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Market Analysis and Trading Strategies with Bayesian Networks

KC Chang
George Mason University
Dept. of Systems Engineering and Operations Research
Fairfax, VA, U.S.A.
kchang@gmu.edu

Zhi Tian
George Mason University
Dept. of Electrical and Computer Engineering
Fairfax, VA, U.S.A.
ztian1@gmu.edu

Abstract – This paper examines the application of data fusion and probabilistic reasoning for investment decision and its performance evaluation. Specifically, Bayesian networks are used to model the qualitative and quantitative relationships between various factors that affect the dynamics of equity index (S&P 500) for predictive analysis. The resulting assessments are applied to trading decisions utilizing derivatives such as S&P futures and options. The simulated trading performance results demonstrate the effectiveness of the Bayesian network approach.

Keywords: financial markets, Bayesian networks, S&P futures, futures options trading, return and risk analysis.

1 Introduction

In the global marketplace, trading and investment are the cornerstones of economic growth and job creation. Investors have to sift through a large amount of financial information in order to decipher the market behavior, predict future market directions, and make trading decisions in hope for good portfolio returns. Given the complexity of the markets and the high stake of trading decisions, financial engineering and risk analysis have become an important research field.

In the financial markets, there are two main schools of thought for portfolio modeling, market analysis and stock selection. Fundamental analysis looks into economic factors to make subjective judgments on the qualitative relationship between portfolio and market returns, whereas technical analysis uses quantitative historical data of a security to predict its future price movement. To judiciously utilize both qualitative and quantitative information, data fusion has emerged as a highly relevant and promising paradigm for financial engineering. In particular, Bayesian networks are well suited for data fusion of financial data, because they not only provide graphical models for fundamental analysts to intuitively capture their knowledge of economic factors and visualize the market trends, but also offer powerful probabilistic reasoning tools for decision making and risk analysis from complicated large-scale inference networks with multivariate nodes [1-8].

The applicability of Bayesian networks for portfolio modeling and analysis has been demonstrated recently [6] [8]. Some examples from the oil industry and gold mining industry are given to illustrate the use of Bayesian Network models to describe not only qualitative information such as belief, judgments, and fundamentals of the industry, but also quantitative data such as historical stock prices and movements. The inference results yield the posterior probabilistic distributions of the portfolio returns, which can be used for both trading decision and risk analysis. In contrast, many traditional technical analysis methods only yield the summary statistics such as mean and variance of the expected portfolio returns. Most of these methods are applied for market analysis only, without explicit linking to active stock selection. Recently, Bayesian networks have been applied for stock selection [1-4]. They focus on simple buy-or-sell trading strategies, and typically use the tool for short-term trading such as day trading or weekly trading. These simulated tests on short-term trading were less influenced by subjective judgments, but mainly relies on historical data; therefore, the results mainly reflect the technical capability of Bayesian networks.

This paper employs a Bayesian network (BN) approach for both predictive market analysis and trading. The focus is on long-term investments, using not only asset buy-or-sell but also options trading strategies. Since long-term trading rides out the market fluctuation, our BN framework is designed to model the inter-dependent relationships among market factors and make asset allocation decisions for long-term investment and risk mitigation. In doing so, the built-in capabilities of Bayesian networks in performing both fundamental and technical analyses are utilized.

Extensive simulations and testing are carried out using real-world financial data. Our Bayesian network performs predictive analysis of the Equity index (S&P 500) data from 1990 to 2014, and use the predicted market movements to support trading decisions on S&P futures and options. Considerable profit potential are demonstrated.

1.1 Prior Art and Related Work

There are two basic categories of market or investment analysis techniques, namely, fundamental analysis and technical analysis. Fundamental analysis relies on economical and financial indicators to qualitatively evaluate the value of a business, predict its future stock valuation, and assess its credit risks. Analysts capture their knowledge, speculation, and insights of the market into fundamental analysis, but do not have a systematic way of incorporating the historical market data. In contrast, technical analysis is considered a quantitative approach, in which numeric indicators of the stock market, such as stock prices, moving average, momentum, and volume dynamics, are modeled and analyzed to predict future market movements. Many mathematical and statistical methods have been used for quantitative analysis, such as time series analysis, regression analysis, finite difference methods, and Monte Carlo simulations [13].

Quantitative analysis is predominately popular in recent years because of the increasing availability and accessibility of a large amount of financial data, sometimes free from the Internet. Meanwhile, advances in numerical tools and computing devices have led to fast algorithms that can process large-scale dynamic financial data at manageable complexity. Nevertheless, technical analysis incurs several main drawbacks and criticisms. It focuses on quantitative relationships among economic variables, but does not provide an easy venue for market analysts to incorporate their subjective judgment or special knowledge that may considerably affect the portfolio and stock picking decisions. While mathematical models used for financial data analysis have become increasingly sophisticated, they are not as competent in capturing the intrinsic risk correlations and inter-dependency among a large number of business players on the market.

In view of the pros and cons of both fundamental and technical analyses, Bayesian networks emerge as an attractive framework for portfolio modeling and analysis [1-8]. A Bayesian network offers a graphical representation that allows an analyst to intuitively capture market factors in the graphical model and visualize the relationships among the variables in the model. As such, judgmental factors in qualitative analysis can be fully incorporated. Utilizing the basic graph structure, quantitative information from historical data provides training, analysis, and testing of the graphical model to generate the probabilistic distributions of portfolio returns. It provides a powerful tool to explicitly capture the dependence among market factors as well as the sensitivity of portfolio returns, which are essential for principled risk control.

Most of the work on Bayesian networks for financial analysis focuses on portfolio risk analysis. In [6][8], the semantics of Bayesian networks are established to model portfolio returns. The output of the Bayesian network is the marginal or mode of the posterior joint distributions of the market variables of interest. The inference results describe the portfolio returns, which match well with the actual portfolio returns for a set of test data obtained over an arbitrary period from 1996 to 1998 [8].

The aforementioned work focused on portfolio modeling and analysis, but did not discuss how these analytical outcomes impact the performance of stock selection. In [7], a Bayesian network is used as a modeling tool for stock picking, and the investment "skills" of a Bayesian network are evaluated using HUGIN software. The evaluation is done using the financial data from the Danish stock market, for which only a simple Bayesian model is designed using buy-or-sell trading recommendations. In [1-2], Bayesian networks are considered for day trading, in which the market trend is predicted for stock prices on a daily basis. The random variables in the Bayesian network represent the up and down of daily stock prices, which are used to predict the next-day trend and make the buy-or-sell decisions for one day. In [3], a variant of dynamic Bayesian networks, termed hierarchical hidden Markov model, is proposed for semi-supervised learning of predicting market directions.

These stock trading strategies are simple buy or sell decisions, which are often used for short-term day trading. In our work, we consider not only buy-or-sell strategies, but also option trading strategies that are used for investing over a longer period of time. Such investment strategies are typically more concerned with market fundamentals, which can ride out the downtrends and short-term market fluctuation. As such, our study accentuates the capability of Bayesian networks in accurately reflecting fundamental analysis using domain knowledge and historical data. In contrast, portfolio analysis for day trading reflects the technical analysis capability of Bayesian networks [1-2].

2 Bayesian Networks for Data Fusion in Market Analysis

Bayesian networks (BNs) are acyclic directed graph which include nodes and arcs. Each node in the network represents a random variable and the arcs between nodes represent their probabilistic relationship [14]. The network topology describes the conditional dependency between the variables and the network as a whole represent the joint probability of all the variables. Each variable could be a discrete or continuous variable and the relationship between nodes could be probabilistic or deterministic.

Once a Bayesian network is built to model a domain specific problem, one of the main purposes is to compute the conditional probability of a particular node given the observed evidence from other nodes. This "fusion" process, also called probabilistic inference, can be executed in an efficient "message passing" manner and is one of the main advantages of applying the BN modeling tool.

2.1 S&P Predictive Model

We now build a Bayesian network model to describe the S&P dynamics based on several highly relevant factors [4-

5][7-8]. To do so, we first construct the network topology from domain knowledge and subject matter experts [9-12], similar to the fundamental analysis process. Then, we illustrate the technical analysis aspect in which the conditional probabilities between children and parent nodes of the directed arcs in the network are learned statistically from the historical data.

There are many factors affecting S&P market dynamics. Some of the most obvious examples include interest rate (LIBOR), consumer price index (CPI), unemployment rate (UR), money supply (MS), housing start (HS), and implied volatility index (VIX). Note that interest rate is an important driver of the nation's economy. Stock market tends to go higher when the interest rate is low which also makes other alternative investment such as fixed income treasuries less attractive. CPI measures the inflation level of the consumer price and is highly relevant to the overall economics and equity market.

Unemployment rate is also an important factor that could affect the market direction. A high unemployment rate might be bad for the market depending on other factors such as government momentary policy. In addition, money supply and housing start may have some impact on the market direction as well. Finally the implied volatility index (VIX, also called the fear and greed index) measures the S&P volatility based on the S&P one-month options and might be a good indicator of market direction. High volatility implies a potential down trend and vice versa.

Taking into account of aforementioned key economic factors, we construct an exemplary Bayesian network to model the S&P dynamics, as shown in Figure 1. This model has been obtained by working with subject matter experts and validated by historical data. Among the factors mentioned earlier, some of them (money supply and housing start) are eliminated due to their weak correlations to the S&P future directions. In the network, each node is modeled as a binary random variable with two state values: "up" or "down". Note that in Figure 1, SP2 indicates the potential future S&P state ("up" or "down") for the next trading period and SP1 represents the S&P state in the current trading period. The remaining variables in the network represent their corresponding states in the current period. For example, an "up" state for VIX (VX1) indicates the implied volatility has gone up in the current time period.

The purpose of the BN model is to predict S&P market direction for the next trading period in order to facilitate trading decision or risk management. The network parameters (conditional probabilities of a child node given its parents) are learned from the historical data [15]. The historical data is organized in a sliding widow manner so that a series of past data can be used to train the model for predicting the probability of the market trend at the next time period.

Specifically, we collected the historical data from 1990 to 2014, where the S&P and VIX data are obtained from the yahoo finance web site and the rest are downloaded from Federal Reserve economic data repository [16]. Specifically, we set the trading cycle to be one month and the training window size to be 180 months. In other words, we use the historical data from the past 180 months to train the model and apply it to predict the market direction for the next trading month. We repeat the prediction process from 2005 to 2014 for 120 trading months. An example set of conditional probability tables (CPT) in the network learned from the historical data is shown in Table 1.

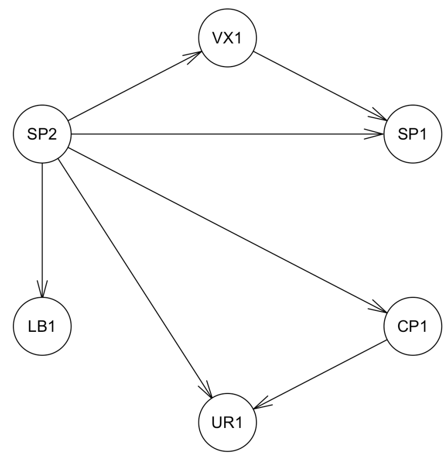

Figure 1. A Bayesian Network for S&P Dynamic Model

Table 1. An example set of CPTs leanred from historical data

SP2 = "up"	SP2 = "down"
0.61	0.39

SP2	VX1 = "up"	VX1 = "down"
"up"	0.42	0.58
"down"	0.50	0.50

SP2	VX1	SP1 = "up"	SP1 = "down"
"up"	"up"	0.31	0.69
"up"	"down"	0.84	0.16
"down"	"up"	0.26	0.74
"down"	"down"	0.88	0.11

SP2	LB1 = "up"	LB1 = "down"
"up"	0.42	0.58
"down"	0.40	0.60

SP2	UR1 = "up"	UR1 = "down"
"up"	0.31	0.69
"down"	0.40	0.60

SP2	UR1	CP1 = "up"	CP1 = "down"
"up"	"up"	0.79	0.21
"up"	"down"	0.76	0.24
"down"	"up"	0.68	0.32
"down"	"down"	0.95	0.05

2.2 Trading S&P Futures and Options

S&P futures and their options are traded in several financial markets, such as the Chicago Mercantile Exchange (CME) [17] and the CME electronic GLOBEX platform [18]. There are several common trading practices on the markets [21], as quoted below from Wikipedia.

- Long: a long position is to buy or own a underlying entity (e.g., asset, index, or interest rate futures);
- Short: a short position is to sell or owe;
- Put: a put option gives the owner of the put, the right, but not the obligation, to sell an asset (the underlying) at a specific price (the strike), by a pre-determined date (the expiration or maturity date) to a given party (the seller of the put). Put options are most commonly used in the stock market to protect against the decline of a stock price below a specific price.
- Call: a call option gives the buyer of the call, the right, but not the obligation, to buy an agreed quantity of the underlying from the seller (or "writer") of the call option before a certain time (the expiration date) at a certain strike price.

S&P futures is one of the most liquid futures markets in the world. One can long or short the futures contracts as long as there is a counter party who is willing to take the opposite side. Similarly, the S&P futures options market is extremely liquid and popular. One could long or short the put or call options depending on the goals of the trading strategies.

By writing (selling) the put options when the market is expected to go higher would result in the options expiring worthlessly and therefore the seller could keep the collected premium. Similarly, the seller could keep the premium collected by writing the call options if the market does not go up. However, while the potential loss of buying options is limited by the premium paid, shorting options could be very risky because the loss is only limited by the market actions. For example, shorting a call option while the market continues going up could result in a severe loss.

2.3 Options Pricing Model

To derive the fair option price, a common practice is to assume that the underlying asset follows a geometric Brownian motion (GBM) model with constant drift and volatility, described by the following stochastic differential equation:

$$dS = \mu\, S\, dt + \sigma\, S\, dW \tag{1}$$

where S is the asset price, μ is the drift parameter, σ is the volatility, and W is a wiener process or Brownian motion. With the assumed model, a closed-form options pricing model has been developed [19-20], as follows:

$$\begin{aligned} c &= S_0\, N(d_1) - K\, e^{-rT} N(d_2) \\ p &= K\, e^{-rT}\, N(-d_2) - S_0\, N(-d_1) \end{aligned} \tag{2}$$

where

$$\begin{aligned} d_1 &= \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma\sqrt{T}} \\ d_2 &= \frac{\ln(S_0 / K) + (r - \sigma^2 / 2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \end{aligned} \tag{3}$$

In eqn. (2), c is the price of a call option, p is for put option, S_0 is the current asset price, K is the strike price, T is the maturity (expiration) time, and σ is the asset volatility. This popular Back-Scholes-Merton (BSM) option pricing model had revolutionized the derivative industry for the last several decades.

Note that the value of an option consists of both time value and intrinsic value. While the intrinsic value depends on the relative strike price to the asset value, the time value always reduces to zero at the expiration. As mentioned earlier, an important assumption behind the derivation of the BSM pricing model is that the price of the underlying asset follows a GBM model with constant drift and volatility.

However, since the stock crash of October 1987, the volatility of stock index options implied by the market prices has been observed to be "skewed" in the sense that the volatility became a function of strike and expiration instead of remaining a constant. This phenomenon referred to as the "volatility smile" has since spread to other markets [21]. Because the original BSM model can no longer account for the smile, investors have to use more complex models to value and hedge their options. In this paper, for the purpose of evaluating the trading performance, we will emulate the option prices subject to the smile phenomenon by utilizing the historical implied volatility index (VIX) data and approximate the volatility smile as a quadratic function of moneyness[1] [22].

2.4 Trading Process

As mentioned earlier, the target node (SP2) in the Bayesian network shown in Figure 1 represents the one-step prediction of the S&P market in the next trading cycle. We use a monthly cycle to synthesize the trading process. At the beginning of each month, we simulate the trading on the S&P futures and options markets based on the strategies derived from the BN model predictions. We use historical end-of-the-day S&P settlement prices and the options pricing model (Section 2.3) to emulate the filled-prices of the transactions. We assume no transaction cost and no slippage.

[1] Moneyness is the relative position of the current price of an underlying asset with respect to the strike price of a derivative.

3 Test and Simulation

We apply two trading strategies based on the BN prediction. The first strategy is "Long and Short" where we either long the S&P index futures if the prediction is "up" or short the futures if the prediction is "down". The second strategy is "Options Writing" where we either short the S&P futures put options when the predicted trend is expected to be up or short the call options when the trend is expected to be down. We compare the trading performances to the naïve buy-and-hold strategy.

3.1 Long and Short Strategy

In the "Long and Short" (LS) strategy we either long the S&P if the prediction is "up" (posterior probability of SP2 is "up" is greater than 0.5) or short the S&P if the prediction is "down" (posterior probability of SP2 is "down" is greater than 0.5). We apply a monthly trading cycle to test the strategy. On the first trading day of each month, if the predicted trend is up for the coming month, 100% of the equity will be committed to a long position of the S&P futures for the entire month. Similarly, 100% equity will be committed to a short S&P futures position for the entire month if the prediction is down.

3.2 Options Writing Strategy

With "Options Writing" (OW) strategy, we either short the S&P put options when the trend is expected to be up (posterior probability of SP2 is "up" is greater than 0.5) or short the call options when the trend is expected to be down (posterior probability of SP2 is "down" is greater than 0.5). We also apply a monthly trading cycle to be consistent with the monthly expiration options. Specifically, if the market is predicted to be up, we will short the at-the-money[2] (ATM) S&P put options expiring in the coming month; and if the market prediction is down, we will short the corresponding S&P ATM call options. We will keep the options until expiration before repeating the same process in the next trading cycle.

Note that the options could expire out of the money[3] (OTM), and therefore become worthless. In that case, the premium collected by the seller becomes the profit and the positions will be closed automatically by the exchange. On the other hand, if the options expire in the money (ITM), the options will have to be settled in cash in the sense that the sellers have to pay the market price to "buy" back the options they sold. In that case, if the market price is higher than the premium collected, the seller will incur a loss.

[2] When the option strike price is equal to the current price of the underlying asset.

[3] The strike of a call option is above the market price or the strike of a put option is below the market price of the underlying asset.

3.3 Simulated Trading

Figure 2 shows the historical data of the relevant factors in the BN model. Since there are only limited historical options prices with specific strikes and expirations available in the public domain, we simulate the options filled-prices based on the model described earlier. Specifically, options prices are obtained by utilizing the BSM model given the S&P price, risk-free interest rate, volatility, and an expiration date of 4 trading weeks after writing the options. The S&P prices are based on historical data and served as the ATM strike prices. Risk-free interest is based on historical 3-month LIBOR data and volatility is based on the historical implied volatility index (VIX). However, as mentioned earlier, it is well known that true volatility is not a constant but a function of strike and expiration (volatility smile and surface). To obtain a more realistic options price, we develop a smile model and adjust the option price accordingly as described in Section 2.3. The results have been validated against the available market data and proved to be reasonably accurate.

3.4 Performance Results

Figure 3 summarizes the performance of the LS strategy. As shown in the figure, the strategy longs the market most of the time, while shorts the market only about 10% of the time based on the assessed probabilities obtained by the Bayesian network model. The performance of this dynamic trading strategy is significantly better than the buy-and-hold approach while with smaller risk/volatility.

Figure 4 shows the performance of the OW strategy. As shown in the figure, while the rate of return is lower than that of the LS strategy over the 10 years period, the percentage of positive return is much higher (75% vs. 63%). The rate of return of this dynamic trading strategy is also much higher then the buy-and-hold approach while with smaller risk/volatility. Note that one could easily employ leverage in options trading. Typically, the trader is allowed to have up to 10 to 1 leverage by the exchange. For example, with a 2 to 1 leverage (OW-II, selling twice as many as options contracts per capital) and a 4 to 1 leverage (OW-IV), the performances are shown in Figures 5 and 6, respectively. The results show that with higher leverage, the returns are much better while the risk is also increased. The strategy is clearly very effective and flexible. Depending on the risk aptitude of the investor, one could conceivably adjust the leverage level to meet a range of desirable investment goals with different risk-reward trade-offs.

Table 2 summarizes the trading results. In the table, two additional performance metrics, maximum drawdown and Sharpe ratio, are given for comparison. Specifically, drawdown is defined as the peak-to-trough decline during a specific period of an investment. The Sharpe ratio is a measure for calculating risk-adjusted return. It is the average return earned in excess of the risk-free rate over the return volatility (standard deviation). It can be seen from the table that LS and OW perform much better than the naïve buy-and-hold policy. The BN-based strategies not only offer higher returns over the 10-year period, but also exhibit smaller drawdowns and volatilities. Furthermore, by employing leveraged options writing strategies, the performance can be adapted for different risk/reward levels. For example, an aggressive investor might decide to employ a higher leverage ratio than a conservative one.

Table 2. Performance Comparison

	Positive Return	Rate of Return (10 yrs)	Maximum Drawdown	Sharpe Ratio
Buy-Hold	62.2%	74.3%	52.6%	0.39
LS (BN)	63.0%	168.5%	38.8%	0.69
OW-I (BN)	74.8%	110.6%	24.4%	0.80
OW-II (BN)	74.8%	307.0%	45.1%	0.86
OW-IV (BN)	74.8%	1,012.6%	75.7%	0.89

4 Conclusion

We have developed a data fusion approach using Bayesian networks for predicting market directions to support investment and trading decisions. The Bayesian network is constructed from both historical data and domain knowledge of several relevant financial factors. The resulting model is applied to predict the S&P direction for the next trading cycle. Several trading strategies are implemented based on the market predictions. The results of the simulated trading using these strategies over a 10-year period show considerable potential equity gains. The BN-based strategies significantly outperform the naïve buy-and-hold policy, which demonstrate the potential and effectiveness of the BN approach.

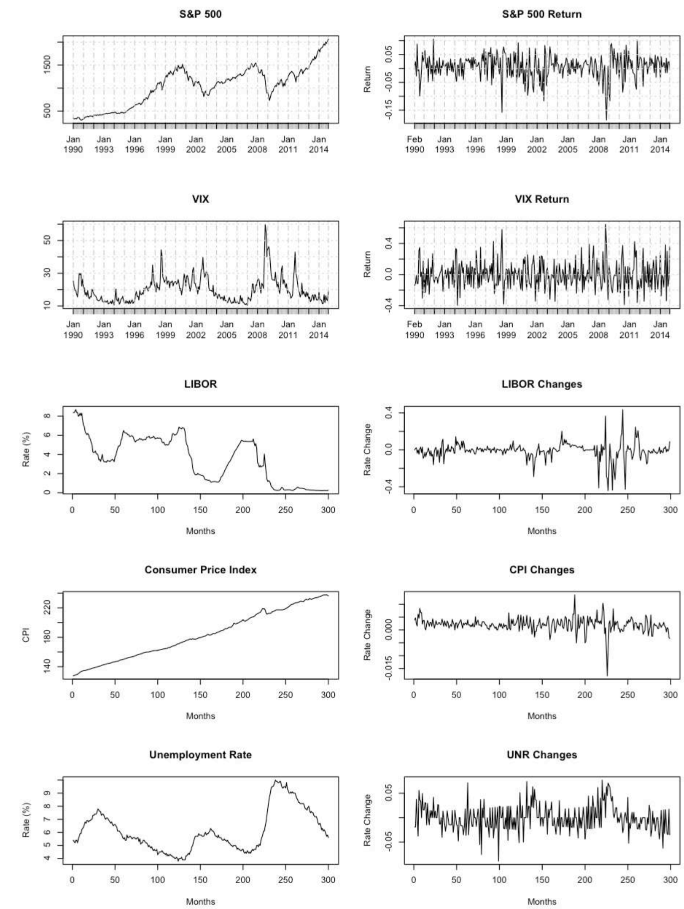

Figure 2. Historical Data of Relevant Factors

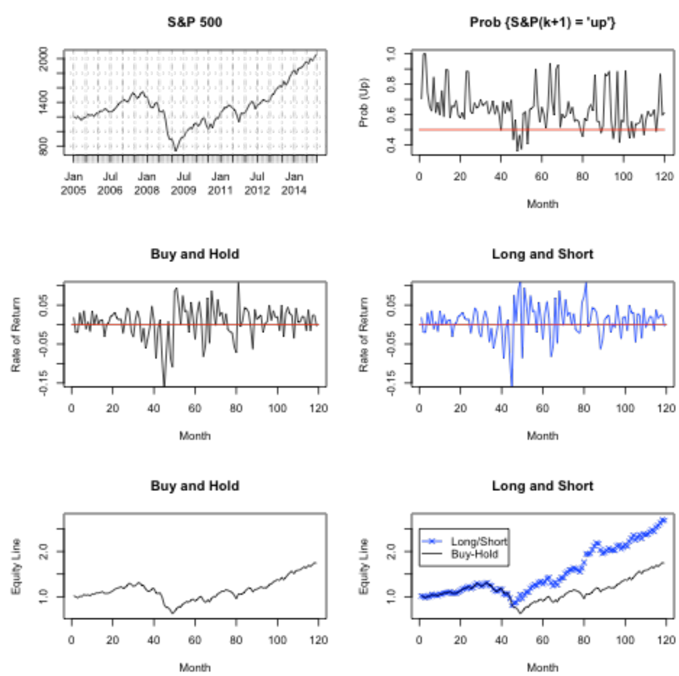

Figure 3. Long and Short Trading Performance

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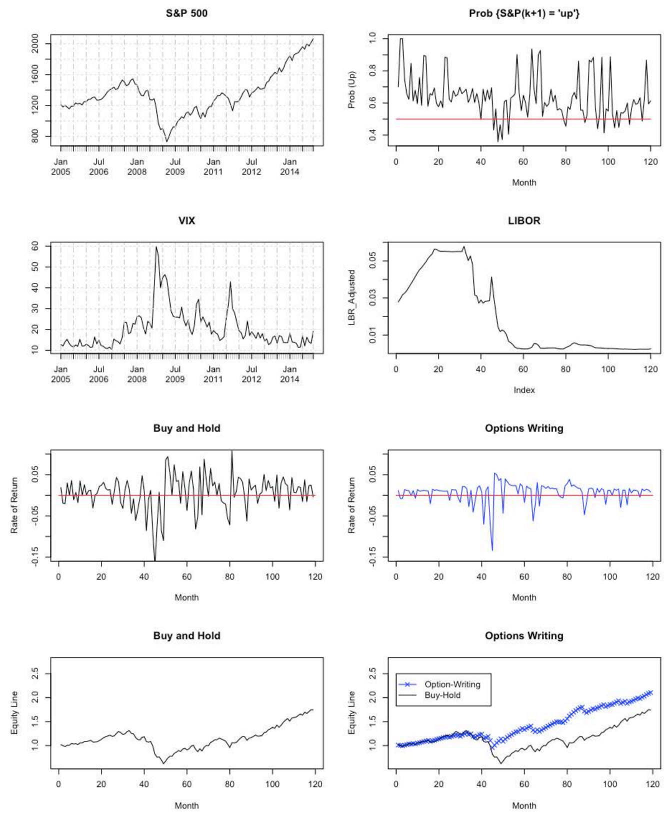

Figure 4. Options Writing Strategy with No Leverage (OW-I)

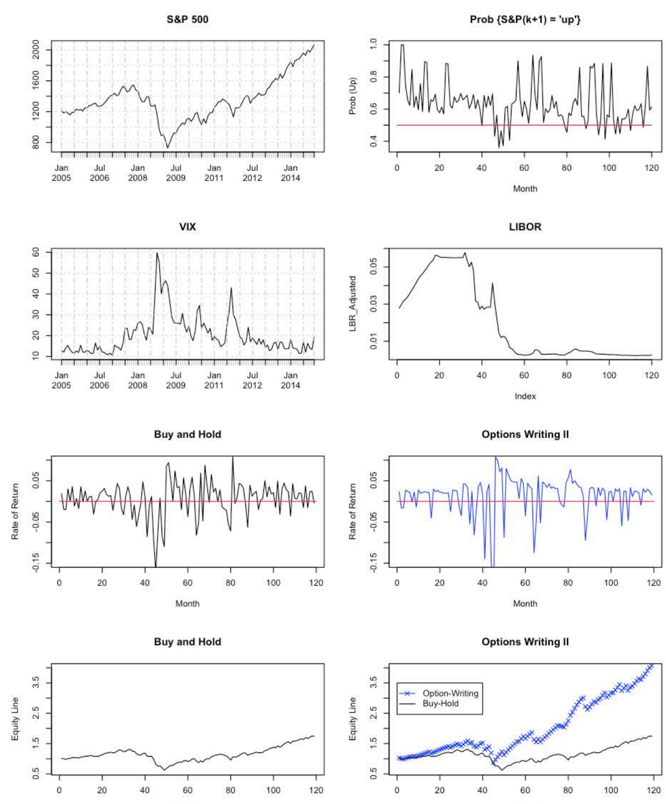

Figure 5. Options Writing with 2 to 1 Leverage (OW-II)

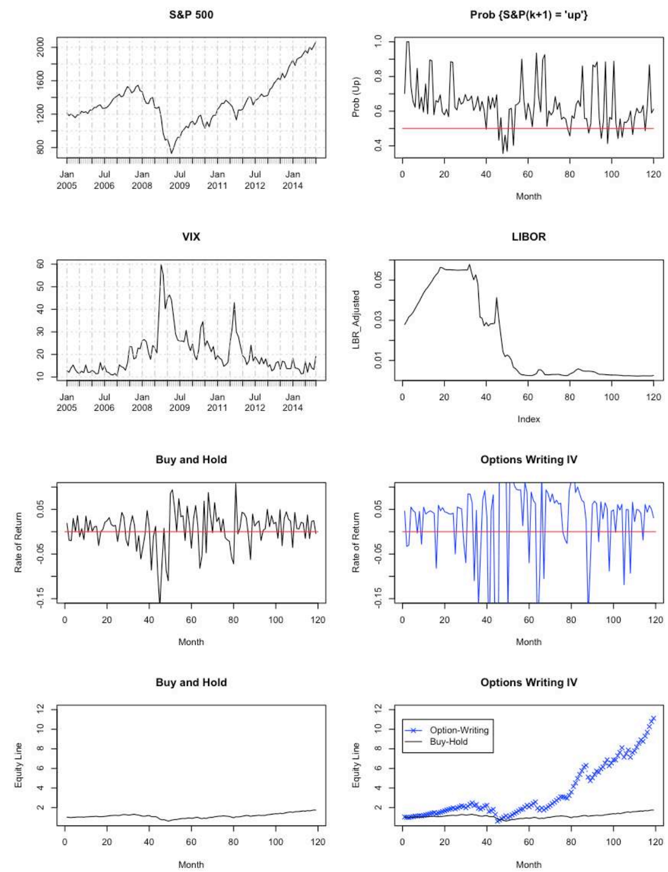

Figure 6. Options Writing with 4 to 1 Leverage (OW-IV)

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