \text{if } \mathcal{K} \models \exists R(g(d)) \\
0 & \text{if } \mathcal{K} \models \neg \exists R(g(d)) \\
\text{unknown} & \text{otherwise}
\end{cases}
\]
Join data with knowledge - example

<table>
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Association Rules

Definition

Given a dataset \( D \) made by a set of attributes \( \{A_1, \ldots, A_n\} \)

- an **itemset** of \( D \) is an expression

\[
f_{i_1} = v_1 \land \cdots \land f_{i_k} = v_k
\]  

(1)

- the **support** of an itemset is the number of tuples (rows) in \( D \) that match the itemset.

- **Association rule** is an expression

\[
f_{i_1} = v_1 \land \cdots \land f_{i_k} = v_k \Rightarrow f_{i_{k+1}} = v_{k+1} \land \cdots \land f_{i_n} = v_n
\]  

(2)

- The **confidence** of (2) is the fraction of cases in \( D \) that match the conclusion amongst the one that matches the premises.

\[
conf(\theta \Rightarrow \varphi) = \frac{support(\theta \land \varphi)}{support(\theta)}
\]
Semantically enriched association rules

Let \((\mathcal{K}, D, g)\) be a grounded ontology A **semantically enriched association rule** is an association rule defined on the semantically enriched dataset \(D^{+K}\).
Learning association rules

- Association rules are learned by
  - finding the frequent itemsets w.r.t. a given support threshold,
  - extracting the rules from the frequent itemsets satisfying a given confidence threshold.

- The first subproblem is the most challenging/expensive

- We use the standard Apriori algorithm
  - key assumption: a set of variables is frequent only if all its subsets are frequent,
  - itemsets are built iteratively, incrementing the length at each step.
### Learning SEAR: example

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### Examples of SEARs

\[f_{\text{leads}} = 1, f_{\text{year}} = 2011 \Rightarrow f_{\text{Salary}} = 30–40K\]
\[\text{conf} = 4/4 = 1.00\]
\[\Rightarrow f_{\text{Person}} = 1\]
\[\text{conf} = 8/9 = 0.88\]
\[\Rightarrow f_{\text{Project}} = 0\]
\[\text{conf} = 8/9 = 0.88\]

\[f_{\text{Salary}} = 30–40K \Rightarrow f_{\text{leads}} = 1\]
\[\text{conf} = 4/5 = 0.80\]

\[f_{\text{leads}} = 1 \Rightarrow f_{\text{Salary}} = 30–40K\]
\[\text{conf} = 4/5 = 0.80\]

\[f_{\text{Salary}} = 20–30K \Rightarrow f_{\text{leads}} = 0\]
\[\text{conf} = 3/4 = 0.75\]

\[f_{\text{year}} = 2012, f_{\text{Salary}} = 30–40K \Rightarrow f_{\text{leads}} = 1\]
\[\text{conf} = 2/3 = 0.66\]
Concrete contributions

1. defined the notion of **ontology grounded on a set of data**, also called grounded ontologies
2. introduced **semantically enriched association rules** to represent knowledge induced from grounded ontologies
3. defined a **simple algorithm for learning semantically enriched association rules** from grounded ontologies.
Concrete contributions

1. defined the notion of ontology grounded on a set of data, also called grounded ontologies
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Concrete contributions

4. defined a new reasoning task, called the most plausible model that computes the most common model of an ontology w.r.t., a set of semantically enriched association rules
5. proposed a first approximated tableaux algorithm to compute the most plausible model
The reasoning task

Definition (Inference Problem)

Given: \( D, K, \) the set \( R \) of SEARs, an data item \( d_0 \), a concept \( C_0 \), and the grounding \( g(d_0) = x_0 \),

Determine: the model \( \mathcal{I}_r \) for \( K \cup \{C_0(x_0)\} \) representing the most plausible model for \( K \cup \{C_0(x_0)\} \) w.r.t., \( g \) and \( R \).
Plausibility ordering

Definition (Plausibility ordering (first attempt))

Let \((\mathcal{K}, \{d\}, g)\) be a grounded ontology:

\[
\begin{align*}
\mathcal{I}, d, g \models f_i = a & \quad \text{if} \quad f_i(d) = a \\
\mathcal{I}, d, g \models f_C = 1 & \quad \text{if} \quad g(\mathcal{I}) \in C^\mathcal{I} \\
\mathcal{I}, d, g \models f_C = 0 & \quad \text{if} \quad g(\mathcal{I}) \notin C^\mathcal{I} \\
\mathcal{I}, d, g \models f_R = 1 & \quad \text{if} \quad g(\mathcal{I}) \in (\exists R \cdot \top)^\mathcal{I} \\
\mathcal{I}, d, g \models f_R = 0 & \quad \text{if} \quad g(\mathcal{I}) \notin (\exists R \cdot \top)^\mathcal{I} \\
\mathcal{I}, d, g \models \phi_1 \land \cdots \land \phi_n & \quad \text{if} \quad \mathcal{I}, d, g \models \phi_i \quad \text{for} \quad 1 \leq i \leq n \\
\mathcal{I}, d, g \models \phi \Rightarrow \psi & \quad \text{if} \quad \mathcal{I}, d, g \not\models \phi \text{ or } \mathcal{I}, d, g \models \psi
\end{align*}
\]
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Let \((\mathcal{K}, \{d\}, g)\) be a grounded ontology:

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\begin{align*}
\mathcal{I}, d, g & \models f_i = a \quad \text{if} \quad f_i(d) = a \\
\mathcal{I}, d, g & \models f_C = 1 \quad \text{if} \quad g(d)^\mathcal{I} \in C^\mathcal{I} \\
\mathcal{I}, d, g & \models f_C = 0 \quad \text{if} \quad g(d)^\mathcal{I} \notin C^\mathcal{I} \\
\mathcal{I}, d, g & \models f_R = 1 \quad \text{if} \quad g(d)^\mathcal{I} \in (\exists R. \top)^\mathcal{I} \\
\mathcal{I}, d, g & \models f_R = 0 \quad \text{if} \quad g(d)^\mathcal{I} \notin (\exists R. \top)^\mathcal{I} \\
\mathcal{I}, d, g & \models \phi_1 \land \cdots \land \phi_n \quad \text{if} \quad \mathcal{I}, d, g \models \phi_i \text{ for } 1 \leq i \leq n \\
\mathcal{I}, d, g & \models \phi \Rightarrow \psi \quad \text{if} \quad \mathcal{I}, d, g \not\models \phi \text{ or } \mathcal{I}, d, g \models \psi
\end{align*}
\]

\[
p(\mathcal{I}, d, g, \alpha) = \begin{cases} 
0 & \text{If } \mathcal{I}, d, g \models \alpha \\
2 \times \text{conf}(\alpha) - 1 & \text{Otherwise}
\end{cases}
\]
Definition (Plausibility ordering (first attempt))

Let \((\mathcal{K}, \{d\}, g)\) be a grounded ontology:

- \(\mathcal{I}, d, g \models f_i = a\) if \(f_i(d) = a\)
- \(\mathcal{I}, d, g \models f_C = 1\) if \(g(d)^\mathcal{I} \in C^\mathcal{I}\)
- \(\mathcal{I}, d, g \models f_C = 0\) if \(g(d)^\mathcal{I} \notin C^\mathcal{I}\)
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- \(\mathcal{I}, d, g \models \phi_1 \land \cdots \land \phi_n\) if \(\mathcal{I}, d, g \models \phi_i\) for \(1 \leq i \leq n\)
- \(\mathcal{I}, d, g \models \phi \Rightarrow \psi\) if \(\mathcal{I}, d, g \not\models \phi\) or \(\mathcal{I}, d, g \models \psi\)

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\end{cases}
\]

\(\mathcal{I} \preceq \mathcal{J}\) iff
\[
\sum_{\alpha \in AR} p(\mathcal{I}, d, g, \alpha) \geq \sum_{\alpha \in AR} p(\mathcal{J}, d, g, \alpha)
\]
Tableaux

Let $C_0$ be an $\mathcal{ALC}$-concept in NNF. In order to test satisfiability of $C_0$, the ALC tableaux algorithm starts with $\mathcal{A}_0 := \{C_0(x_0)\}$, and applies the following rules:

<table>
<thead>
<tr>
<th>Rule</th>
<th>Condition</th>
<th>$\rightarrow$ Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rightarrow \sqcap$</td>
<td>$C \sqcap D(x) \in \mathcal{A}$</td>
<td>$\mathcal{A} := \mathcal{A} \cup {C(x), D(x)}$</td>
</tr>
<tr>
<td>$\rightarrow \sqcup$</td>
<td>$C \sqcup D(x) \in \mathcal{A}$</td>
<td>$\mathcal{A} := \mathcal{A} \cup {C(x)}$ or $\mathcal{A} \cup {D(x)}$</td>
</tr>
<tr>
<td>$\rightarrow \exists$</td>
<td>$\exists R. C(x) \in \mathcal{A}$</td>
<td>$\mathcal{A} := \mathcal{A} \cup {R(x, y), C(y)}$</td>
</tr>
<tr>
<td>$\rightarrow \forall$</td>
<td>$\forall R. C(x), R(x, y) \in \mathcal{A}$</td>
<td>$\mathcal{A} := \mathcal{A} \cup {C(y)}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>$\mathcal{A} := \mathcal{A} \cup {C(y)}$</td>
</tr>
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</table>

Indeterministic Choices

In applying $\rightarrow \sqcup$ rule we have to do a choice, either we expand $\mathcal{A}$ with $C(x)$ or with $D(x)$. Depending on the choice we can generate a more or less plausible model.
**ALC Tableaux**

Let $C_0$ be an $\mathcal{ALC}$-concept in NNF. In order to test satisfiability of $C_0$, the ALC tableaux algorithm starts with $\mathcal{A}_0 := \{C_0(x_0)\}$, and applies the following rules:

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Indeterministic Choices

In applying $\rightarrow_{\square}$ rule we have to do a choice, either we expand $\mathcal{A}$ with $C(x)$ or with $D(x)$. Depending on the choice we can generate a more or less plausible model.
Example (Simple)

\[ C \sqsubseteq A \sqcup B \]

\[ f_{\text{Salary}}(d) = 27 \]

\[ f_{\text{Salary}} = 10\text{–}20 \Rightarrow A \quad \text{conf} = 0.77 \]

\[ f_{\text{Salary}} = 20\text{–}30 \Rightarrow B \quad \text{conf} = 0.66 \]

\[ g(d) = x_0 \]
**Example (Simple)**

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\[ g(d) = x_0 \]

There are three possible models:

<table>
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<tr>
<th>( \Delta^I )</th>
<th>( A^I )</th>
<th>( B^I )</th>
<th>( C^I )</th>
<th>( \sum_{\alpha \in AR} p(I, d, g, \alpha) )</th>
</tr>
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<tbody>
<tr>
<td>( I_1 )</td>
<td>{x_0}</td>
<td>{x_0}</td>
<td>\emptyset</td>
<td>{x_0}</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>{x_0}</td>
<td>\emptyset</td>
<td>{x_0}</td>
<td>{x_0}</td>
</tr>
<tr>
<td>( I_3 )</td>
<td>{x_0}</td>
<td>{x_0}</td>
<td>{x_0}</td>
<td>{x_0}</td>
</tr>
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</table>

The plausibility ordering is \( I_2 \prec I_1 \), \( I_3 \prec I_1 \).
A complete simple example

Example (The most plausible model for Eva)

<table>
<thead>
<tr>
<th>ID</th>
<th>E_ID</th>
<th>salary</th>
<th>year</th>
<th>Person</th>
<th>Prj</th>
<th>leads</th>
<th>worksFor</th>
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<td>E05</td>
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\[ T \sqsubseteq Person \sqcup Prj \]

\[ T(Eva) \]

\[ Person \sqcup Prj(Eva) \]
A complete simple example

Example (The most plausible model for Eva)

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\[
\top \sqsubseteq \text{Person} \sqcup \text{Prj} \\
\Rightarrow f_{\text{Person}} = 1[0.88] \\
\top(Eva) \\
\text{Person} \sqcup \text{Prj}(Eva) \\
\text{Person}(Eva)
\]
A complete simple example

Example (The most plausible model for Eva)

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\[ \top \subseteq \text{Person} \uplus \text{Prj} \]

\[ \Rightarrow f_{\text{Person}} = 1[0.88] \]

\[ \text{Person} \subseteq \exists \text{worksFor} \cdot \text{Prj} \uplus \exists \text{leads} \cdot \text{Prj} \]

\[ \top(\text{Eva}) \]

\[ \text{Person} \uplus \text{Prj}(\text{Eva}) \]

\[ \text{Person}(\text{Eva}) \]

\[ \exists \text{worksFor} \cdot \text{Prj} \uplus \exists \text{leads} \cdot \text{Prj} \]
A complete simple example

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\[
\top \subseteq \text{Person} \sqcup \text{Prj} \\
\Rightarrow f_{\text{Person}} = 1[0.88] \\
\text{Person} \subseteq \exists \text{worksFor.Prv} \sqcup \exists \text{leads.Prv} \\
f_{\text{Salary}}(\text{Eva}) = 30–40 \\
f_{\text{Salary}} = 30–40 \Rightarrow f_{\text{leads}} = 1 \\
\top(\text{Eva}) \\
\text{Person} \sqcup \text{Prj}(\text{Eva}) \\
\text{Person}(\text{Eva}) \\
\exists \text{worksFor.Prv} \sqcup \exists \text{leads.Prv} \\
\exists \text{leads.Prv}(\text{eva})
A complete simple example

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\[
\top(\text{Eva})
\]

\[
\text{Person} \sqcup \text{Prj}(\text{Eva})
\]

\[
\text{Person}(\text{Eva})
\]

\[
\exists \text{worksFor}. \text{Prj} \sqcup \exists \text{leads}. \text{Prj}
\]

\[
\exists \text{leads}. \text{Prj}(\text{eva})
\]

\[
\text{leads}(\text{eva}, p), \text{ Prj}(p)
\]
Conclusions & Future Work

**Conclusions:**
- Proposed a framework for learning association rules from hybrid sources of information
- Preliminary ideas on exploitation of the learnt association rules during the deductive reasoning process

**Future works:**
- Experimental evaluation of the proposed preliminary methodology
- Extension and consolidation of the theoretical framework to include also relations between objects.