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# Determination, Separation, and Tracking of an Unknown Time Varying Number of Maneuvering Sources By Bayes Joint Decision-Estimation

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***Abstract*—This paper proposes a solution for the problem of determination, separation, and tracking of an unknown time-varying number of maneuvering sources based on a mixture of signals received by some omnidirectional sensors located at different places. To our knowledge, this problem has not been addressed in its full range: what has been addressed is limited to some simplified versions (e.g., with a fixed known number of sources) or with sensor arrays, which is different from our case (with omnidirectional sensors). This problem has one decision and two estimation subproblems: determine the number of sources (decision), estimate signals of different sources (separation), and estimate state vector of the sources (tracking), while the number of sources can change with time and their dynamic models are uncertain. These three subproblems are highly interrelated that solving one requires solutions of the other two. Therefore, they have to be considered jointly. Optimal Bayes joint decision and estimation (JDE) based on a generalized Bayes risk can handle such problems. However, here we have several additional difficulties, including *two* interrelated estimation subproblems, dynamic model uncertainty, correlated states of different sources, dependent dynamic models of different sources, nonlinearity of observation model, and two involved Markov process types (one for the number of sources and the other for the dynamic model). An approximate linear minimum mean square error estimator is derived to deal with the interrelated estimation subproblems. Having considered all the aforementioned issues, Bayes JDE required terms are derived based on a recursive calculation of some key terms. The proposed method is theoretically solid and simple for implementation. It is examined by simulations.**

**Keywords:** Joint decision and estimation, target tracking, LMMSE estimation, Markov process, source separation.[1]

## I. Introduction

The problem of jointly deciding on the number of concurrent sources, separating their signals, and tracking their state is a complicated task, which is of great interest in different applications including surveillance, teleconferencing, cocktail party, etc. One may categorize the existing literature on source separation and tracking roughly into three classes from the viewpoint of the assumptions on modeling of the problem. The first class assumes separation of signals of some static sources [1], [2], [3]. Generally, it is assumed that mixtures of signals from different sources are received by different sensors located at different places. Then the goal is to find an inverse of the mixing operator in order to recover the signals propagated by different sources. It is assumed that sources do not move. In the second class it is assumed that sources can move, but the number of sources is fixed and known. The third class addresses a more general problem in which the number of sources is unknown and may be time-varying [4], [5], [6], [7]; however, it is often assumed that there is no dynamic model uncertainty for the motion of the sources. Also, from the viewpoint of observation model and sensors, the existing publications are mostly based on sensor arrays located at different places, which can capture much more information about sources than omnidirectional sensors; however, they are more complicated and expensive.

The problem in this paper belongs to the third class. The problem is simultaneously deciding on the number of concurrent sources while the number may change over time, estimating their signals (separation), and tracking their state in the presence of dynamic model uncertainty based on observations received by omnidirectional sensors located at different places. We are not aware of any existing publication considering the whole problem, although there are papers on simplified versions of this problem (e.g., with a known fixed number of sources) or based on sensor arrays rather than omnidirectional sensors. Most publications on this topic are based on sensor arrays. Source separation and tracking tasks in this problem are different from those with sensor arrays. For example, in the latter after obtaining directions of arrival, tracking and separation become two separate tasks, meaning that first tracking can be done based on directions of arrival (similar to multitarget tracking) and then signals can be estimated using the results of tracking. Also, one may use beamforming and spatial filtering based on array processing for separation of signals. For omnidirectional sensors, however, an observation received by a sensor is a mixture of signals from all the sources plus noise. So, an observation corresponding to a specific source can not be distinguished from another. Therefore, one is supposed to estimate states and signals of sources jointly. Thus, this problem has three major subproblems: decision about the number of sources, estimation of source signals, and estimation of the sources states. In other words, this problem includes one decision subproblem and two estimation subproblems. These three subproblems are highly interrelated. Therefore, they have to be considered jointly. Furthermore, there are additional difficulties in this problem: There is uncertainty about the dynamic model of each source (unlike most publications in the third class) and given observations, dynamic models of different sources are not generally independent; given observations, states of different sources are not generally

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independent; observation models are nonlinear. According to the dimensions of the working space and the speed of signal propagation, it is assumed for simplicity that propagation of signals is instantaneous, and reverberations are negligible in the working space. This problem is naturally complex and has multiple uncertainties. Thus, computational complexity is an issue in this problem. So, computationally demanding methods are not desirable for either the whole structure of the problem (e.g., joint density estimation for determining the number of sources and estimating their signals and states) or a part of the problem (e.g., particle filter for nonlinear filtering). Therefore, for different parts of the problem we need approaches which are good in both performance and computation.

Bayes Joint decision and estimation based on a generalized Bayes risk [8] can optimally handle problems in which decision and estimation affect each other [9], [10]. Unlike some other methods for solving such problems (e.g., density estimation based methods), Bayes JDE directly addresses the desired goal — joint decision and point estimation. This is suitable for our problem. In other words, based on Bayes JDE we directly obtain a point estimate and do not first estimate the whole posterior density (which is a hard task) and then obtain a point estimate. However, Bayes JDE has not been applied to solving this problem. In other words, it is a new framework in the area of source separation and tracking. Although Bayes JDE can conceptually handle our problem, there are several difficulties in the derivation of its required terms for this problem, two of which are interrelated estimation subproblems and measurement nonlinearity. In order to deal with these two issues a linear minimum mean square error (LMMSE) estimator is derived (using an approximation) for state estimation in the presence of unknown signals and with a nonlinear observation model. The derived LMMSE estimator is new for this problem. The LMMSE estimator, which is the best linear estimator and has a low computational complexity, is derived in two steps: first, we obtain a prior state estimate of the sources in the presence of unknown signals based on a linear approximation, and then the final estimates of the states and signals are derived from their posterior joint density. Another point in derivation of the LMMSE estimator is: given observations, states of different sources are generally correlated and can not be considered separately. So, this correlation must be considered in the derivation.

In order to deal with changes in the number of sources, we model the sequence of the number of sources as a Markov process. Also, we model the evolution of the dynamic model of the sources (as a group) by Markov processes to cope with the dynamic model uncertainty. Multiple model approach can not be easily applied to this problem the way it is applied in a typical target tracking problem. One reason is that two types of Markov processes are involved in this problem, and another reason is that given observations, dynamic models of different sources are not generally independent. Thus, another issue in the derivation of the Bayes JDE required terms (including posterior state estimate given a hypothesis, and posterior hypothesis probability) is that two types of Markov processes are involved in this problem. Derivations of the essential terms based on these two types of Markov processes are new results in this problem. Finally, having considered all the issues, the required terms in the JDE framework are derived based on a recursion of some key terms.

The paper is organized as follows. Section II is for problem description and modeling. Section III considers a simple version of the whole problem in which the number of sources is fixed and known and the dynamic models are also known. An LMMSE estimator is derived for state estimation in the presence of unknown signals. Also, the signal estimator is presented. Section IV considers the whole problem. In this section a recursive JDE is presented for simultaneous decision about the number of concurrent sources and estimation of their states and signals. Due to space limitation, we omit details of the derivation of equations presented in section III and IV. Simulations are presented in section V.

## II. Problem Description and Modeling

Consider a working space with $N$ omnidirectional sensors located at different places and $M_k$ moving sources at time $k$. Each source's motion follows its own dynamic model and the number of sources is unknown and may change with time. A source may appear or disappear in time. Propagation of a signal from each source is omnidirectional and signals from different sources are independent. Also, observation noises of different sensors are Gaussian and uncorrelated. Each sensor receives a mixture (a linear combination) of signals from multiple sources. The coefficient corresponding to the signal from source $m \in \{1, ..., M_k\}$ received at sensor $n \in \{1, ..., N\}$ is a nonlinear function of the source location $p^{so}_{m,k} = (x^{so}_{m,k}, y^{so}_{m,k}, z^{so}_{m,k})$ and sensor location $p^{se}_n = (x^{se}_n, y^{se}_n, z^{se}_n)$. Note that we do not consider time index for the sensors. However, sensors can move as far as we know their locations. Let $s_{m,k}$ be the signal of source $m$ at time $k$, and $v_{n,k}$ the observation noise of sensor $n$ at time $k$. The observation model is

$$o_{n,k} = \sum_{m=1}^{M_k} h(p^{se}_n, p^{so}_{m,k}) s_{m,k} + v_{n,k} \quad , \quad n = 1, ..., N \quad (1)$$

where $o_{n,k}$ is the observation received by sensor $n$ at time $k$ and $h(p^{se}_n, p^{so}_{m,k})$ is the mixing coefficient for the signal from source $m$ at sensor $n$ at time $k$. We assume there are enough sensors to avoid the unobservability problem. Equation (1) can be written in matrix form as

$$O_k = H_k S_k + V_k \quad (2)$$

where $O_k = [o_{1,k}, o_{2,k}, ..., o_{N,k}]'$ is the observation vector, $H_k = [h_{1,k} \quad h_{2,k} \quad \dots \quad h_{M_k,k}]$ is the matrix of coefficients where $h_{m,k} = [h(p^{se}_1, p^{so}_{m,k}), h(p^{se}_2, p^{so}_{m,k}), \dots, h(p^{se}_N, p^{so}_{m,k})]'$, $S_k = [s_{1,k}, s_{2,k}, ..., s_{M_k,k}]'$ is the vector of source signals, and $V_k = [v_{1,k}, v_{2,k}, ..., v_{N,k}]'$ is the observation noise vector. In addition, we have

$$O_k = \sum_{m=1}^{M_k} h_{m,k} s_{m,k} + V_k \quad (3)$$

Let $X_{m,k}$ be the state vector of source $m$ at time $k$. The linear dynamic motion of source $m$ in the Cartesian coordinates can be modeled as

$$X_{m,k} = F_{m,k} X_{m,k-1} + G_{m,k} w_{m,k-1} \quad (4)$$

where $w_{m,k-1}$ is zero mean Gaussian noise with variance $Q_{m,k-1}$. We assume the dynamic noises of all the sources have the same variance ($Q_{m,k} = Q_k$). In a 3D space we consider dynamic models along different axes being decoupled. So,

$$F_{m,k} = diag(F^x_{m,k}, F^y_{m,k}, F^z_{m,k})$$
$$G_{m,k} = [G^x_{m,k}{}', G^y_{m,k}{}', G^z_{m,k}{}']'$$

For example, for a nearly constant velocity model along $x$ axis for source $m$ at time $k$ we have

$$F^x_{m,k} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \quad G^x_{m,k} = [\frac{T^2}{2}, T]' \tag{5}$$

We call all the concurrent sources together a *meta source* [2], and when we want to emphasize the number of sources in a meta source we denote it as $\mathrm{MS}_j$, which means a meta source consisting of $j$ sources. So, the evolution of the state vector of the meta source can be written as

$$X_k = F_k X_{k-1} + \Upsilon_k \tag{6}$$

where $F_k = diag(F_{1,k}, F_{2,k},..., F_{M_k,k})$, $\Upsilon_k = [\Upsilon'_{1,k}, \Upsilon'_{2,k},... , \Upsilon'_{M_k,k}]'$, and $\Upsilon_{m,k} = G_{m,k} w_{m,k-1}$. Also, $X_k = [X'_{1,k}, X'_{2,k}, ..., X'_{M_k,k}]'$ is the meta source state vector. The signal from source $m$ is assumed to be Gaussian with mean $\mu_m$ and variance $\sigma^2_m$: $s_{m,k} \sim N(\mu_m, \sigma^2_m)$. Since signals from different sources are independent, for the signal vector we have

$$S_k \sim N(\mu, P_S) \tag{7}$$

where $P_S = diag(\sigma^2_1, \sigma^2_2, ..., \sigma^2_{M_k})$ and $\mu = [\mu_1, \mu_2, ..., \mu_{M_k}]'$. We assume that the signals parameters $\mu$ and $P_S$ are known. The observation noise for all sensors is $V \sim N(0, R)$, where $R$ is the covariance matrix. As mentioned, each mixing coefficient is usually assumed to be a function of the positions of the corresponding source and sensor. Let

$$r_{m,n,k} = \left[(x^{se}_n - x^{so}_{m,k})^2 + (y^{se}_n - y^{so}_{m,k)})^2 + (z^{se}_n - z^{so}_{m,k})^2\right]^{\frac{1}{2}}$$

Then, according to the propagation model of the signals, the following model is considered for the mixing coefficients:

$$h(p^{se}_n, p^{so}_{m,k}) = g(r_{m,n,k}) = \frac{1}{\sqrt{r^2_{m,n,k} + d^2}} \tag{8}$$

where $d$ is a constant in order to prevent the denominator from going to zero when a source is very close to a sensor. When a source is very close to a sensor the corresponding coefficient between the source and sensor is almost 1. Thus, we simply assume $d = 1$.

## III. Determination, Separation, and Tracking of an Unknown Time-Varying Number of Maneuvering Sources

We present the proposed method for the whole problem in two steps in this section. In the first step (subsection A), a solution for the problem of separation and tracking of a known fixed number of non-maneuvering sources is presented. Then in subsection B, the whole problem is addressed.

[2]As we will see, since the observation vector is a mixture of data from multiple sources, tracking of the sources (specifically the update step) should be done considering all of them together. So, this name is useful.

### *A. Separation and Tracking of a Known Fixed Number of Non-maneuvering Sources*

In this subsection, it is assumed that the number of sources is known and fixed, and dynamic models of sources are known. In other words, in equation (3) source signals and their mixing coefficients are unknown. $h_{m,k}$ is a vector-valued nonlinear function of the position of source $m$ at time $k$. Thus, equation (3) is nonlinear in the meta source state vector $X_k$, and the signals $s_{m,k}$ are also unknown. The goal is to estimate the state vector $X_k$ and the signal vector $S_k$. To do so, an idea is: considering $S_k$ as a nuisance parameter, estimate $X_k$, and then estimate $S_k$ based on the estimate of $X_k$. However, it is not easy to handle the integrals involved in this approach. In addition, the integrals do not have a closed form and an approximation is needed. Most integral approximation methods are computationally demanding for such a problem. Furthermore, there are different kinds of uncertainties involved in our problem (regarding the source states, source signals, number of sources, and dynamic model for each source) which make it complicated already. Therefore, a less computationally complex approach is desired. We derive an LMMSE estimator [11], [12] of $X_k$ in the presence of unknown signals, based on an approximation. Then, based on the estimate of $X_k$ as prior information, we estimate $X_k$ and $S_k$ based on their joint posterior density.

The LMMSE meta source state estimator is based on the following equations

$$\begin{aligned} \hat{X}^p_k &= E^*[X_k|\hat{X}_{k-1}, O_k] \\ &= \hat{X}_{k|k-1} + C_{[X,O]_{k|k-1}} C^{-1}_{O_{k|k-1}} (O_k - \hat{O}_{k|k-1}) \end{aligned} \tag{9}$$

$$\begin{aligned} P^p_k &= E[(X_k - \hat{X}^p_k)(X_k - \hat{X}^p_k)'] \\ &= P_{k|k-1} - C_{[X,O]_{k|k-1}} C^{-1}_{O_{k|k-1}} C'_{[X,O]_{k|k-1}} \end{aligned} \tag{10}$$

where "$E$" denotes expectation operator, $E^*[X_k|O^k]$ denotes the LMMSE estimator of $X_k$ given observations $O^k$ (observations up to time $k$) [11], $\hat{X}_{k-1}$ is available from time $k-1$ (21), and the superscript "p" indicates that this is a preliminary step for estimation. Here,

$$\begin{aligned} \tilde{X}_k &= X_k - \hat{X}_{k|k-1} \\ \tilde{O}_k &= O_k - \hat{O}_{k|k-1} \\ \hat{X}_{k|k-1} &= E^*[X_k|\hat{X}_{k-1}] \\ \hat{O}_{k|k-1} &= E^*[O_k|\hat{X}_{k-1}] \\ P_{k|k-1} &= cov(\tilde{X}_k) \\ C_{O_{k|k-1}} &= cov(\tilde{O}_k) \\ C_{[X,O]_{k|k-1}} &= cov(\tilde{X}_k, \tilde{O}_k) \end{aligned}$$

In the following, we calculate the required terms in the LMMSE estimator. For state prediction we have

$$\begin{aligned} \hat{X}_{k|k-1} &= F_k \hat{X}_{k-1} \\ P_{k|k-1} &= F_k P_{k-1} F'_k + G_k Q_{k-1} G'_k \end{aligned}$$

where $G_k = [G'_{1,k}, G'_{2,k}, ..., G'_{M,k}]'$. For observation prediction

$$\hat{O}_{k|k-1} \approx \sum_{m=1}^{M} \hat{h}_{m,k|k-1} \mu_m \tag{11}$$

where a Taylor series expansion has been used to calculate the mean of $h_{m,k}$, $\mu$ was defined in (7), and $\hat{h}_{m,k|k-1} = h_{m,k}|_{X_k=\hat{X}_{k|k-1}}$. Also, it should be noticed that since the number of sources is assumed known and fixed in this subsection, we drop time index $k$ for $M_k$. It can be shown that

$$C_{[X,O]_{k|k-1}} \approx \sum_{m=1}^{M} P_{k|k-1}\breve{H}'_{m,k}\mu_m \tag{12}$$

$$\begin{aligned} C_{O_{k|k-1}} \approx& \sum_{m=1}^{M}\Big[\sigma_m^2 \hat{h}_{m,k|k-1}\hat{h}'_{m,k|k-1} \\ &+ \sigma_m^2 \breve{H}_{m,k}P_{k|k-1}\breve{H}'_{m,k}\Big] \\ &+ \sum_{m=1}^{M}\sum_{l=1}^{M}\Big[\breve{H}_{m,k}P_{k|k-1}\breve{H}'_{l,k}\mu_m\mu_l\Big] + R \end{aligned} \tag{13}$$

where $\breve{H}_{m,k} = \frac{\partial h_{m,k}}{\partial X'_k}|_{X_k=\hat{X}_{k|k-1}}$. Thus, the LMMSE meta source state estimator, based on the approximation above, is obtained by substituting the terms into (9) and (10).

Now, meta source signal and state can be estimated based on their joint posterior density given observations and the available prior information about the state as follows:

$$p(X_k, S_k|O^k, \mathcal{P}_k) \propto p(X_k|S_k, O^k, \mathcal{P}_k)p(S_k|O^k, \mathcal{P}_k) \tag{14}$$

where $\mathcal{P}_k = \{\hat{X}_k^p, P_k^p\}$ denotes the prior information available based on the output of the LMMSE estimator (9) and (10). Then, we have

$$\begin{aligned} p(S_k|O^k, \mathcal{P}_k) &= \frac{p(O_k|S_k, O^{k-1}, \mathcal{P}_k)}{p(O_k|O^{k-1}, \mathcal{P}_k)}p(S_k|O^{k-1}, \mathcal{P}_k) \\ &\propto N(O_k; \hat{H}_k^p S_k, R)N(S_k; \mu, P_S) \propto N(S_k; \hat{S}_k, \hat{P}_{S,k}) \end{aligned} \tag{15}$$

where $\hat{H}_k^p = H_k|_{\hat{X}_k^p}$ is an estimate of the mixing coefficient matrix in (2), and

$$\hat{S}_k = \mu + \hat{P}_{S,k}\hat{H}_k^{p\prime}R^{-1}(O_k - \hat{H}_k^p\mu) \tag{16}$$

$$\hat{P}_{S,k} = P_S - P_S\hat{H}_k^{p\prime}(\hat{H}_k^p P_S \hat{H}_k^{p\prime} + R)^{-1}\hat{H}_k^p P_S \tag{17}$$

The final point estimate of the meta source state can be obtained based on the posterior density (14) conditioned on the estimated signal vector $\hat{S}_k$ as

$$\begin{aligned} &p(X_k|\hat{S}_k, O^k, \mathcal{P}_k) \propto \\ &p(O_k|X_k, O^{k-1}, \hat{S}_k, \mathcal{P}_k)p(X_k|O^{k-1}, \hat{S}_k, \mathcal{P}_k) \end{aligned} \tag{18}$$

Therefore, similar to (9) and (10) we can calculate the LMMSE estimator of $X_k$, but this time based on the new prior which is the output of the previous LMMSE estimator $\{\hat{X}_k^p, P_k^p\}$:

$$\begin{aligned} \hat{X}_k &= E^*[X_k|\hat{X}_{k-1}, O_k, \hat{S}_k, \mathcal{P}_k] \\ &= \hat{X}_k^p + \breve{C}_{[X,O]_{k|k-1}}\breve{C}^{-1}_{O_{k|k-1}}\breve{O}_k \end{aligned} \tag{19}$$

$$\begin{aligned} P_k &= E\Big[(X_k - \hat{X}_k)(X_k - \hat{X}_k)'|\hat{S}_k, \mathcal{P}_k\Big] \\ &= P_k^p - \breve{C}_{[X,O]_{k|k-1}}\breve{C}^{-1}_{O_{k|k-1}}\breve{C}'_{[X,O]_{k|k-1}} \end{aligned} \tag{20}$$

where

$$\begin{aligned} \breve{X}_k &= X_k - \hat{X}_k^p \\ \breve{O}_k &= O_k - E^*[O_k|\hat{X}_{k-1}, \hat{S}_k, \mathcal{P}_k] \\ \breve{C}_{[X,O]_{k|k-1}} &= cov(\breve{X}_k, \breve{O}_k|\hat{S}_k, \mathcal{P}_k) \\ \breve{C}_{O_{k|k-1}} &= cov(\breve{O}_k|\hat{S}_k, \mathcal{P}_k) \end{aligned}$$

Then, the final estimate of $X_k$ is

$$\hat{X}_k = \hat{X}_k^p + A_k B_k\Big(O_k - \sum_{m=1}^{M}\hat{h}_{m,k}\hat{s}_{m,k}\Big) \tag{21}$$

$$P_k = P_k^p - A_k B_k A'_k \tag{22}$$

with

$$\begin{aligned} A_k &= \sum_{l=1}^{M} P_k^p \breve{H}^p_{l,k}{}'\hat{s}_{l,k} \\ B_k &= \Big(\sum_{m=1}^{M}\sum_{l=1}^{M}\Big[\breve{H}^p_{m,k}P_k^p\breve{H}^p_{l,k}{}'\hat{s}_{l,k}\hat{s}_{m,k}\Big] + R\Big)^{-1} \end{aligned}$$

where $\hat{s}_{m,k}$ is the $m$th element of $\hat{S}_k$ in (16), $\breve{H}^p_{m,k} = \frac{\partial h_{m,k}}{\partial X'_k}|_{X_k=\hat{X}_k^p}$, and $\hat{h}_{m,k} = h_{m,k}|_{X_k=\hat{X}_k^p}$.

Therefore, with estimation of $X_k$, the solution for the problem of separation and tracking of a known fixed number of non-maneuvering sources is complete. In the next subsection, we consider the whole problem.

### *B. Determination, Separation, and Tracking of Unknown Time Varying Number of Maneuvering Sources*

In this subsection, the problem presented in subsection A is extended to the case in which the number of sources is unknown and time-varying and the meta source dynamic model is uncertain. As explained, this is a joint decision and estimation (JDE) problem. We solve it based on optimal Bayes JDE method [8].

***Optimal Bayes JDE:*** We briefly explain the optimal Bayes JDE framework [8] for a generic problem. Consider $\mathcal{N}$ hypotheses $\{\mathcal{H}^1, \mathcal{H}^2, ..., \mathcal{H}^{\mathcal{N}}\}$ and $\mathcal{M}$ decisions $\{\mathcal{D}^1, \mathcal{D}^2, ..., \mathcal{D}^{\mathcal{M}}\}$. The Bayes JDE risk is

$$\bar{R} = \sum_{i}^{\mathcal{M}}\sum_{j}^{\mathcal{N}}\Big(\alpha_{ij}c_{ij} + \beta_{ij}E[C^e_{ij}(u,\hat{u})|\mathcal{D}^i, \mathcal{H}^j]\Big)P(\mathcal{D}^i, \mathcal{H}^j) \tag{23}$$

where $u$ is the estimand and $\hat{u}$ is its estimate. $c_{ij}$ is the cost of the $i$th decision while the $j$th hypothesis is true. $\alpha_{ij}$ and $\beta_{ij}$ are weights of decision and estimation costs, respectively. $C^e_{ij}(u,\hat{u})$ is estimation cost fucntion.

***Optimal JDE Solution:*** To minimize $\bar{R}$ in (23), the optimal decision $\mathcal{D}$ is

$$\mathcal{D} = \mathcal{D}^i \quad if \quad C_i(o) \le C_l(o), \forall l \tag{24}$$

where $o$ is the observation and the cost is given by

$$C_i(o) = \sum_{j=1}^{\mathcal{N}}\Big(\alpha_{ij}c_{ij} + \beta_{ij}E[C^e_{ij}(u,\hat{u})|\mathcal{D}^i, \mathcal{H}^j]\Big)P(\mathcal{H}^j|o) \tag{25}$$

Given any set of regions $\{\Gamma_1, ..., \Gamma_M\}$ of the data space, the optimal estimate is available. For simplicity, we consider the set of decision regions forming a partition of the data space. Then the optimal estimator based on (23) with $C^e_{i,j}(u, \hat{u}) = (u - \hat{u})'(u - \hat{u})$ is given by

$$\hat{u} = \sum_{i=1}^{\mathcal{M}} 1(o; \Gamma_i)\check{u}_i \tag{26}$$

$$\check{u}_i = \sum_{j=1}^{\mathcal{N}} E(X|o, \mathcal{H}_j) \frac{\beta_{ij} P(\mathcal{H}_j|o)}{\sum_{l=1}^{N} \beta_{il} P(\mathcal{H}_l|o)}, \quad o \in \Gamma_i$$

$$1(o; \Gamma_i) = \begin{cases} 1 & \text{o} \in \Gamma_i \\ 0 & \text{else} \end{cases}$$

and $\check{u}_i$ is undefined if $o \notin \Gamma_i$. The optimal Bayes joint decision-estimate $(\mathcal{D}, \hat{u})$ is the joint of the above optimal decision and estimate.

***Determination, separation, and tracking of an unknown time-varying number of maneuvering sources based on a recursive Bayes JDE:*** Provided that the number of sources is known, the meta source state and signal vectors can be estimated based on the approach presented in subsection A ((21) and (22)). Since this is a problem with a dynamic system, it is better to use a recursive version of the Bayes JDE [10] for implementation. We assume that at most $\mathcal{M}$ concurrent sources are possible in the working space and model the sequence of the number of sources by a Markov process having known initial probabilities and known transition probabilities:

$$P(\mathcal{H}^i_k|\mathcal{H}^j_{k-1}) = [\Pi_{\mathcal{H}}]_{ji} \quad i, j \in \{1, ..., \mathcal{M}\}$$

where $\mathcal{H}^i_k$ denotes the event that the number of concurrent sources at time $k$ is $i$, and $\Pi_{\mathcal{H}}$ is the Markov transition probability matrix. For simplicity, we assume that at each time at most one change occurs in the number of sources, although an extension of our formulation to any number of changes is straightforward. Also, in order to deal with the uncertainty in a meta source dynamic model we define a Markov process to model the evolution of the dynamic model [13], where the initial probabilities are assumed known. Also, transition probabilities are known and for an $\text{MS}_j$ are

$$P(m^u_k|m^v_{k-1}) = [\Pi^j_m]_{vu} \quad u, v \in \{1, ..., (N_d)^j\}$$

where $m^u_k$ denotes the event that the dynamic model of the $\text{MS}_j$ at time $k$ is the $u$th one. $N_d$ is the number of possible dynamic models for each source. One can consider each possible dynamic model of an $\text{MS}_j$ as a combination of dynamic models of $j$ different sources. In the Bayes JDE framework for this problem, we assume that the set of possible decisions is the same as that of hypotheses. Based on (23), the estimation cost function should be determined for the cases in which the decisions are correct, and also the cases in which they are incorrect. We consider the estimation cost function as

$$C^e_{ij}(X_k, \hat{X}_k) = \frac{1}{j}(X_k - \hat{X}_k)'(X_k - \hat{X}_k), \quad i = j$$

and $C^e_{ij} = \gamma$ for $i \neq j$, where $\gamma$ is a design parameter. Also, normalization by the number of sources $j$ is used because we do not want to have a higher cost for tracking more sources [14].

It should be noticed that the Bayes JDE required terms should be appropriately derived (considering two types of Markov processes) so that the terms can be recursively calculated. Due to space limitation we skip the details of the derivations and just present the final results. Through the following steps we explain our recursive Bayes JDE for the problem of determination, separation, and tracking of an unknown time-varying number of maneuvering sources.

1) Initialization:
   $\hat{X}^{\mathcal{H}^j, m^u}_0 \equiv \hat{X}^{j,u}_0$, $\forall j$, $\forall u$: State estimate of $\text{MS}_j$ with the $u$th dynamic model at time 0, where $j \in \{1, 2, ..., \mathcal{M}\}$ and $u \in \{1, 2, ..., (N_d)^j\}$.
   $P^{j,u}_0$, $\forall j$, $\forall u$: Corresponding error covariance.
   $P(\mathcal{H}^j_0)$, $\forall j$: Probability that the number of concurrent sources is $j$ at time 0.
   $\xi^{ij}_0 = E(C^e_{ij}(X_0, \hat{X}_0)|\mathcal{D}^i_0, \mathcal{H}^j_0)$: Expected estimation cost if the truth is $j$ and the decision is $i$ about the number of concurrent sources, where $i \in \{1, 2, ..., \mathcal{M}\}$.
   $P(m^u_0|\mathcal{H}^j_0)$: Probability of the dynamic model $u$ being the true one for $\text{MS}_j$ at time 0.
2) Assume the following terms are available from the previous time $k-1$:

$$\hat{X}^{j,u}_{k-1}, P^{j,u}_{k-1}, P(\mathcal{H}^j_{k-1}|O^{k-1}), P(m^u_{k-1}|O^{k-1}, \mathcal{H}^j_{k-1})$$

   along with the posterior cost for decision at time $k-1$:

$$C_i(O^{k-1}) = \sum_{j=1}^{\mathcal{M}} c^{ij}_{k-1} P(\mathcal{H}^j_{k-1}|O^{k-1}) \tag{27}$$
$$c^{ij}_{k-1} = \alpha_{ij} c_{ij} + \beta_{ij} \xi^{ij}_{k-1}$$

   The decision regions based on decision costs (27) are denoted as $\{\Gamma^1_{k-1}, ..., \Gamma^{\mathcal{M}}_{k-1}\}$, which is a partition, where $\Gamma^i_{k-1}$ denotes the region for decision $i$ at time $k-1$.
3) After receiving $O_k$, the terms in step 2 should be updated to obtain

$$\hat{X}^{j,u}_k, P^{j,u}_k, P(\mathcal{H}^j_k|O^k), P(m^u_k|O^k, \mathcal{H}^j_k) \tag{28}$$

4) To modify the decision regions, hypothesis probabilities in (27) are updated in (28). So, we have

$$C^*_i(O^k) = \sum_{j=1}^{\mathcal{M}} c^{ij}_{k-1} P(\mathcal{H}^j_k|O^k) \tag{29}$$

   where $C^*$ denotes the intermediate cost in which the hypothesis probabilities have been updated to $P(\mathcal{H}^j_k|O^k)$), but the costs $c^{ij}_{k-1}$ have not been updated to $c^{ij}_k$ yet. Decision regions are also modified based on the intermediate cost as $\{\Gamma^{*1}_k, ..., \Gamma^{*\mathcal{M}}_k\}$, where

$$\Gamma^{*i}_k = \{O_k : C^*_i(O^k) \leq C^*_l(O^k), \forall l\}$$

5) According to our estimation cost function, it can be shown that the expected estimation cost conditioned on a hypothesis and decision is calculated as

$$\xi^{ij}_k = \frac{1}{j} E\Big[(X_k - \hat{X}_k)'(X_k - \hat{X}_k)\Big|\mathcal{D}^i_k, \mathcal{H}^j_k\Big], \quad i = j$$

and it is equal to $\gamma$ for $i \neq j$, where $E\Big[(X_k - \hat{X}_k)'(X_k - \hat{X}_k)\Big|\mathcal{D}_k^i, \mathcal{H}_k^j\Big]$ for $i = j$ is available via the tracking filter.

6) Decision regions are updated using the updated decision costs

$$C_i(O^k) = \sum_{j=1}^{\mathcal{M}} c_k^{ij} P(\mathcal{H}_k^j | O^k)$$
$$c_k^{ij} = \alpha_{ij} c_{ij} + \beta_{ij} \xi_k^{ij}$$
$$\Gamma_k^i = \{O_k : C_i(O^k) \leq C_l(O^k), \forall l\}$$

7) The final decision-estimate is $(\mathcal{D}_k^d, (\hat{X}_k, P_k))$, where

$$C_d(O^k) \leq C_i(O^k) \quad \forall i \neq d$$

$$\hat{X}_k = \arg\min_X \sum_{j=1}^{\mathcal{M}} \Big(\beta_{dj} E[C_{dj}^e(X_k, X) | \mathcal{D}_k^d, \mathcal{H}_k^j] \cdot P(\mathcal{D}_k^d, \mathcal{H}_k^j)\Big)$$

Based on our estimation cost function it can be shown that $\hat{X}_k = \hat{X}_k^d$, $P_k = P_k^d$, which can be calculated based on the terms in (28). The superscript $d$ means "decision $d$ has been made about the number of concurrent sources".

8) For the next time $k+1$, go to step 2, with $X_k^{j,u}$, $P_k^{j,u}$, $P(\mathcal{H}_k^j | O^k)$, and $P(m_k^u | O^k, \mathcal{H}_k^j)$.

## IV. Performance Evaluation Measures

Some measures are considered for performance evaluation of the proposed method. The first one is joint performance measure (JPM) [10] for simultaneous evaluation of decision and estimation. We also consider the average over the outputs of the decision part over different Monte Carlo runs to evaluate decision performance. Furthermore, we use average Euclidean-error (AEE) measure [15] for evaluating the estimation part (source tracking and signal estimation). However, it should be noticed that JPM is the best one because in this problem joint performance is the goal. The JPM is the expectation of a distance between observation vector and the predicted observation vector by the JDE algorithm as follows:

$$\rho(O_k, \hat{O}_{k|k-1}) = E[d(O_k, \hat{O}_{k|k-1})] \tag{30}$$

where $d$ is the Euclidean distance between two vectors. Joint performance measure (30) can be computed using sample average over $N_{JDE}$ number of predicted observations generated by JDE and $N_{MC}$ Monte Carlo runs of the algorithm as

$$\rho(O_k, \hat{O}_{k|k-1}) \approx \frac{1}{N_{MC}} \frac{1}{N_{JDE}} \sum_{j=1}^{N_{MC}} \sum_{i=1}^{N_{JDE}} d(O_k^j, \hat{O}_{k|k-1}^{i,j}) \tag{31}$$

Also, AEE for position or signal estimate evaluation is

$$d_{AEE}(Y_k, \hat{Y}_k) = \frac{1}{N_{MC}} \sum_{j=1}^{N_{MC}} d(Y_k^j, \hat{Y}_k^j) \tag{32}$$

where $Y$ can be, for example, the position or signal vector of a meta source, and the superscript $j$ denotes the $j$th Monte Carlo run. When the determined number of sources in a meta source is correct, AEE computation is straightforward. However, when the determined number of sources is incorrect, AEE is computed as follows. For example, if the true number ($n$) of sources is smaller than the estimated one ($m > n$), we choose a subset (including $n$ sources) of the estimated meta source which leads to less AEE in position. Of course, if the true number of sources is larger than the estimated one, for some of the sources there is no AEE computation.

## V. Simulations

In order to demonstrate performance of the proposed method we consider the following scenario. In our proposed method we assume that the problem is observable. Therefore, in the simulations, for observability of the problem first we considered several sensors located at different places. However, later based on simulation results it turned out that for our setting in this scenario, the method can work even based on 8 omnidirectional sensors located at $(0, 0)$, $(0, 50)$, $(0, 100)$, $(50, 0)$, $(50, 100)$, $(100, 0)$, $(100, 50)$, and $(100, 100)$. But generally for observability and better performance in different cases, one may use more sensors. For simplicity, we assume that there are at most two concurrent sources, and the diagonal elements of the transition probability matrix of the Markov process for the number of concurrent sources are 0.99 and off-diagonals 0.01. Also, to handle the uncertainty in the dynamic model, for each source we consider three models along each axis: one nearly constant velocity ($M_1$) and two nearly constant acceleration models with accelerations -1 ($M_2$) and 1 ($m/s^2$) ($M_3$), respectively [16]. The transition probability matrix of a Markov process corresponding to the dynamic model of a meta source is determined considering the transition probability matrix for the dynamic model of each source along each axis as

$$\begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.05 & 0.95 & 0 \\ 0.05 & 0 & 0.95 \end{bmatrix}$$

where the probability of direct transition from $M_2$ to $M_3$ (and vice versa) is zero. In the simulated scenario, source 1 is present at the beginning (Fig. 1). Then, at time 10 source 2 appears and is present until time 20 when it disappears at time 21. From time 1 to 10 source 1 moves at a nearly constant velocity, at time 11 it changes to have a nearly constant acceleration until time 27 when it switches back to a nearly constant velocity motion. Source 2 follows a model with a constant acceleration all the time during its presence. The parameter values in this scenario are as follows. Sampling interval is 0.5 (second), the dynamic noise standard deviation is 0.003, and observation noise standard deviation for each sensor is 0.2. We consider signals of different sources being independent with the same Gaussian distribution $N(220, 3500)$. These are the parameters of signals at the sources, not at the sensors, because a sensor receives signal from a source multiplied by the corresponding coefficient (attenuation) plus some observation noise. Decision cost coefficients in JDE are $c_{ij} = 1$ for $i \neq j$ and 0 otherwise. For relative weights of decision and estimation we consider $\alpha_{ij} = 1$ for every $i$ and $j$ and $\beta_{ij} = 0.8$ for $i = j$ and 0.7 otherwise. The number of predicted observations generated by Bayes JDE at each time (for JPM computation) is $N_{JDE} = 10$, the number of Monte

Carlo runs is $N_{MC} = 100$. The design parameter $\gamma$ in the estimation cost function is set to 8. In the derivation of the Bayes JDE equations, we assume that at each time at most one change occurs in the number of concurrent sources. Then, a prior disappearance probability of each source is required. These probabilities can be automatically calculated according to the distances of the sources to the exit area of the working space (one can also incorporate other aspects into these probabilities). In other words, the farther from the exit area, the less probable to disappear. In the simulation, we consider disappearance probability being inversely proportional to the distance to the exit area. Also, the exit area is around the corner $(100, 0)$. Moreover, in the derivation of the Bayes JDE equations a prior density for the initial state $(x_0, \dot{x}_0, y_0, \dot{y}_0)$ of a newborn source is required. In the simulation, we assume that this prior density is a multivariate Gaussian with mean equal to the true value and covariance matrix $diag(10, 1, 10, 1)$ (the entrance area is around the origin corner). Also, prior probabilities of the number of sources (at the beginning) are set to be equal for both hypotheses.

We are not aware of any other existing method proposed for solving this problem of determination, separation, and tracking of an unknown time varying number of maneuvering sources using omnidirectional sensors. In order to compare performance of our method with a benchmark, we run an algorithm in which the number of sources is known at each time.

Fig. 2 shows the JPM result for our Bayes JDE based method in comparison with the algorithm that knows the number of sources (ideal). As it can be seen, at the beginning the error of JDE is larger than that of the ideal one, because JDE has no prior information about the number of sources. Then, the difference between the two algorithms becomes negligible until time 10 when the number of sources changes. Due to this change, JDE error increases since its prediction about the number of sources is not good at time 10, but its performance gets better soon. Since the ideal algorithm knows the number of sources all the time, its error does not change much. The reason for the relative increase in the error of the ideal algorithm at time 10 is that estimating signals and states of two concurrent sources is more difficult than that of one source. Then, at time 21 source 2 disappears and the performance of JDE degrades. However, JDE recognizes the change in the number of sources within a couple of time steps and again its performance gets better. Except for time steps at which the number of sources changes and a couple of the transient time steps after a change, the difference between JDE and the ideal algorithm is small.

Fig. 3 shows the average decision output of JDE (over Monte Carlo runs) about the number of concurrent sources at each time. Fig. 4 shows AEE of position estimates and Fig. 5 shows AEE of signal estimates for the proposed method in comparison with the ideal one. AEE computation for different cases is based on section IV. Therefore, when there exist two sources and JDE does not decide correctly, there is no AEE computation for one of the sources. Therefore, it is clear that AEE of the estimates, or decision output, can not illustrate the whole performance of the method well, and JPM is the best measure for this problem.

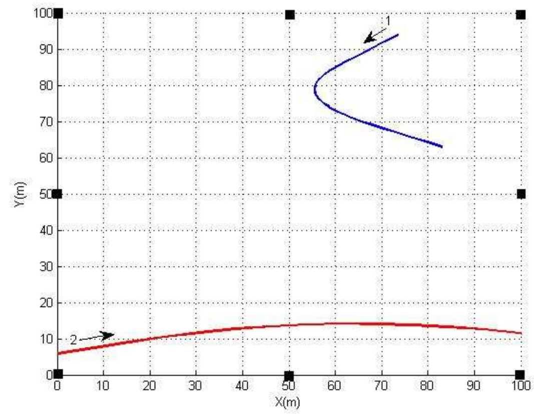


Figure 1. Working space with two sources

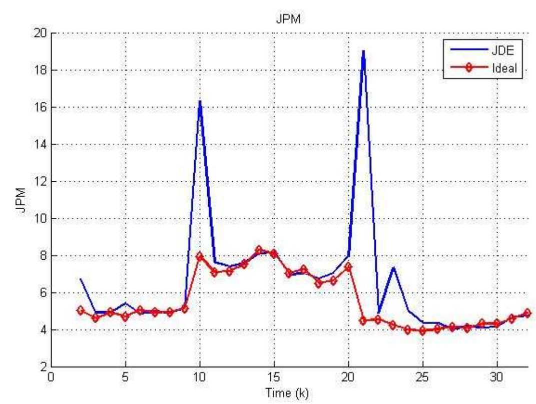


Figure 2. Joint performance measure, JDE in comparison with ideal algorithm that knows the number of sources over time. Changes in the number of sources happen at time 10 and 21.

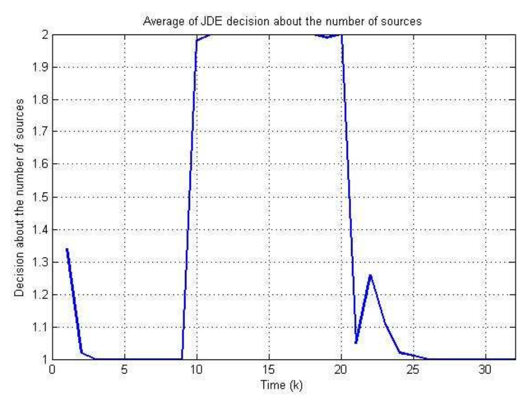


Figure 3. Average over JDE decision about number of sources over time. The number of sources: one (time 1-9), two (time 10-20), one (time 21 on)

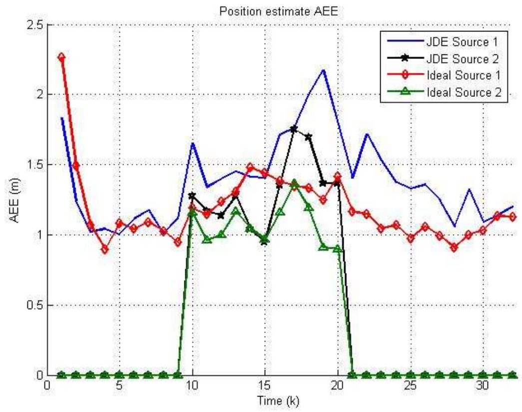


Figure 4. AEE of position estimates of sources. Source 2 exists only over time 10 to 20.

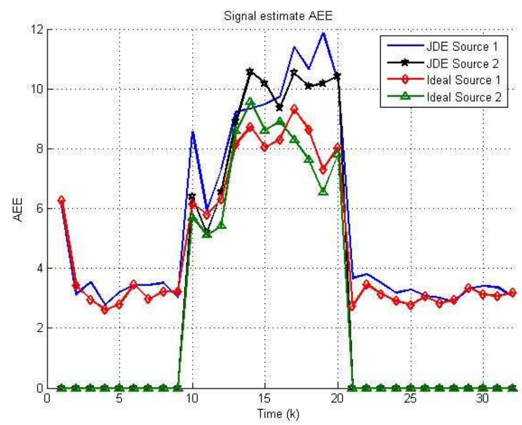


Figure 5. AEE of signal estimates of sources. Source 2 exists only over time 10 to 20.

## VI. SUMMARY

A solution has been proposed for the problem of determination, separation, and tracking of an unknown time-varying number of maneuvering sources using observations received by omnidirectional sensors. This problem is complex and thus computationally demanding approaches are not desirable here. In other words, a theoretically solid and easily implementable approach is desirable. Determination of the number of sources and estimation of their state and signal vectors by minimizing Bayes joint decision and estimation risk is theoretically solid and simple. So, Bayes JDE is quite desirable for this problem. Also, the Bayes JDE method is flexible enough to address the problem. Moreover, Bayes JDE is a new approach in the area of source separation and tracking.

An LMMSE estimator has been derived for the estimation part of the problem, which is a new result for this problem. This estimator is theoretically solid and the best among linear estimators. According to the results, the approximation is adequate for this case, although a higher order approximation is possible without much difficulty. The derived estimators for the state and signal vectors are not complex at all, which is desirable for this problem. The multiple model approach is a powerful method for handling dynamic model uncertainty of a maneuvering target. However, it is not easily applicable in this problem due to the uncertainty in the number of sources. Therefore, the corresponding equations for handling changes in the number of sources and dealing with maneuvers of the sources have been jointly derived in a recursive form which is also a new result for this problem.

In order to evaluate performance of the proposed method, a comprehensive joint performance measure (JPM) has been used, since other measures can not quantify the whole performance and they just consider some aspects separately [15]. The results show that the proposed method is effective and performs well in comparison with the ideal one. Therefore, the proposed method has different desirable properties we have been looking for: theoretically solid and simple for implementation.

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