DeLorean: A Reasoner for Fuzzy OWL 1.1

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Outline

1. Introduction
2. Fuzzy $\mathcal{SROIQ}$ and Its Crisp Representation
3. DeLorean Reasoner
4. Use Case: A Fuzzy Wine Ontology
5. Conclusions and Future Work

Outline

Fuzzy Description Logics

- **Description Logics** (DLs) are a family of logics for representing structured knowledge, very useful as ontology languages.
- DLs are not appropriate for **fuzzy/vague/imprecise** knowledge for which a clear and precise definition is not possible.
  - An inn is a cheap and small hotel.
  - Patient001's Serotonin Level is quite low.
  - English is generally spoken in Canada.
  - I do not like flamenco very much.
- Since **fuzzy logic** and fuzzy set theory are suitable formalisms for the management of these types of knowledge, **fuzzy DLs** extend DLs with fuzzy logic.

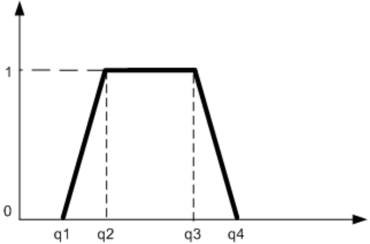
Our contributions

- We report on the implementation of **DeLorean reasoner**, the first one that supports the fuzzy DL $\mathcal{SROIQ}(\mathbf{D})$ (a fuzzy OWL 1.1).
  - Several limitations of the current standard language for ontology representation OWL have been identified, and several extensions have been proposed. Among them, **OWL 1.1** (OWL 2) is its most likely immediate successor.
  - The broad acceptance of the OWL 1.1 will largely depend on the availability of editors, reasoners, and other tools.
  - The reasoner is based on a reduction to crisp DLs.
- We describe the use of some **optimizations**, among them handling of superfluous elements before applying crisp reasoning.
- We present a **preliminary evaluation** of the performance of the optimizations in the size of the reduced crisp ontologies.

Outline

Fuzzy logic

- Fuzzy set theory and fuzzy logic (Zadeh, 1965) manage imprecise and vague knowledge.
- In classical set theory elements either belong to a set or not.
- In fuzzy set theory elements can belong to **some degree in** $[0, 1]$.
  - 0 means no-membership.
  - 1 means full membership.
  - A value between 0 and 1 represents the extent to which an element can be considered as an element of the fuzzy set.
- α**-cut**: Set of elements that belong to a fuzzy set with degree α.
- Example: **Trapezoidal** membership function of a fuzzy set:



Families of fuzzy logics

- All crisp set **operations** are extended to fuzzy sets.
  - Intersection: t-norm function.
  - Union: t-conorm function.
  - Complement: negation function.
  - Implication: implication function.
- Fuzzy operators are grouped in **families**:

Family	t-norm $\alpha \otimes \beta$	t-conorm $\alpha \oplus \beta$	negation $\ominus \alpha$	implication $\alpha \Rightarrow \beta$
Zadeh	$\min\{\alpha, \beta\}$	$\max\{\alpha, \beta\}$	$1 - \alpha$	$\max\{1 - \alpha, \beta\}$
Łukasiewicz	$\max\{\alpha + \beta - 1, 0\}$	$\min\{\alpha + \beta, 1\}$	$1 - \alpha$	$\min\{1 - \alpha + \beta, 1\}$
Gödel	$\min\{\alpha, \beta\}$	$\max\{\alpha, \beta\}$	$\begin{cases} 1, & \alpha = 0 \\ 0, & \alpha > 0 \end{cases}$	$\begin{cases} 1 & \alpha \leq \beta \\ \beta, & \alpha > \beta \end{cases}$
Product	$\alpha \cdot \beta$	$\alpha + \beta - \alpha \cdot \beta$	$\begin{cases} 1, & \alpha = 0 \\ 0, & \alpha > 0 \end{cases}$	$\begin{cases} 1 & \alpha \leq \beta \\ \beta / \alpha, & \alpha > \beta \end{cases}$

- Different families have **different logical properties**.

The Fuzzy DL $\mathcal{SROIQ}$

- Vocabulary of the language:
  - **Individuals**: `fernando`.
  - **Concepts**, fuzzy sets of individuals: `Tall`.
  - **Roles**, fuzzy binary relations over individuals: `isFriendOf`.
- **Complex definitions** of concepts and roles can be built.
- A **fuzzy interpretation** $\mathcal{I}$ is a pair $(\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$.
  - $\Delta^{\mathcal{I}}$ is a non empty set, the interpretation domain
  - $\cdot^{\mathcal{I}}$ is an interpretation function mapping:
    - Each concept onto a function $C^{\mathcal{I}} : \Delta^{\mathcal{I}} \to [0, 1]$
    - Each role onto a function $R^{\mathcal{I}} : \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \to [0, 1]$
- A **fuzzy knowledge base** consists of fuzzy axioms organized in:
  - A fuzzy **ABox** $\mathcal{A}$: extensional knowledge about individuals.
  - A fuzzy **TBox** $\mathcal{T}$: intensional knowledge about concepts.
  - A fuzzy **RBox** $\mathcal{R}$: intensional knowledge about roles.
- Most reasoning tasks are reducible to **KB consistency**.

Complex fuzzy concept and roles

Constructor	Syntax	Semantics
(top concept)	$\top$	1
(bottom concept)	$\bot$	0
(atomic concept)	A	$A^{\mathcal{I}}(a)$
(concept conjunction)	$C \sqcap D$	$C^{\mathcal{I}}(a) \otimes D^{\mathcal{I}}(a)$
(concept disjunction)	$C \sqcup D$	$C^{\mathcal{I}}(a) \oplus D^{\mathcal{I}}(a)$
(concept negation)	$\neg C$	$\ominus C^{\mathcal{I}}(a)$
(universal quantification)	$\forall R.C$	$\inf_{b \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(a,b) \Rightarrow C^{\mathcal{I}}(b)\}$
(existential quantification)	$\exists R.C$	$\sup_{b \in \Delta^{\mathcal{I}}} \{R^{\mathcal{I}}(a,b) \otimes C^{\mathcal{I}}(b)\}$
(fuzzy nominals)	$\bigcup_{i=1}^{m} \{\alpha_i / o_i\}$	$\sup_{i \mid a \in \{o_i^{\mathcal{I}}\}} \alpha_i$
(at-least restriction)	$\geq n$ S.C	$\sup_{b_1,\ldots,b_m \in \Delta^{\mathcal{I}}} [(\otimes_{i=1}^{m} \{S^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i)\}) \bigotimes (\otimes_{j<k} \{b_j \neq b_k\})]$
(at-most restriction)	$\leq n$ S.C	$\inf_{b_1,\ldots,b_{n+1} \in \Delta^{\mathcal{I}}} [(\otimes_{i=1}^{n+1} \{S^{\mathcal{I}}(a,b_i) \otimes C^{\mathcal{I}}(b_i)\}) \Rightarrow (\oplus_{j<k} \{b_j = b_k\})]$
(local reflexivity)	$\exists S.\mathtt{Self}$	$S^{\mathcal{I}}(a,a)$
(atomic role)	R_A	$R_A^{\mathcal{I}}(a,b)$
(inverse role)	R^{-}	$R^{\mathcal{I}}(b,a)$
(universal role)	U	1

Fuzzy axioms in the ABox, TBox and RBox

Axiom	Syntax	Semantics
(concept ass.)	$\langle a : C \bowtie \alpha \rangle$	$C^{\mathcal{I}}(a^{\mathcal{I}}) \bowtie \alpha$
(role ass.)	$\langle (a, b) : R \bowtie \alpha \rangle$	$R^{\mathcal{I}}(a^{\mathcal{I}}, b^{\mathcal{I}}) \bowtie \alpha$
(inequality ass.)	$\langle a \neq b \rangle$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$
(equality ass.)	$\langle a = b \rangle$	$a^{\mathcal{I}} = b^{\mathcal{I}}$
(GCI)	$\langle C \sqsubseteq D \triangleright \alpha \rangle$	$\inf_{a \in \Delta^{\mathcal{I}}} \{C^{\mathcal{I}}(a) \Rightarrow D^{\mathcal{I}}(a)\} \triangleright \alpha$
(RIA)	$\langle R_1 R_2 \ldots R_n \sqsubseteq R' \triangleright \alpha \rangle$	$\sup_{b_1 \ldots b_{n+1} \in \Delta^{\mathcal{I}}} \bigotimes [R_1^{\mathcal{I}}(b_1, b_2), \ldots, R_n^{\mathcal{I}}(b_n, b_{n+1})] \Rightarrow R^{\mathcal{I}}(b_1, b_{n+1}) \triangleright \alpha$
(transitive)	$\mathtt{trans}(R)$	$\forall a, b \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a, b) \geq \sup_{c \in \Delta^{\mathcal{I}}} R^{\mathcal{I}}(a, c) \otimes R^{\mathcal{I}}(c, b)$
(disjoint)	$\mathtt{dis}(S_1, S_2)$	$\forall a, b \in \Delta^{\mathcal{I}}, S_1^{\mathcal{I}}(a, b) \otimes S_2^{\mathcal{I}}(a, b) = 0$
(reflexive)	$\mathtt{ref}(R)$	$\forall a \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a, a) = 1$
(irreflexive)	$\mathtt{irr}(S)$	$\forall a \in \Delta^{\mathcal{I}}, S^{\mathcal{I}}(a, a) = 0$
(symmetric)	$\mathtt{sym}(R)$	$\forall a, b \in \Delta^{\mathcal{I}}, R^{\mathcal{I}}(a, b) = R^{\mathcal{I}}(b, a)$
(asymmetric)	$\mathtt{asy}(S)$	$\forall a, b \in \Delta^{\mathcal{I}}$, if $S^{\mathcal{I}}(a, b) > 0$ then $S^{\mathcal{I}}(b, a) = 0$

Fuzzy operators supported

We consider two families of fuzzy operators because they make possible the reduction to a crisp KB (others may not be suitable).

1. $Z\ \mathcal{SROIQ}$:
   - **Minimum t-norm**, $\alpha \otimes \beta = \min\{\alpha, \beta\}$
   - **Maximum t-conorm**, $\alpha \oplus \beta = \max\{\alpha, \beta\}$
   - **Łukasiewicz negation**, $\ominus\alpha = 1 - \alpha$
   - **Kleene-Dienes implication**: $\alpha \Rightarrow \beta = \max\{1 - \alpha, \beta\}$
2. $G\ \mathcal{SROIQ}$:
   - Minimum t-norm, $\alpha \otimes \beta = \min\{\alpha, \beta\}$
   - Maximum t-conorm, $\alpha \oplus \beta = \max\{\alpha, \beta\}$
   - **Gödel negation**, $\ominus\alpha = \begin{cases} 1, & \alpha = 0 \\ 0, & \alpha > 0 \end{cases}$
   - **Gödel implication** in GCIs and RIAs: $\alpha \Rightarrow \beta = \begin{cases} 1 & \alpha \leq \beta \\ \beta & \alpha > \beta \end{cases}$

Idea of the reduction to a crisp DL

1. Compute the **set of degrees of truth** which must be considered: Degrees γ in the fuzzy KB and their complementaries $1 - \gamma$.
   - $\ominus\gamma$ is $1 - \gamma$.
   - $\gamma_1 \otimes \gamma_2$ and $\gamma_1 \oplus \gamma_2$ are γ_1 or γ_2.
   - $\gamma_1 \Rightarrow \gamma_2$ is $1 - \gamma_1$ or γ_2.
2. For each fuzzy atomic concept A and role R, add **new crisp elements** (α-cut and strict α-cut).
   - $\mathbf{A}_{>0}, \mathbf{A}_{>0.25}, \mathbf{A}_{\geq 0.25}, \ldots, \mathbf{R}_{>0}, \mathbf{R}_{>0.25}, \mathbf{R}_{\geq 0.25} \ldots$
3. Add **new axioms** to preserve their semantics.
   - $\mathbf{A}_{\geq 0.25} \sqsubseteq \mathbf{A}_{>0}, \mathbf{A}_{>0.25} \sqsubseteq \mathbf{A}_{\geq 0.25}, \ldots, \mathbf{R}_{>0.25} \sqsubseteq \mathbf{R}_{\geq 0.25}, \ldots$
4. Reduce **fuzzy axioms** using the new crisp elements.

- The size of the crisp KB is **quadratic**.
  - The size is linear under a fixed set of degrees.
- The reduction **preserves reasoning**.
  - Consistency of the fuzzy KB and the crisp KB are equivalent.

Example of reduction

Reduction of $\mathcal{K} = \{ \langle \textbf{StGenevieveTexasWhite} : \textbf{WhiteWine} \geq 0.5 \rangle \}$

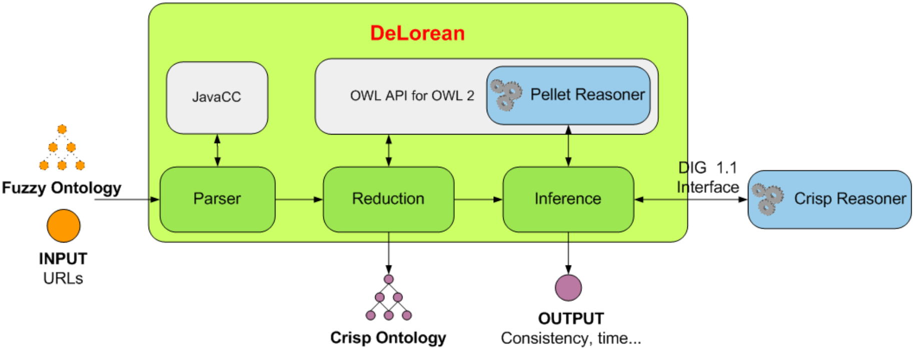
1. **Degrees of truth** to be considered:
   $\{0, 0.5, 1\}$
2. New **crisp elements**:
   $\text{WhiteWine}_{>0}, \text{WhiteWine}_{\geq 0.5}, \text{WhiteWine}_{>0.5}, \text{WhiteWine}_{\geq 1}$
3. New **axioms**:
   $\textbf{WhiteWine}_{\geq 0.5} \sqsubseteq \textbf{WhiteWine}_{>0}$,
   $\textbf{WhiteWine}_{>0.5} \sqsubseteq \textbf{WhiteWine}_{\geq 0.5}$,
   $\textbf{WhiteWine}_{\geq 1} \sqsubseteq \textbf{WhiteWine}_{>0.5}$
4. **Reduction** of every axiom in the KB:
   $\mathit{crisp}(\mathcal{K}) = \text{StGenevieveTexasWhite} : \rho(\text{WhiteWine}, \geq 0.5) =$
   $\textbf{StGenevieveTexasWhite} : \textbf{WhiteWine}_{\geq 0.5}$

Outline

Main Features

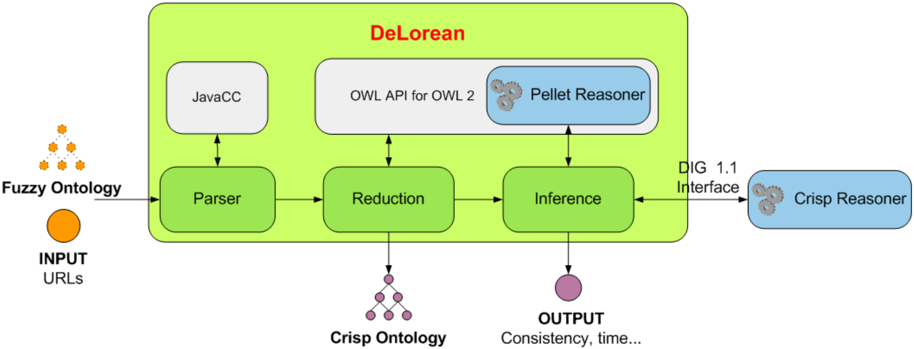
- **Prototype** implementation:
  - DELOREAN: DEscription LOgic REasoner with vAgueNess.
- **First version**:
  - Based on Jena API, using Java, JavaCC and DIG 1.1 interface.
  - Logic supported: $\mathcal{SHOIN}$ (OWL DL) under Zadeh family.
  - Possibility of using any crisp reasoner thanks to the DIG interface.
  - First reasoner that supported a fuzzy extension of OWL DL.
  - Optimization of the number of new elements and axioms.
- **Current version**:
  - Based on OWL API for OWL 2, using Java, JavaCC and DIG 1.1.
  - Logic: $\mathcal{SROIQ}(\mathbf{D})$ (OWL 1.1), **Zadeh and Gödel families**.
  - Uses PELLET crisp reasoner, or any DIG-complaint DL-reasoner if the expressivity is limited to $\mathcal{SHOIQ}$.
  - First reasoner that supports **a fuzzy extension of OWL 1.1**.
  - Implements all the optimizations that we will describe here.

Architecture



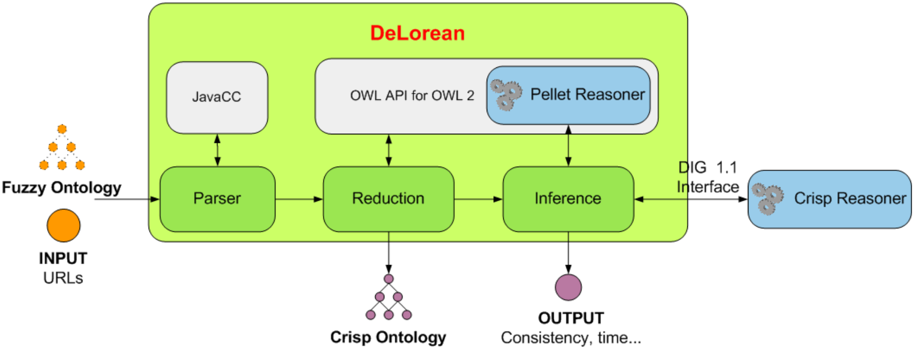
- The **Parser** reads an input file with a fuzzy ontology and translates it into an internal representation.
- We could use any language to encode the fuzzy ontology, as long as a parser can understand it. We could even have several parsers.
- It is also possible to import OWL 1.1 ontologies, which are saved into a text file which the user can edit and extend, e.g. adding membership degrees.

Architecture



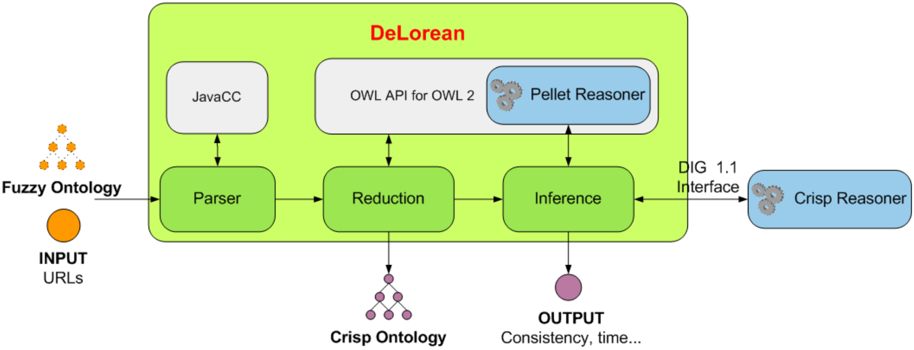
- The **Reduction** module implements the reduction algorithm.
- It builds an OWL API model with an equivalent crisp ontology which can be exported to an OWL 1.1 file.
- The implementation also takes into account all the described optimizations.

Architecture



- The **Inference** module tests this ontology for consistency, using either PELLET or any crisp reasoner through the DIG interface.
- Crisp reasoning does not take into account superfluous elements.

Architecture



- A very simple **User interface** manages inputs and outputs:
  - Inputs: path of the input fuzzy ontology, path of the output ontology and reasoner used.
  - Outputs: result of the consistency test and the elapsed time.

Optimizations

Optimizing the number of new elements and axioms

- Some previous works use two more atomic concepts:

$$\mathtt{A}_{\leq\beta}, \mathtt{A}_{<\alpha}$$

and some additional axioms:

$$\begin{array}{cc}
\mathtt{A}_{<\gamma_k} \sqsubseteq \mathtt{A}_{\leq\gamma_k}, & \mathtt{A}_{\leq\gamma_i} \sqsubseteq \mathtt{A}_{<\gamma_{i+1}}, \\
\mathtt{A}_{\geq\gamma_k} \sqcap \mathtt{A}_{<\gamma_k} \sqsubseteq \bot, & \mathtt{A}_{>\gamma_i} \sqcap \mathtt{A}_{\leq\gamma_i} \sqsubseteq \bot, \\
\top \sqsubseteq \mathtt{A}_{\geq\gamma_k} \sqcup \mathtt{A}_{<\gamma_k}, & \top \sqsubseteq \mathtt{A}_{>\gamma_i} \sqcup \mathtt{A}_{\leq\gamma_i}
\end{array}$$

but it has been shown that they are unnecessary.

Optimizing GCI reductions

- In some particular cases, the reduction of fuzzy GCIs can be optimized.
- For example, in **range** role axioms of the form $\langle \top \sqsubseteq \forall \mathtt{R}.\mathtt{C} \geq 1 \rangle$, **domain** role axioms of the form $\langle \top \sqsubseteq \forall \mathtt{R}^{-}.\mathtt{C} \geq 1 \rangle$ and **functional** role axioms of the form $\langle \top \sqsubseteq \leq 1\ \mathtt{R}.\top \geq 1 \rangle$:

$$\kappa(\langle \top \sqsubseteq \mathtt{D} \bowtie \gamma \rangle) = \top \sqsubseteq \rho(\mathtt{D}, \bowtie \gamma)$$

- In **disjoint concept** axioms of the form $\langle \mathtt{C} \sqcap \mathtt{D} \sqsubseteq \bot \geq 1 \rangle$:

$$\kappa(\mathtt{C} \sqsubseteq \bot \bowtie \gamma) = \rho(\mathtt{C}, > 0) \sqsubseteq \bot$$

- If the resulting TBox contains $\mathtt{A} \sqsubseteq \mathtt{B}$, $\mathtt{A} \sqsubseteq \mathtt{C}$ and $\mathtt{B} \sqsubseteq \mathtt{C}$, then $\mathtt{A} \sqsubseteq \mathtt{C}$ is unnecessary (it can be deduced from the other axioms).

Allowing crisp concepts and roles

- Let $\mathtt{A}$ be a fuzzy concept, and $\mathcal{N}$ the set of relevant degrees of a fuzzy KB.
- We need $\mathcal{N} - 1$ concepts of the form $\mathtt{A}_{\geq \alpha}$ and another $\mathcal{N} - 1$ concepts of the form $\mathtt{A}_{> \beta}$ to represent it.
- We also need $2 \cdot (|\mathcal{N}| - 1) - 1$ axioms to preserve the semantics of these elements.
- In real applications not all concepts and roles will be fuzzy.
- If $\mathtt{A}$ is declared to be crisp, we just need **1 concept** to represent it and **no new axioms**.
- Of course, this optimization requires some manual intervention.
- The case for fuzzy roles is exactly the same.

Optimizations

Reasoning ignoring superfluous elements

- The reduction is currently designed to promote reusing.
- The reduction of a fuzzy KB with the axiom

$$\text{StGenevieveTexasWhite} : \text{WhiteWine} \geq 0.75$$

  creates several concepts but in fact only $\text{WhiteWine}_{\geq 0.75}$ is used.
- $\text{WhiteWine}_{>0}$, $\text{WhiteWine}_{\geq 0.25}$, $\text{WhiteWine}_{>0.25}$, $\text{WhiteWine}_{\geq 0.5}$, $\text{WhiteWine}_{>0.5}$, $\text{WhiteWine}_{>0.75}$, $\text{WhiteWine}_{\geq 1}$ are **superfluous** in the sense that they **cannot cause a contradiction**.
- For a satisfiability test of the crisp reduction, we can avoid the terminological axioms where they appear.
- But if additional axioms are added to the KB, some concept and roles **may no longer be superfluous**!!

Outline

A Fuzzy Wine Ontology

- We consider now a fuzzy extension of the well-known **Wine ontology**[1], a highly expressive ontology, in $\mathcal{SHOIN}(\mathbf{D})$.
- In an empirical evaluation of the reductions of fuzzy DLs to crisp DLs, P. Cimiano et al. wrote that *"the Wine ontology showed to be completely intractable both with the optimized and unoptimized reduction even using only* 3 *degrees"*.
- However, the authors only considered the optimization of the number of new elements and axioms.
- We will show that the **rest of the optimizations**, specially the (natural) assumption that there are some crisp elements, **reduce significantly the size of the resulting crisp ontology**.

[1] http://www.w3.org/TR/2003/CR-owl-guide-20030818/wine.rdf

A fuzzy extension of the ontology

- We have **added a degree to the axioms**.
- Given a variable set of degrees $\mathcal{N}$, the degrees of truth for **fuzzy assertions are randomly chosen** in $\mathcal{N}$.
  - Having fuzzy assertions are of the form $\langle \tau \triangleright \beta \rangle$ with $\beta \neq 1$ reduces the number of superfluous concepts, so we are in the worst case from the point of view of the size of the resulting crisp ontology.
  - In practice, we will be often able to say that an individual fully belongs to a fuzzy concept, or that two individuals are fully related by means of a fuzzy role.
- In the case of **fuzzy GCIs and RIAs**, the degree is always 1 **in special GCIs** (namely concept equivalences and disjointness, domain, range and functional role axioms) or if there is a crisp element in the left side; **otherwise**, the degree is 0.5.
  - Using 1 produces the worst case from the point of view of the size of the resulting crisp ontology.

Crisp concepts and roles

- A careful analysis of the fuzzy KB brings about that most of the concepts and the roles should indeed be interpreted as crisp.
- For example, some of the subclasses of the class Wine refer to the type of grape used, which is a crisp concept, or to a well-defined geographical origin of the wines.
  - In other applications there could exist examples of fuzzy regions, but this is not our case.
- We identified **50 (out of 136) fuzzy concepts** in the ontology:
  - Some examples: WineColor, RedWine, RoseWine, WhiteWine, RedTableWine, WhiteTableWine, WineSugar, SweetWine, DryWine, DryRedWine, DryRiesling, DryWhiteWine, WineBody, FullBodiedWine, WineFlavor, WineTaste . . .
- Furthermore, we identified **5 (out of 16) fuzzy roles**:
  - hasColor, hasSugar, hasBody, hasFlavor, hasWineDescriptor.

Measuring the importance of the optimizations

- We omitted the concrete role yearValue.
- We focus the experimentation in Zadeh family, but using Gödel implication in the semantics of fuzzy GCIs and RIAs.
- Next page shows some metrics of the ontologies obtained when reduction of the fuzzy ontology, applying different optimizations.
  1. Column “Original” shows some metrics of the **original** ontology.
  2. “None” considers the reduction obtained after applying **no optimizations**.
  3. “(NEW)” considers the reduction obtained after optimizing the **number of new elements and axioms**.
  4. “(GCI)” considers the reduction obtained after optimizing **GCI reductions**.
  5. “(C/S)” considers the reduction obtained after allowing **crisp** concepts and roles **and** ignoring **superfluous** elements.
  6. Finally, “All” applies **all the** previous **optimizations**.

Measuring the importance of the optimizations

	Original	None	(NEW)	(GCI)	(C/S)	All
Individuals	206	206	206	206	206	206
Named concepts	136	2176	486	2176	800	191
Abstract roles	16	128	128	128	51	20
Concept assertions	194	194	194	194	194	194
Role assertions	246	246	246	246	246	246
Inequality assertions	3	3	3	3	3	3
Equality assertions	0	0	0	0	0	0
New GCIs	0	4352	952	4352	1686	324
Subclass axioms	275	1288	1288	931	390	390
Concept equivalences	87	696	696	696	318	318
Disjoint concepts	19	152	152	19	152	19
Domain role axioms	13	104	104	97	104	97
Range role axioms	10	80	80	10	80	10
Functional role axioms	6	48	48	6	48	6
New RIAs	0	136	119	136	34	34
Sub-role axioms	5	40	40	40	33	33
Role equivalences	0	0	0	0	0	0
Inverse role axioms	2	16	16	16	2	2
Transitive role axioms	1	8	8	8	1	1

Measuring the importance of the optimizations

- The size of the **ABox** is always the same, because every axiom in the fuzzy ABox generates one axiom in the reduced ontology.
- Using **all optimizations**, the number of new GCIs and RIAs preserving the semantics of the new elements is reduced:
  - From 4352 to 324 GCIs (7.44%).
  - From 136 to 34 RIAs (25%).
- **Optimizing GCI reductions** reduces disjoint concepts, domain, range and functional role axioms:
  - From 152 to 19 disjoint concepts(12.5 %).
  - From 104 to 97 domain axioms (93.27 %).
  - From 80 to 10 range axioms (12.5 %).
  - From 48 to 6 functional axioms (12.5 %).
- In domain role axioms, the optimization does not work very well because it needs an inverse role and this happens only in one of the axioms.

Measuring the importance of the optimizations

- Combining the optimization of **GCI reductions** with that of **crisp elements** reduces the number of new axioms:
  - From 1288 to 390 subclass axioms (30.28 %).
  - From 696 to 318 concept equivalences (45.69 %).
  - From 40 to 33 sub-role axioms (82.5 %).
- Inverse and transitive roles are reduced thanks to the optimization of **crisp elements**:
  - From 16 to 2 inverse role axioms (12.5%).
  - From 8 to 1 transitive role axioms (12.5%).

Measuring the importance of the optimizations

- The following table shows the **influence of the number of degrees on the size** of the resulting crisp ontology, as well as on the reduction time when **all the optimizations** are used.

	Crisp	3	5	7	9	11	21
Number of axioms	811	1166	1674	2182	2690	3198	5738
Reduction time	-	0.343	0.453	0.64	0.782	0.859	1.75

- Times are shown in seconds.
- The **reduction time is small enough** to make possible **recomputing** the reduction of an ontology when necessary, thus allowing superfluous concepts and roles in the reduction to be avoided.

Outline

Conclusions and Future Work

- We have presented DeLorean, the **more expressive fuzzy DL reasoner** (it supports fuzzy OWL 1.1).
- DeLorean implements several interesting **optimizations**:
  - Definition of crisp concepts and roles.
  - Handling superfluous concepts and roles before reasoning.
- A preliminary evaluation shows that these optimizations **help to reduce significantly the size** of the resulting ontology.
- In **future work**, we would like:
  - To measure the usefulness of the optimizations with respect to the reasoning time.
  - To rely on the hypertableau reasoner HermiT, which seems to outperform other DL reasoners.
  - To compare DeLorean against other fuzzy DL reasoners.

Thank you very much for your attention