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# Treatment of Biased and Dependent Sensor Data in Graph-based SLAM

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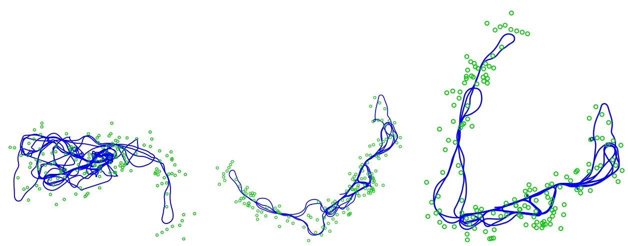
***Abstract*—A common approach to attack the simultaneous localization and mapping problem (SLAM) is to consider factor-graph formulations of the underlying filtering and estimation setup. While Kalman filter-based methods provide an estimate for the current pose of a robot and all landmark positions, graph-based approaches take not only the current pose into account but also the entire trajectory of the robot and have to solve a nonlinear least-squares optimization problem. Using graph-based representations has proven to be highly scalable and very accurate as compared with traditional filter-based approaches. However, biased measurements as well as unmodeled correlations can lead to a sharp deterioration in the estimation quality and hence require careful consideration. In this paper, a method to incorporate biased or dependent measurement information is proposed that can easily be integrated into existing optimization algorithms for graph-based SLAM. For biased sensor data, techniques from ellipsoidal calculus are employed to compute the corresponding information matrices. Dependencies among noise terms are treated by a generalization of the covariance intersection concept. The treatment of both biased and correlated sensor data rest upon the inflation of the involved error matrices. Simulations are used to discuss and evaluate the proposed method.**

***Index Terms*—SLAM, bias, correlations, covariance intersection, combined stochastic and set-membership models.**

## I. Introduction

Simultaneous localization and mapping (SLAM) has evolved into a central component of mobile robotics [1], [2] as it enables a robot to autonomously perform navigation and path-planning tasks in unknown environments. Many early and popular methods have employed filtering techniques to approach the SLAM problem. A filtering method, which is often based on the extended Kalman filter (EKF), provides an estimate that comprises the current pose of the robot and all landmark positions. Due to the sheer size of the corresponding matrix, updating and bookkeeping of this matrix often renders filtering methods intractable in large-scale environments. Much effort has been devoted to reduce the computational complexity. For instance, compressed versions of the EKF [3], combinations with covariance intersection (CI) [4], and also submap splitting and joining algorithms [5] can be named as approaches to cope with large environments. A viable alternative to filtering methods is the use of graph-based formulations of the SLAM problem, where not only the current pose but also the entire history is stored in the state vector. Although considering the entire trajectory seems at first to be cumbersome and contradictory, graph-based methods benefit from the conditional independence of the measurements, which can be exploited to obtain an extremely sparse representation of the joint error matrix. As a second major advantage, graph-based formulations are typically more robust to nonlinear process and sensor models. While filtering methods cannot revise previous linearization choices, graph-based methods essentially represent smoothing algorithms and, as such, provide smoothed estimates over the entire trajectory and all measurements. These smoothing approaches have proven to be highly efficient for large-scale environments.



(a) Effect of bias. (b) Naive solution. (c) Proposed method.

Fig. 1: Victoria park data set with (simulated) biased range measurements. In each case, the $g^2o$ framework [6] has been used.

Graph-based solutions to the SLAM problem consider a factored representation of the full joint probability distribution. For the FastSLAM algorithm [7], the problem structure of SLAM is reformulated as a Bayesian network, to which Rao-Blackwellized particle filters are applied. Factor graphs allow for the use of routines for nonlinear least squares optimization [8], such as Gauss-Newton iterations or the Levenberg-Marquardt algorithm, i.e., the computation of an optimal estimate can be posed as a nonlinear optimization problem. Such graph-based formulations have become a very popular way of modeling the SLAM problem. Nodes in the graph represent robot poses and landmarks while an edge between two nodes encodes odometry data or landmark observations, respectively. Important instances of graph-based methods are $\sqrt{\text{SAM}}$ [8], its incremental variant iSAM [9], and the g$^2$o framework [6].

SLAM approaches are primarily concerned with the question of how to compute an optimal estimate from noisy odometry and landmark measurements. However, there are a number of different challenges to take care of, which give rise to as many further developments. An important problem is to determine a good initial guess in order to solve the nonlinear least-squares problem. In [10], a decomposition into subproblems that can be solved robustly and more efficiently is used to find an initial guess and finally the global minimum. The incorporation of prior information about landmark positions is discussed in [11]. A related problem is the identification and treatment of data association outliers. Methods that feature an increased robustness to these outliers are based on a scaling of the covariance matrices [12], [13] or use switchable constraints [14], [15] that reduce the influence of association errors. Another typical assumption that is likely to be violated refers to the use of Gaussian factors, and [16] demonstrate how to also treat non-Gaussian noise terms in graph-based SLAM. In particular, nonlinearities introduced by the process model are investigated in [17].

In general, the efficiency of graph-based optimization algorithms heavily relies on the fundamental assumption of conditional independence, which is violated if sensor noise is correlated or systematic effects are present. Fig. 1(a) illustrates how biased landmark observations can impact the optimization result. For EKF-SLAM, [18] employ state augmentations to cope with measurements affected by a bias. Covariance inflation techniques have been proposed in [19] for the purpose of treating correlated factors as independent ones. In this paper, a concept for a systematic and robust treatment of biased and dependent sensor data is proposed, which continues the work in [19] and also integrates techniques from ellipsoidal calculus [20]. Against the background of filtering problems, a consistent generalization of the Kalman filter algorithm [21], [22] can be employed to cope with such perturbations of sensor data. Consequently, the aim of this paper is to apply this concept from filtering theory within the context of graph-based SLAM. As it can be seen in Fig. 1(b), a naive characterization of the bias by an increased error covariance matrix is not sufficient. The proposed concept, which is subject of the subsequent sections, leads to the improved result depicted in Fig. 1(c).

## II. Nonlinear Least Squares Optimization for Graph-based SLAM

In graph-based SLAM, an estimate for the entire trajectory and the positions of all landmarks is to be computed. In order to pose SLAM as an optimization problem, the factorized joint probability distribution

$$P(X|U,Z) = \underbrace{\prod_i P(\underline{x}_{i+1}|\underline{x}_i,\underline{u}_i)}_{\text{odometry measurements}} \underbrace{\prod_{ij} P(\underline{x}_j|\underline{x}_i,\underline{z}_{ij})}_{\text{loop closure/landmark meas.}} \quad (1)$$

is considered, where the set $X$ encompasses the entire trajectory $\underline{x}_0,\ldots,\underline{x}_N$ of the robot and the positions $\underline{x}_j$ of all landmarks. The set $U$ contains the odometry data $\underline{u}_i$ in the process model

$$\underline{x}_{i+1} = f_i(\underline{x}_i,\underline{u}_i) + \underline{\boldsymbol{w}}_i \ , \quad (2)$$

and $Z$ is the the set of observations. Each observation $\underline{z}_{ij}$ is related to the pose $\underline{x}_i$ and landmark position $\underline{x}_j$ by the sensor model

$$\underline{z}_{ij} = h_{ij}(\underline{x}_i,\underline{x}_j) + \underline{\boldsymbol{v}}_{ij} \ . \quad (3)$$

The noise terms $\underline{\boldsymbol{w}}_i$ and $\underline{\boldsymbol{v}}_{ij}$ are assumed to be Gaussian with covariance matrices $\mathbf{C}_i^w$ and $\mathbf{C}_{ij}^v$, respectively. If $\underline{x}_j$ is considered to be another robot's pose at an earlier time step instead of a landmark, $\underline{z}_{ij}$ is a loop-closure constraint, i.e., $\underline{z}_{ij}$ is a pose-to-pose measurement. A factor graph that only contains odometry and loop-closure constraints reduces to a pose graph. In fact, each factor graph can be transformed into a pose graph as a landmark being observed from different poses can be translated into a loop-closure constraint. In the following, we use the term loop closure for both landmark and pose-to-pose measurements.

In order to solve the SLAM problem, we strive for the set $X^*$ that maximizes the joint probability (1). With the normally distributed noise terms $\underline{\boldsymbol{w}}_i$ and $\underline{\boldsymbol{v}}_{ij}$, the factors in (1) are all Gaussian probability densities, and the optimization problem can be rewritten as

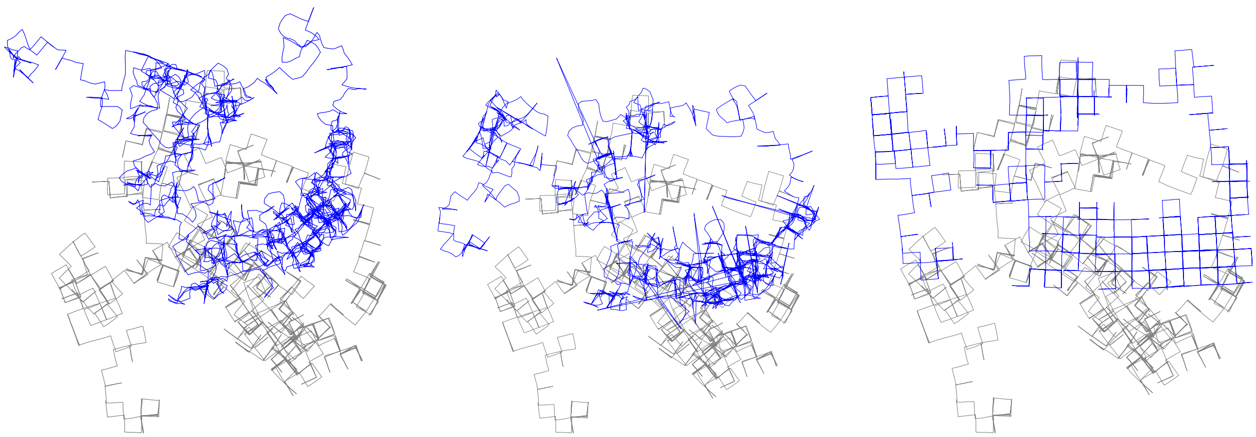
$$\begin{aligned} X^* &= \arg\max_X P(X|U,Z) \\ &= \arg\min_X \log P(X|U,Z) \\ &= \arg\min_X \sum_i \|\underline{x}_{i+1} - f_i(\underline{x}_i,\underline{u}_i)\|^2_{\mathbf{C}_i^w} \\ &\qquad + \sum_{ij} \|\underline{z}_{ij} - h_{ij}(\underline{x}_i,\underline{x}_j)\|^2_{\mathbf{C}_{ij}^v} \ , \end{aligned} \quad (4)$$

where the notation $\|\underline{x}\|^2_{\mathbf{C}} = \underline{x}^{\mathrm{T}}\mathbf{C}^{-1}\underline{x}$ has been used. Due to the typically nonlinear system and sensor models $f_i$ and $h_{ij}$, optimization methods like Gauss-Newton iterations or the Levenberg-Marquardt algorithm have to be employed to solve this least squares problem.

In general, solutions to (4) are derived and studied at a more abstract level [6]. Each type of constraints, be it an odometry or landmark observation, is characterized by an error

$$\mathbf{e}_{ij} \equiv \mathbf{e}_{ij}(\underline{x}_i,\underline{x}_j) \equiv \mathbf{e}_{ij}(\underline{x}_i,\underline{x}_j,\underline{z}_{ij}) \ ,$$

which has the error covariance matrix $\mathrm{E}[\mathbf{e}_{ij}\mathbf{e}_{ij}^{\mathrm{T}}] = \mathbf{C}_{ij}$ and where $\underline{z}_{ij}$ represents a virtual measurement between state



(a) Effect of bias. (b) Naive method. (c) Proposed method.

Fig. 2: Different solutions for the Manhattan3500 data set. Loop-closure measurements are affected by a (simulated) bias. The initial estimate is shown in gray.

components $\underline{x}_i$ and $\underline{x}_j$. For the observation model (3), the error has the form

$$\mathbf{e}_{ij}(\underline{x}_i, \underline{x}_j, \underline{z}_{ij}) = \underline{z}_{ij} - h_{ij}(\underline{x}_i, \underline{x}_j) = \underline{\boldsymbol{v}}_{ij} \tag{5}$$

and, for the process model (2), the error becomes

$$\mathbf{e}_{i(i+1)}(\underline{x}_i, \underline{x}_{i+1}, \underline{z}_{i(i+1)}) = \underline{x}_{i+1} - f_i(\underline{x}_i, \underline{u}_i) = \underline{\boldsymbol{w}}_i \ ,$$

where the odometry data $\underline{z}_{i(i+1)} = \underline{u}_i$ serve as virtual measurement. The objective function consequently reads

$$F(X) = \sum_{<i,j>\in\mathcal{G}} \underbrace{\mathbf{e}_{ij}(\underline{x}_i, \underline{x}_j)^{\mathrm{T}} \, \mathbf{\Omega}_{ij} \, \mathbf{e}_{ij}(\underline{x}_i, \underline{x}_j)}_{=:F_{ij}(X)} \ , \tag{6}$$

where the set $\mathcal{G}$ contains the pairs of indices for which a constraint exist and $\mathbf{\Omega}_{ij} = \mathbf{C}_{ij}^{-1}$ is the information matrix of the error, i.e., the each error term has the covariance matrix

$$\mathrm{E}[\mathbf{e}_{ij}\mathbf{e}_{ij}^{\mathrm{T}}] = \mathbf{C}_{ij} = \mathbf{\Omega}_{ij}^{-1} \ .$$

In the optimization routines, first-order Taylor series expansion are typically used to linearize the error functions $\mathbf{e}_{ij}(\underline{x}_i, \underline{x}_j)$. Due to the smoothing effect, graph-based methods have shown to be highly efficient for a number of nonlinear models like for instance bearing-only sensors. As it is discussed in the following section, conditional independence of sensor data is a major prerequisite for these approaches.

## III. Graph-based SLAM with Biased and Dependent Measurement Data

The factorization of the joint probability distribution (1) plays a key role in graph-based SLAM. Biased and dependent data violate this substantial assumption at the early beginning. More precisely, the loop closure constraints in (1), i.e., multiple observations of poses or landmarks, represented by $P(\underline{x}_j|\underline{x}_i, \underline{z}_{ij})$ have the product representation

$$P(X_{\mathcal{J}_i}|\underline{x}_i, Z_{i\mathcal{J}_i}) = \prod_{j\in\mathcal{J}_i} P(\underline{x}_j|\underline{x}_i, \underline{z}_{ij}) \tag{7}$$

as sensor noise is supposed to be conditionally independent given the state $\underline{x}_i$. In the above equation, $\mathcal{J}_i = \{j_1, \dots j_n\}$ contains all indices with $\langle i, j_1\rangle, \dots, \langle i, j_n\rangle \in \mathcal{G}$, i.e, the set $X_{\mathcal{J}_i}$ subsumes all landmarks or poses $\underline{x}_j$ that are observed from pose $\underline{x}_i$ and the set $Z_{i\mathcal{J}_i}$ represents the corresponding measurements. The assumption (7) of independence is not tenable if a systematic error is affecting measurements or correlated noise is present. Without the factored decomposition (7), the joint probability distribution (1) then has the undesirable form

$$P(X|U,Z) \propto \prod_i P(\underline{x}_{i+1}|\underline{x}_i, \underline{u}_i) \prod_i P(X_{\mathcal{J}_i}|\underline{x}_i, Z_{i\mathcal{J}_i}) \ , \tag{8}$$

which prohibits a sparse representation of the optimization problem.

The product probability representation (7) is directly linked to the assumption

$$\mathrm{E}[\mathbf{e}_{ik}\mathbf{e}_{il}^{\mathrm{T}}] = \mathbf{0}$$

for $k \neq l, k, l \in \mathcal{J}_i$. With respect to the objective function (6), this sparseness assumption is inevitable for efficient implementations of factor graphs. In the case of (8) that a factorization is not possible, non-zero cross-covariance terms are present, i.e., $\mathrm{E}[\mathbf{e}_{ik}\mathbf{e}_{il}^{\mathrm{T}}] \neq \mathbf{0}$. Hence, we have to consider the joint error vector

$$\mathbf{e}_{i\mathcal{J}_i} = \begin{bmatrix} \mathbf{e}_{ij_1} \\ \vdots \\ \mathbf{e}_{ij_n} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{v}}_{ij_1} \\ \vdots \\ \underline{\boldsymbol{v}}_{ij_n} \end{bmatrix} \ , \tag{9}$$

whose joint covariance matrix

$$\mathrm{E}[\mathbf{e}_{i\mathcal{J}_i}\mathbf{e}_{i\mathcal{J}_i}^{\mathrm{T}}] = \mathbf{C}_{i\mathcal{J}_i} = \mathbf{\Omega}_{i\mathcal{J}_i}^{-1} \tag{10}$$

lacks of a block-diagonal structure. In consequence, the missing conditional independence in (8) leads to

$$F_{i\mathcal{J}_i}(X) = \mathbf{e}_{i\mathcal{J}_i}^{\mathrm{T}} \mathbf{\Omega}_{i\mathcal{J}_i} \mathbf{e}_{i\mathcal{J}_i} \neq \sum_{j\in\mathcal{J}_i} \mathbf{e}_{ij}^{\mathrm{T}} \mathbf{\Omega}_{ij} \mathbf{e}_{ij} \ , \tag{11}$$

which renders the minimization of (6) significantly more difficult. In particular, the entire error function (9) has to be linearized. In the subsequent subsections, we examine how to restore the sparse structure in (10) for biased and correlated loop-closure measurements.

### A. Treatment of Bias (Set-membership Perspective)

In the presence of a bias $\mathbf{b}_{ij}$ that effects the sensor model (3), the error (5) becomes

$$\mathbf{e}_{ij}(\underline{x}_i, \underline{x}_j) = \underline{\boldsymbol{v}}_{ij} + \mathbf{b}_{ij} \ .$$

In general, the bias cannot be identified but can be conservatively represented by its membership to a set, which is chosen large enough to include the bias. Accordingly, the bias is assumed to be unknown but bounded by an ellipsoid, i.e., $\mathbf{b}_{ij} \in \mathcal{E}(\underline{0}, \mathbf{B}_{ij})$, where the notation

$$\mathcal{E}(\hat{\underline{c}}, \mathbf{X}) = \left\{ \underline{x} \mid (\hat{\underline{c}} - \underline{x})^{\mathrm{T}} \mathbf{X}^{-1} (\hat{\underline{c}} - \underline{x}) \leq 1 \right\}$$

is used. We first consider the case of single biased measurement from pose $\underline{x}_i$. The error covariance matrix can then conservatively be bounded by

$$\mathrm{E}[\mathbf{e}_{ij}\mathbf{e}_{ij}^{\mathrm{T}}] = \mathrm{E}[\underline{\boldsymbol{v}}_{ij}\underline{\boldsymbol{v}}_{ij}^{\mathrm{T}}] + \mathbf{b}_{ij}\mathbf{b}_{ij}^{\mathrm{T}} \leq \mathbf{C}_{ij} + \mathbf{B}_{ij} \ , \qquad (12)$$

where $\mathbf{A} \leq \mathbf{B}$ means that the difference $\mathbf{B} - \mathbf{A}$ is positive semidefinite. The corresponding component of the objective function (6) becomes

$$F_{ij} = \mathbf{e}_{ij}^{\mathrm{T}} \bar{\boldsymbol{\Omega}}_{ij} \mathbf{e}_{ij} = \mathbf{e}_{ij}^{\mathrm{T}} (\mathbf{C}_{ij} + \mathbf{B}_{ij})^{-1} \mathbf{e}_{ij} \ , \qquad (13)$$

with information matrix $\bar{\boldsymbol{\Omega}}_{ij} := (\mathbf{C}_{ij} + \mathbf{B}_{ij})^{-1}$. This modified information matrix corresponds to the common approach to incorporate additional uncertainties, i.e., the covariance matrix is simply increased by an additional matrix term. However, in the situation that multiple measurements from pose $\underline{x}_i$ are affected by a bias, Fig. 1(b) and Fig. 2(b) reveal that the modification (13) is not sufficient.

If multiple measurements from pose $\underline{x}_i$ are affected by bias terms $\mathbf{b}_{ij_1}, \ldots, \mathbf{b}_{ij_n}$, the loop-closure components (7) cannot be factorized anymore, and the joint distribution is of the form (8). The joint error vector (9) becomes

$$\mathbf{e}_{i\mathcal{J}_i} = \begin{bmatrix} \mathbf{e}_{ij_1} \\ \vdots \\ \mathbf{e}_{ij_n} \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{v}}_{ij_1} + \mathbf{b}_{ij_1} \\ \vdots \\ \underline{\boldsymbol{v}}_{ij_n} + \mathbf{b}_{ij_n} \end{bmatrix} ,$$

and the corresponding error matrix (10) yields

$$\mathrm{E}[\mathbf{e}_{i\mathcal{J}_i}\mathbf{e}_{i\mathcal{J}_i}^{\mathrm{T}}] = \begin{bmatrix} \mathbf{C}_{ij_1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{C}_{ij_n} \end{bmatrix} + \begin{bmatrix} \mathbf{b}_{ij_1}\mathbf{b}_{ij_1}^{\mathrm{T}} & \cdots & \mathbf{b}_{ij_1}\mathbf{b}_{ij_n}^{\mathrm{T}} \\ \vdots & \ddots & \vdots \\ \mathbf{b}_{ij_n}\mathbf{b}_{ij_1} & \cdots & \mathbf{b}_{ij_n}\mathbf{b}_{ij_n} \end{bmatrix} . \qquad (14)$$

The bound (12) cannot simply be applied to each component, i.e.,

$$\mathrm{E}[\mathbf{e}_{i\mathcal{J}_i}\mathbf{e}_{i\mathcal{J}_i}^{\mathrm{T}}] \nleq \begin{bmatrix} \mathbf{C}_{ij_1} + \mathbf{B}_{ij_1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{C}_{ij_n} + \mathbf{B}_{ij_n} \end{bmatrix} .$$

In order to derive a bound for the second matrix in (14), the Minkowski sum of ellipsoids has to be computed. Each bias term $\mathbf{b}_{ij_k}$ can be represented in the joint space by the degenerated ellipsoid

$$\begin{bmatrix} \underline{0} \\ \vdots \\ \mathbf{b}_{ij_k} \\ \vdots \\ \underline{0} \end{bmatrix} \in \mathcal{E}\left( \underline{0}, \begin{bmatrix} \mathbf{0} & & \cdots & & \mathbf{0} \\ & \ddots & & & \\ \vdots & & \mathbf{B}_{ij_k} & & \vdots \\ & & & \ddots & \\ \mathbf{0} & & \cdots & & \mathbf{0} \end{bmatrix} \right) \qquad (15)$$

As discussed, for instance, in [20] and [23], the sum of the bias terms (15) then lies in the outer ellipsoid

$$\begin{bmatrix} \mathbf{b}_{ij_1} \\ \vdots \\ \mathbf{b}_{ij_n} \end{bmatrix} \in \mathcal{E}\left( \underline{0}, p_{\mathrm{sum}} \begin{bmatrix} \frac{1}{p_{ij_1}}\mathbf{B}_{ij_1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{1}{p_{ij_n}}\mathbf{B}_{ij_n} \end{bmatrix} \right) \qquad (16)$$

with $p_{ij_k} > 0$ and $p_{\mathrm{sum}} = p_{ij_1} + \ldots + p_{ij_n}$. A trace-minimal approximation is given by $p_{ij_1} := \mathrm{trace}(\mathbf{B}_{ij_1})$ while other criteria such as the determinant require numerical methods. If it is the same bias affecting each measurement, the factor that inflates each shape matrix reduces to $p_{\mathrm{sum}} \cdot \frac{1}{p_{ij_1}} = n$. Eventually, the error matrix now has the bound

$$\mathrm{E}[\mathbf{e}_{i\mathcal{J}_i}\mathbf{e}_{i\mathcal{J}_i}^{\mathrm{T}}] \leq \begin{bmatrix} \mathbf{C}_{ij_1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{C}_{ij_n} \end{bmatrix} + \begin{bmatrix} \frac{p_{\mathrm{sum}}}{p_{ij_1}}\mathbf{B}_{ij_1} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \frac{p_{\mathrm{sum}}}{p_{ij_n}}\mathbf{B}_{ij_n} \end{bmatrix} =: \bar{\boldsymbol{\Omega}}_{i\mathcal{J}}^{-1} ,$$

which meets the sparsity assumption for graph-based SLAM. The term (11) retains the sum representation

$$F_{i\mathcal{J}_i}(X) = \mathbf{e}_{i\mathcal{J}_i}^{\mathrm{T}} \bar{\boldsymbol{\Omega}}_{i\mathcal{J}_i} \mathbf{e}_{i\mathcal{J}_i} = \sum_{j \in \mathcal{J}_i} \mathbf{e}_{ij}^{\mathrm{T}} \big(\mathbf{C}_{ij} + \tfrac{p_{\mathrm{sum}}}{p_{ij}}\mathbf{B}_{ij}\big)^{-1} \mathbf{e}_{ij}$$
$$= \sum_{j \in \mathcal{J}_i} F_{ij}(X) \ . \qquad (17)$$

This concept represents a generalization of the combined stochastic and set-membership Kalman filter in [21] and offers the advantage that the treatment of biased sensor data can directly be integrated into existing implementations of graph-based SLAM. In particular, the problem structure in (6) can be preserved, and only the information matrices need to be adapted according to (17). The derivation also shows that purely set-membership errors can be treated within graph-based SLAM.

### B. Treatment of Dependence (Bayesian Perspective)

Dependencies in the joint probability $P(X_{\mathcal{J}_i} | \underline{x}_i, Z_{i\mathcal{J}_i})$ can arise when observations to different poses or landmarks have correlated errors, i.e., $\mathrm{E}[\underline{\boldsymbol{v}}_{ij_k}\underline{\boldsymbol{v}}_{ij_l}] \neq \mathbf{0}$ for $k \neq l$, and hence, the product form (7) does not hold. However, this problem can be resolved by employing a weighted geometric mean [24], which can be regarded as a generalization of the covariance intersection algorithm [4]. If the conditional independence (7) does not hold, the conservative product representation

$$\tilde{P}(X_{\mathcal{J}_i}|\underline{x}_i, Z_{i\mathcal{J}_i}) \propto \prod_{j\in\mathcal{J}_i} P^{\omega_{ij}}(\underline{x}_j|\underline{x}_i, \underline{z}_{ij}) \qquad (18)$$

with $\sum_{j\in\mathcal{J}_i} \omega_j = 1$ and $\omega_j > 0$ can be used. The general idea has also been studied and discussed in [19] for pose graphs. The joint probability density now becomes

$$P(X|U,Z) = \prod_i P(\underline{x}_{i+1}|\underline{x}_i, \underline{u}_i) \prod_{ij} P^{\omega_{ij}}(\underline{x}_j|\underline{x}_i, \underline{z}_{ij})$$

and preserves the desired product structure. After taking the logarithm in (4), the weighted geometric mean (18) leads to

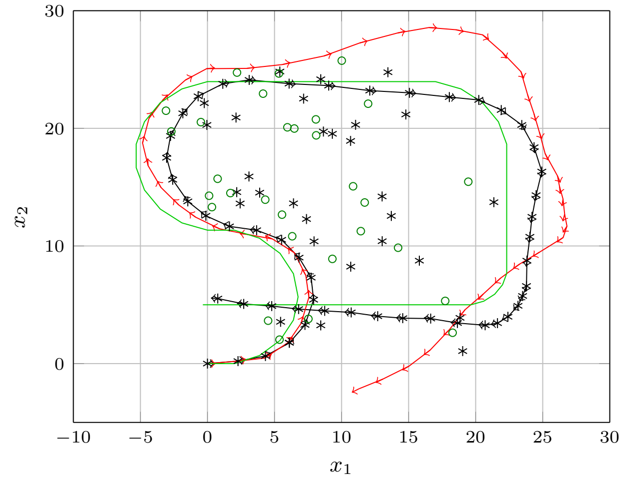
$$F_{i\mathcal{J}_i}(X) = \sum_{j\in\mathcal{J}_i} \omega_j \mathbf{e}_{ij}^{\mathrm{T}} \mathbf{\Omega}_{ij} \mathbf{e}_{ij} = \sum_{j\in\mathcal{J}_i} \mathbf{e}_{ij}^{\mathrm{T}} \left(\tfrac{1}{\omega_j}\mathbf{C}_{ij}\right)^{-1} \mathbf{e}_{ij} \ .$$

for the corresponding partial sums in the objective function (6). The parameter $\omega_{ij}$ can be chosen to minimize the trace of the joint covariance matrix, as in the case of a bias. By setting $\omega_{ij} = \frac{p_{\text{sum}}}{p_{ij}}$, it can immediately be seen that the same inflation technique has been used in (16) for the ellipsoidal shape matrices. More precisely, the treatment of both biased and dependent data relies on the same inflation technique of the corresponding error matrices. This result is expected and desired because a bias term also leads to correlations, as it can be seen in (14). In particular, biased and dependent sensor observations can be treated simultaneously.

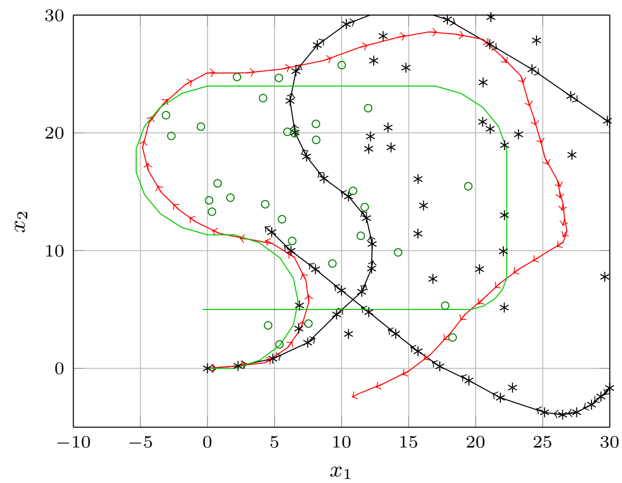
The proposed concept bears resemblance with dynamic covariance scaling [12] for data association outliers. In our work, we focus on biased and dependent landmark and loop-closure constraints and employ inflation techniques as a systematic approach to reestablish a factorized joint probability density such that graph-based optimization algorithms can exploit sparseness.

## IV. Example

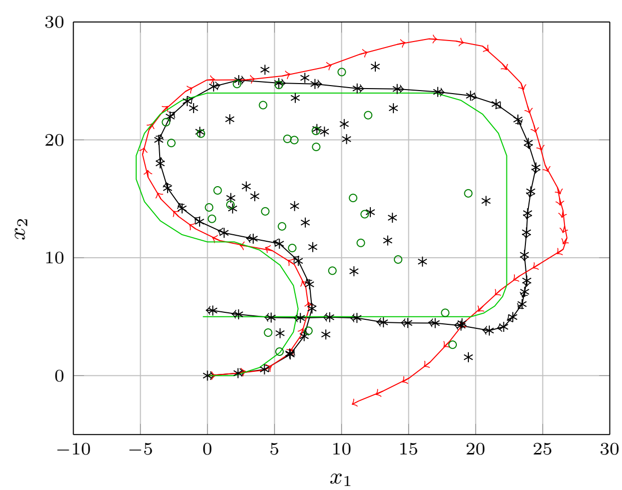
In order to discuss the proposed concept, three examples have been evaluated. Fig. 1 and Fig. 2 illustrate different optimization results with the Victoria park and the Manhattan3500 data set, respectively. The former data set has been altered by introducing a bias of 15m to 25 percent of the range measurements. As shown in Fig. 1(a), the bias leads to a poor estimate of the trajectory and the landmark positions. For Fig. 1(b), a bound on the bias is used to modify the information matrices as in (13). This modification does not take care of the off-diagonal blocks in (14), which are introduced by the bias, and still provides a poor estimate. Fig. 1(c) shows the improved result obtained by the proposed method, where the systematic inflation of the error matrix in (17) has been employed. Similar conclusions can be drawn from Fig. 2, where each loop-closure constraint has been altered by a bias of 2 in $x_1$-direction. Again, a naive consideration of a bound for the bias is not sufficient, as it can be seen in Fig. 2(b). With the proposed method, the enhanced estimate in Fig. 2(c)




(a) Result without bias.




(b) Result with biased range measurements.




(c) Result with biased range measurements that are treated by the proposed method.

Fig. 3: SLAM results for landmark-based tracking.

can be achieved. For both data sets, the $g^2o$ framework [6] has been employed.

A small-scale example is illustrated in Fig. 3, where the GTSAM library [25] has been utilized. Here, an object is tracked over 50 time steps. The actual trajectory and the landmark positions are depicted in green (—). The red (—) trajectory is reported by odometry measurements. At each time step, bearing-range measurements are performed while each landmark within the range of 10 is observed. In each figure, the estimation result is drawn in black (—). Due to the large number of observations at each time step, biased measurements have a strong effect on the estimation result. In Fig. 3(b) and Fig. 3(c), each range measurement is affected by a bias of 2. With the proposed method, the result in Fig. 3(c) is close to the optimal result shown in Fig. 3(a).

The examples demonstrate that biased measurements lead to correlations and have a strong effect on the SLAM result. A naive bound on the bias turns out to be not sufficient. However, a systematic treatment of biased and dependent information can easily be implemented by means of an inflation of the involved matrices, and the proposed concept does not require any specific modifications or customization of the optimization algorithms; only the information matrices have to be adapted to the required bound of the joint error matrix.

## V. Conclusion

Biased and dependent sensor data require careful attention in filtering and smoothing applications. As such, efficient implementations of graph-based SLAM strongly rely on a sparse representation of the dependency structure, which cannot be preserved if measurement errors are biased or correlated. In this paper, the block-diagonal structure of the joint error matrix is retained by employing an inflated matrix bound. In case of biased observations, this bound is obtained by means of a Minkowski sum of ellipsoids. For correlated sensor noise, a generalization of the covariance intersection algorithm has been utilized. Both cases rely on the same parameterization of the inflated matrix bound and can be treated simultaneously. While in this work a trace-minimal parameterization has been chosen, future work will also consider other choices of the parameters. In particular, the choice of a minimal parameter can be included in the optimization routines.

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