Formal Challenges of Uncertainty: Spatial Nearness, Parts, Connectivity, Vagueness

Uncertainty Reasoning for the Semantic Web
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Agenda: Nothing New, Just Review of Some Formal Approaches

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- Challenges
- Mereotopology: Parts + Connectivity
- The Region Connection Calculus (RCC): variants & meaning
- Granularity, Approximation, & Vagueness
- Paradox of the Heap (Sorites)
- How to Solve Sorites?
- Spatial Vagueness: Egg-Yolk Theory
- How Does This Apply to Semantic Web?
- Summary
- References

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Challenges

- Nearness (approximation)
  - Spatial proximity: how do you know when you're close; how do you query? Example: Show me every address close to London
  - The Washington Monument is near/far from the Potomac River
- Parts
  - Space has parts, e.g., regions: how do you know if you are in a region? Example: Is the Bronx part of New York City? Is High Wycombe part of Greater London?
- Connectivity
  - Determining if something is a part of another thing may require them to be connected somehow
- Vagueness
  - Above examples begin to get into this. Is it ontological (world and truth is vague), is it semantic (ways of referring are vague), or is it epistemic (ways of knowing are vague)?

Mereotopology: Parts + Connectivity

- Mereo: parts
- Topology: connectivity
- Most upper ontologies today define Part in terms of mereotopology
- Which sub-theory is primary? Good question

The Region Connection Calculus (RCC): variants & meaning

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- A mereotopological theory of spatial regions
- RCC: Randall et al, 1992
- RCC8: Renz, 2002
- These axioms spawn a family of potential ontologies, distinct reasoning, depending on which you choose

RCC Axioms [1]

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All of these relations use the *connected* relation C, axiomatized to be *reflexive* [∀x.C (x, x)] and *symmetric* [∀x, y.C(x, y) → C(y, x)]. I.e., everything is connected to itself, and if an X is connected to a Y, then Y is connected to X. [2]

Relation	Description	Formal Definition
*DC(x, y)	disconnected	¬C(x, y)
P(x, y)	part of	∀z.C(z, x) → C(z, y)
PP(x, y)	proper part of	P(x, y) ∧ ¬P(y, x)
*EQ(x, y)	equal	P(x, y) ∧ P(y, x)
O(x, y)	overlaps	∃z.P(z, x) ∧ P(z, y)
*PO(x, y)	partially overlaps	O(x, y) ∧ ¬P(x, y) ∧ ¬P(y, x)
DR(x, y)	discrete from	¬O(x, y)
*EC(x, y)	externally connected	C(x, y) ∧ ¬O(x, y)
*TPP(x, y)	tangential proper part	PP(x, y) ∧ ∃z.EC(z, x) ∧ EC(z, y)
*NTPP(x, y)	non-tangential proper part	PP(x, y) ∧ ¬∃z.[EC(z, x) ∧ EC(z, y)]
$P^{-1}(x, y)$	converse of part of	P(y, x)
$PP^{-1}(x, y)$	converse of proper part of	PP(y, x)
$*TPP^{-1}(x, y)$	converse of tangential proper part	TPP(y, x)
$*NTPP^{-1}(x, y)$	converse of non-tangential proper part	NTPP(y, x)

[1] Renz. 2002, p. 42.

[2] Obrst, 2002, p. 1.

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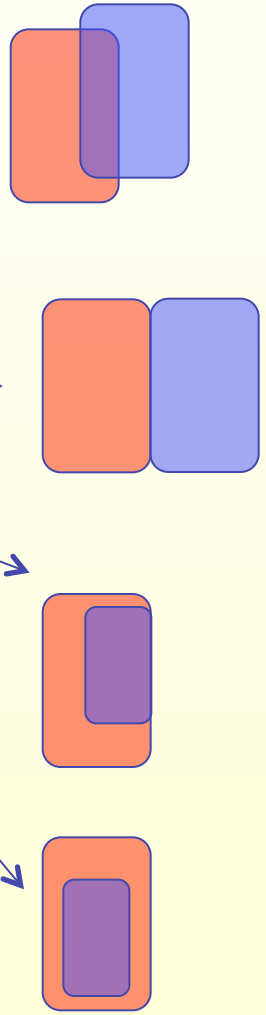
What the RCC Axioms mean

- Disconnected: *DC(x, y)
- Equal: *EQ(x, y)
- Partially Overlaps: *PO(x, y)
- Externally Connected: *EC(x, y)
- Tangential Proper Part: *TPP(x, y)
- Non-Tangential Proper Part: *NTPP(x, y)
- Converse of TPP: *TPP $^{-1}$(x, y)
- Converse of NTPP: *NTPP $^{-1}$(x, y)
- RCC-8 is NP-hard
  - Renz (2002) gives two algorithms for finding a consistent scenario for a set of constraints over the maximal tractable RCC-8 subsets, in O(n^3) time and O (n^2) time, respectively



Semantic Approximation, Granularity, & **Vagueness**

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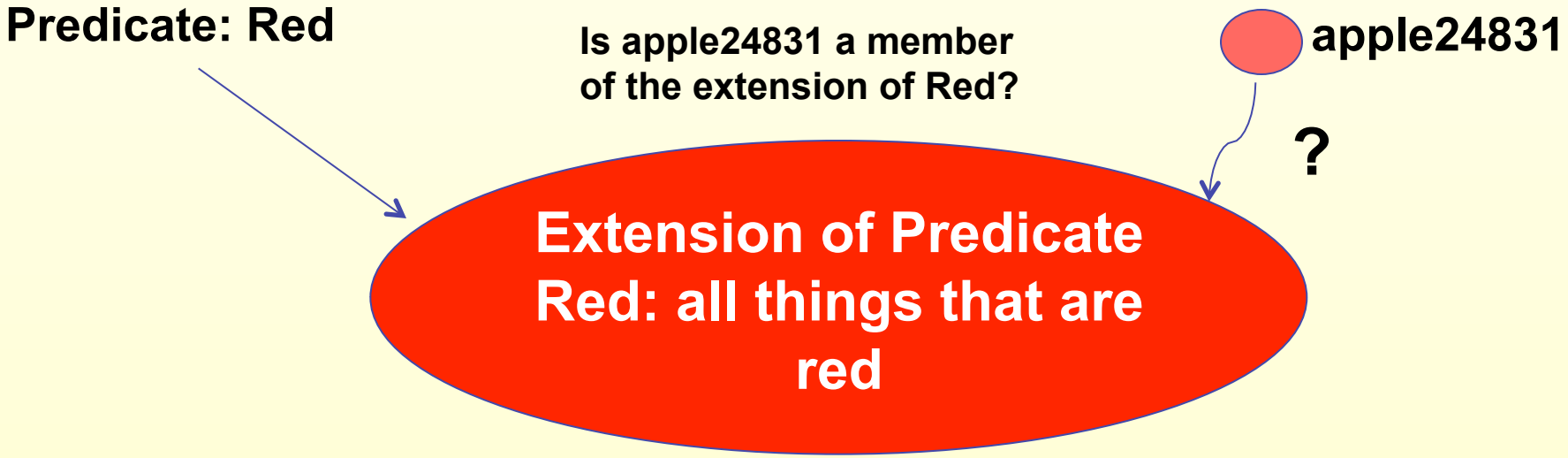
- **Vagueness vs. Imprecision: not same**
- **Vagueness: borderline cases**
- **Vagueness of spatial names & descriptions are not ontological, but instead are semantic (Varzi, 01)**
  - Ontological: the things themselves are vague, i.e., have fuzzy boundaries
  - Semantic: which things are you referring to? ; can be precisified?
  - Epistemic: don't know enough to decide, i.e., a form of ignorance?



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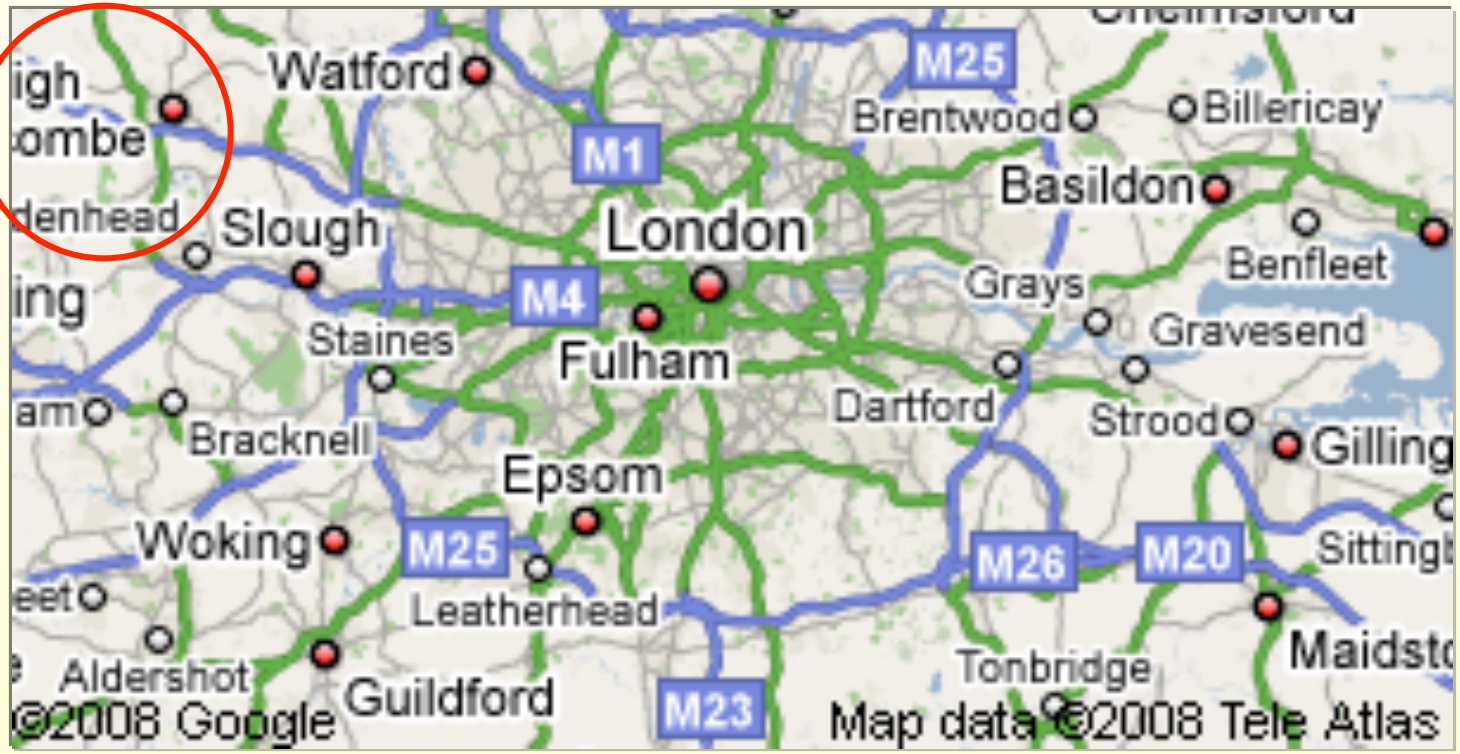
Greater London

- Is High Wycombe in Greater London?
- Is Oxford in Greater London?
- Is Glascow?



From Google Maps

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Multiple Kinds of Vagueness?

- Where does Mt. Everest begin?
  - Peak(s): clearly Mt. Everest
  - 10k feet altitude up Mt. Everest: clearly Mt. Everest
  - 10 feet altitude up Mt. Everest: clearly Mt. Everest
  - 100 feet away from that point? Still Mt. Everest?
  - 1 mile away from that point?
  - 10 miles away from that point?
  - I.e., the foot hills of Everest, the valley floor, etc.
- Semantic Vagueness: What constitutes being a "mountain"? A hill 3k feet high? Ok, how about a hill 2.9k feet high?
  - What is the referent of "Mt. Everest"?
- Mereotopological (part) Vagueness: Where does the mountain begin? Paradox of the heap (Sorites paradox)

Paradox of the Heap (Sorites Paradox)

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- What is a Heap? Example: a heap of sand
  - 1 grain of sand does not make a heap.
    If 1 grain of sand does not make a heap then 2 grains of sand do not.
    If 2 grains of sand do not make a heap then 3 grains do not.
    …
    If 9,999 grains of sand do not make a heap then 10,000 do not.

    10,000 grains of sand do not make a heap.
    - http://plato.stanford.edu/entries/sorites-paradox/
  - Above uses Modus Ponens inference rule
- Try it the other way:
  - 10k grains of sand make a heap
  - Take 1 grain away: 9, 999 grains are still a heap
  - …1 grain is a heap
- Notion of a Heap does not have sharp boundaries

How to Solve Sorites Paradox 1

- **Multi-value logics**
  - 3-valued: true, false, unknown, i.e., borderline statements' truth value is somewhere between true and false
  - Infinite-valued logic ("fuzzy logic"): a real number between 0 and 1
  - Then how do you calculate the truth value of compound statements having intermediate truth values: compositionality principle breaks down?
  - Semantics of a compound statement is the composition of the semantics of its components
- **Supervaluation: borderline cases have no truth value**
  - Retains classical logic's laws, but permits truth value gaps
  - Showing that a statement is not true does not guarantee that the statement is false
  - Is being supertrue (being true under all precisifications) sufficient for being true?
  - A vague predicate (e.g., 'mountain') is one that admits of various alternative "precisifications", i.e., one can always get more precise ("Yul Brynner was bald" is supertrue; "Bill Clinton is bald" is superfalse")

How to Solve Sorites Paradox 2

- Kamp's (1981) Contextualism
  - Boundaries in the extension of vague predicates will never be found
  - "Heap" is never interpreted so as to apply to one heap of sand rather than (an indistinguishable) other heap of sand
- Varzi (2001): Vagueness is semantic
  - Objects in the world have precise boundaries
  - But the terms we use to refer to the objects are vague
  - *De re: (of the thing)* The referent of a term is such that it is indeterminate whether certain chunks of reality lie within its boundaries
  - *De dicto (of the word)*: It is indeterminate whether certain chunks of reality lie within the boundaries of the referent of the term

How to Solve Sorites Paradox 3

- Bittner and Smith's (2001) Unified Theory of Granularity, Vagueness, and Approximation
  - Vagueness is semantic
  - All entities are crisp, but for a given vague term/predicate, there are potentially many, equally good candidates for its referent/ extension
  - Underscore the context dependence of vagueness
  - Using supervaluation, semantic evaluations are applied not to sentences, but to judgments which the sentences express (how they are used in specific contexts)
  - Frame a theory of Granularity Partitions: an ontology of foreground/background structure, to avoid "context"
    - **"Granular partitions are defined as systems of cells conceived as projecting onto reality … like the way in which a bank of flashlights projects onto reality when it carves out cones of light in the darkness"**
  - They develop then a theory of vague granular partitions

Spatial Vagueness: Egg-Yolk Theory

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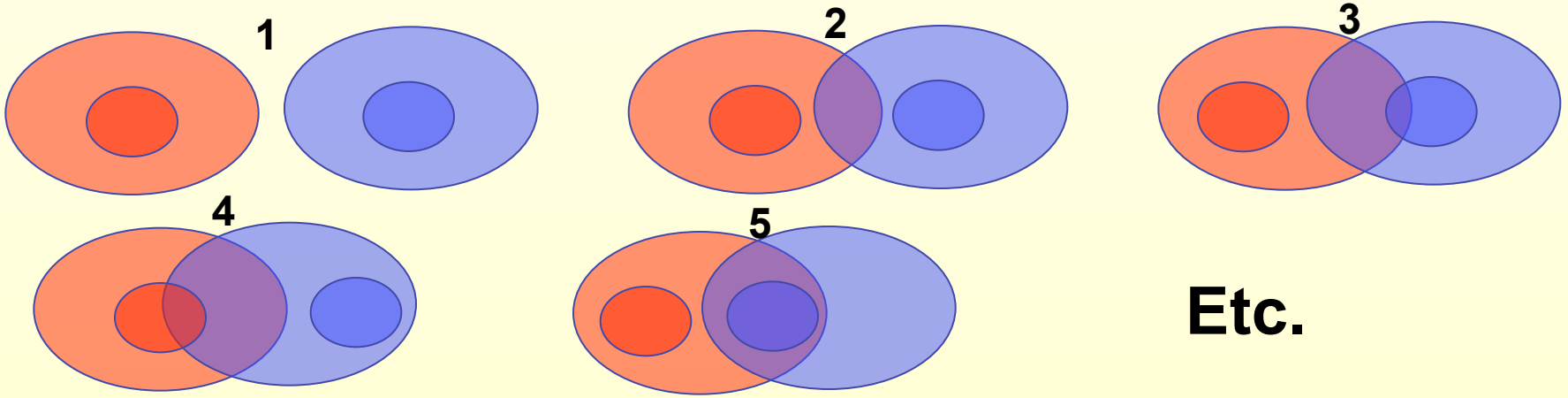
- Crispness of boundaries: Original RCC theory can't address noncrisp regions
- Egg-Yolk theory: originally, degrees of membership in a vague region (Lehman & Cohn, 1994), with 5 relations (RCC-5): DC(x,y), PO(x, y), PP(x,y), EQ(x,y), $TPP^{-1}(x, y)$
- Egg: the outer sub-region; Yolk: the inner sub-region
- "The egg & yolk of an egg-yolk pair are taken to represent conservatively defined limits on the possible complete crispings or precise versions of a vague region" (Cohn & Gotts, 1994)
- RCC-5: 46 possible relations between a pair of egg-yolk pairs



How Does This Apply to the Semantic Web? Many Questions

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- **Lukasiewicz & Straccia (2006, 2008): fuzzy description logics?**
- **Costa & Laskey (2006+): PR-OWL (Bayesian semantics)?**
- **But do fuzziness and probability apply to the ontological or to the epistemological?**
  - I.e., not that entities in the world are fuzzy or vague, so not degrees of truth
  - But that either there is a lack of knowledge (epistemic): not enough is known (we require evidence)
  - Or that the terms/predicates used to refer to entities in the world are vague, i.e. semantic
- **However, our Semantic Web (and other) ontologies as engineering products are approximate**
- **They involve both ontology (referents, the things that exist) and semantics (ways of referring to those things)**
- **Granular partition theory as an alternative to set theory and mereology? Vague reference is always reference to fiat boundaries (human demarcations)? --** Smith & Varzi (2000)

Summary

- We talked about:
  - Nearness (approximation)
  - Parts and Connectivity
  - Mostly about Vagueness
  - How does this apply to the Semantic Web?
- We also talked about some formal theories
  - RCC
  - Brief account of theories of Vagueness
  - Egg-Yolk Theory
  - SNAP and SPAN
  - Impact on Semantic Web?
- We have just barely skimmed the surface!

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Thanks!

- Questions?

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