Extending Fuzzy Description Logics with a Possibilistic Layer

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Motivation

- Classical ontologies are not appropriate to deal with imprecise, vague and uncertain knowledge.
  - Examples: Vague concepts, imprecision in the knowledge, uncertainty in integration or merging of ontologies ...
- Fuzzy and possibilistic logics are orthogonal.
  - **Fuzzy logic**: degrees of truth, e.g. the bottle is half full
  - **Possibilistic logic**: degrees of certainty, e.g. it is possible to degree 0.5 that the bottle is full.
- We propose to build a **possibilistic layer** (uncertain knowledge) on top of a **fuzzy ontology** (vague/imprecise knowledge).
  - Axioms are annotated with possibility and necessity degrees.
  - We reduce it to a possibilistic ontology, which makes possible to use classical reasoners.

Syntax

- A **possibilistic fuzzy knowledge base** is a fuzzy KB where each fuzzy axiom τ is equipped with:
  - A **possibility degree** $(\tau, \Pi\ \alpha)$.
    - α expresses to what extent τ is possible, or
  - A **necessity degree** $(\tau, N\ \alpha)$
    - α expresses to what extent τ is necessary true.
- If no degree is specified, *N* 1 es assumed.

Semantics

- Let $\mathfrak{I}$ be the set of all (fuzzy) interpretations.
- A **possibilistic interpretation** is a mapping $\pi : \mathfrak{I} \rightarrow [0, 1]$ such that $\pi(\mathcal{I}) = 1$ for some $\mathcal{I} \in \mathfrak{I}$.
  - $\pi(\mathcal{I})$ represents the degree to which the world $\mathcal{I}$ is possible.
  - $\mathcal{I}$ is impossible if $\pi(\mathcal{I}) = 0$.
  - $\mathcal{I}$ is fully possible if $\pi(\mathcal{I}) = 1$.
- The **possibility** of an axiom τ is defined as:

$$Poss(\tau) = \sup\{\pi(\mathcal{I}) \mid \mathcal{I} \in \mathfrak{I}, \mathcal{I} \models \tau\}$$

- The **necessity** is defined as:

$$Nec(\tau) = 1 - Poss(\neg\tau)$$

- π satisfies $(\tau, \Pi\gamma)$, denoted $\pi \models (\tau, \Pi\gamma)$, $\pi \models (\tau, \Pi\gamma)$ iff:

$$Poss(\tau) \geq \gamma$$

- π satisfies $(\tau, N\,\gamma)$, denoted $\pi \models (\tau, N\,\gamma)$, iff:

$$Nec(\tau) \geq \gamma$$

Reasoning

- B. Hollunder showed that reasoning within a possibilistic DL can be reduced to reasoning within a classical DL.
- We reduce our **possibilistic fuzzy DL to a possibilistic DL**.
  - A fuzzy KB *fK* can be reduced to a crisp KB $\mathcal{K}(fK)$ and every axiom $\tau \in fK$ is reduced to $\mathcal{K}(\tau)$.
  - **Adding degrees of certainty to *fK* formulae is equivalent to adding degrees of certainty to their reductions**.
  - For every axiom $(\tau, \Pi\gamma) \in pfK$:

$$Poss(\tau) \geq \gamma \text{ iff } Poss(\mathcal{K}(\tau)) \geq \gamma$$

  - For every axiom $(\tau, \Pi\gamma) \in pfK$:

$$Nec(\tau) \geq \gamma \text{ iff } Nec(\mathcal{K}(\tau)) \geq \gamma$$

  - We also need to consider the additional axioms which are created during the reduction of the fuzzy ontology.

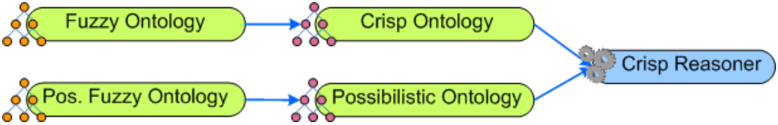
Example

- The axiom $(\langle tom : High \geq 0.5\rangle, N\ 0.2)$ means that it is possible with degree 0.2 that *tom* can be considered a *High* person with (at least) degree 0.5.
- It is reduced into $(\langle tom : High_{\geq 0.5}\rangle, N\ 0.2)$, meaning that it is possible with degree 0.2 that *tom* belongs to the crisp set $High_{\geq 0.5}$.
- The final crisp KB would also need some **additional axioms** (consequence of the reduction of the fuzzy KB):

$$High_{\geq 0.5} \sqsubseteq High_{>0}, High_{>0.5} \sqsubseteq High_{\geq 0.5}, High_{\geq 1} \sqsubseteq High_{>0.5}$$

Final remarks

- F. Bobillo et al. reduces a fuzzy KB to a crisp KB and reasoning is performed by computing **one consistency test** on the crisp KB.
  - Our case is more difficult and needs **several entailment tests**.



- Moreover, how to **represent the possibilistic DL** using a classical DL remains an open issue.

Questions?

Thank you very much for your attention