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Spatial Clutter Measurement Density Estimation in Nonhomogeneous Measurement Spaces

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***Abstract*—Clutter measurement density (CMD) is one of data association parameters, which indicates the number of clutter measurements per unit volume of the measurement space. In probabilistic data association based algorithms, the association probability between a prior estimate and a measurement is proportional to the ratio of target measurement likelihood and CMD. Also the measurement likelihood is used for obtaining the target existence probability for false track discrimination. Although CMD is an important parameter for state estimation as well as track management, it depends on surveillance environments in which the true CMD is rarely known in advance. A clutter measurement density estimator (CMDE) calculates the spatial density of clutter adaptively using measurement information, and provides its estimated CMD to data association algorithms for adaptive target tracking in clutter. A spatial CMDE (SCMDE) selects the measurement with the N-th smallest 2-norm distance from the measurement of interest and evaluates volume of the hypersphere centered at the measurement of interest and touches the selected measurement. The sparsity (inverse of CMD) is obtained from dividing the hypersphere volume by N. It is only applicable to homogeneous measurement spaces of which coordinates have the same unit such as Cartesian coordinates. An improved version of SCMDE which can be utilized in nonhomogeneous measurement spaces with the different coordinate units such as polar coordinates is proposed. By using weighted normal distance that reflects the volume of the nonhomogeneous measurement space, the proposed SCMDE calculates the ellipsoidal volume for each measurement of interest. Performance of the proposed SCMDE is verified by Monte Carlo simulations for various cases.**

***Keywords*—Clutter Measurement Density, Adaptive Target Tracking in clutter, Spatial CMDE**

I. Introduction

Clutter measurement density (CMD) implies the expected number of clutter measurements in the unit volume of measurement space, which is an important parameter to obtain the posterior probability density function of target state in clutter. In [1]–[4], the measurement likelihood is divided by CMD to calculate the measurement likelihood ratio which is used to stochastically discriminate between target detections and the clutter. The measurement likelihood ratio determines target existence probability of the existing tracks for track management [3]–[6]. In realistic situations, CMD is rarely known as prior information.

A CMD estimator (CMDE) is needed for adaptive target tracking in apriori unknown environments. The CMDE may be explicit, implicit or embedded in target tracking algorithms. Most track based CMDEs consider CMD as non-parametric and evaluate CMD using the number of measurements in a validation gate of each existing track and its spatial volume. Since valid measurements which belong to the validation gate of one track may include the target measurements, [7] suggested the conditional mean estimation to get the expected number of target measurements using the probability of target existence by a similar method to Integrated Probabilistic Data Association (IPDA) of [3]. The authors also introduced the number of target measurement estimation based on maximum likelihood method and method of moment for calculating CMD. Although this track based CMDE provides good results in calculating the expected number of clutter measurements in the validation gate, the calculated CMD depends on volume of the validation gate so that the CMD of the interest point may not identical from scan to scan even for uniform cluttered environments.

Multiscan CMDEs are proposed to get more robust performance than single scan CMDEs. In [8], the clutter map consists of several cells composed by partitioning surveillance space. The clutter map counts the falling measurements per each cell during a few scans to average the number of measurements at each cell. The area of cell is a constant so that the CMD estimation of the clutter map is independent to the existing tracks. Also probability hypothesis density filters based CMDE are proposed in [9]. They consider the clutter measurement as the detection of ‘clutter generator’ and estimate its posterior density.

Comparing with the above existing CMDEs, the spatial CMDE (SCMDE) proposed in [10] estimates the inverse of CMD (termed as ‘sparsity’) at each measurement point. The SCMDE is a measurement oriented CMDE that evaluates the sparsity of each measurement. Performance of the existing CMDE depends on the volume of the validation gate while that of the SCMDE depends on the relative distance between measurements. Therefore, the SCMDE is independent of target tracking performance and produces consistent CMD estimates. The sparsity of each measurement is calculated by the mean volume of hypersphere of which radius is Euclidean distance between the measurement and its neighbor. The SCMDE provides the adaptive estimation performance to target tracking filters though the distribution of clutter measurement is nonuniform, which enhances false track discrimination of integrated probabilistic data association filters. However, the limitation of the SCMDE is mentioned in [11] that the algorithm is valid only for homogeneous measurement spaces because it measures the Euclidean distance between measurement pairs.

To eliminate this limitation, we propose the improved version of the SCMDE which can measure CMD in non-homogeneous measurement spaces as well as non-uniform cluttered environments. The proposed algorithm uses a normalized distance between the interesting point and one of its neighbor measurement to calculate the sparse volume of the interesting point in non-homogeneous measurement spaces. When the measurement space is homogeneous, the proposed algorithm is identical to the early version of the SCMDE. The proposed algorithm makes the SCMDE generalized for nonlinear tracking problems, i.e. active sonar/radar tracking.

The relation of clutter measurement sparsity and data association is presented in Section II. The early version of SCMDE in homogeneous measurement spaces is summarized in Section III. Section IV shows the detailed representation of the proposed SCMDE for non-homogeneous measurement spaces. Performance evaluation of the proposed algorithm is carried out by Monte Carlo simulations for various scenarios in Section V followed by conclusions in Section VI.

II. Role of Spatial Clutter Measurement Density in Data Association

In this paper, ρ indicates CMD and sparsity, an inverse of CMD, is denoted by γ. Both are functions of the point of interest z in the measurement space, the sparsity at the interesting point z is defined by

$$\gamma(z) = \frac{1}{\rho(z)} = \frac{V(z)}{m} \tag{1}$$

where $V(z)$ and m denote the interest volume of z and the number of clutter measurement in the area respectively.

Among the variety of existing data association methods, IPDA is selected as an example to explain the role of CMD in data association and false track discrimination.

IPDA is divided into two parts; state estimation and track management. Former part contains track prediction, measurement selection and update which are identical to those of standard PDA [12]. Latter part includes track initiation, track confirmation/termination and track merging. The detailed derivation and expression of IPDA are represented in [3] and [13].

In track update step of the state estimation part, the data association probability between track τ and one of its valid measurement $\mathbf{z}_{k,i}$ is expressed by

$$\beta_{k,i}^{\tau} = \frac{P_D^{\tau} p_{k,i}^{\tau} / \rho(\mathbf{z}_{k,i})}{\Lambda_k^{\tau}} = \frac{P_D^{\tau} p_{k,i}^{\tau} \gamma(\mathbf{z}_{k,i})}{\Lambda_k^{\tau}} \tag{2}$$

where P_D^{τ}, $p_{k,i}^{\tau}$ and Λ_k^{τ} denote the probability of target detection, target measurement likelihood and measurement likelihood ratio of track τ respectively. Using the data association probability, each track provides the relative weight to each measurement for distinguishing (expected) target measurement and clutter measurement stochastically. Due to the fact that the probability is proportional to the inverse of CMD, the incorrect estimation of CMD reduces tracking accuracy.

Measurement likelihood ratio of track τ is defined by

$$\Lambda_k^{\tau} = 1 - P_D^{\tau} P_G^{\tau} + \sum_{l=1}^{m_k^{\tau}} P_D^{\tau} p_{k,l}^{\tau} \gamma(\mathbf{z}_{k,l}) \tag{3}$$

where P_G^{τ} is a gating probability and m_k^{τ} is the number of selected measurements for track τ. The posterior target existence probability for track τ, $P\left\{\chi_k^{\tau} | \mathbf{Z}^k\right\}$, is calculated by

$$P\left\{\chi_k^{\tau} | \mathbf{Z}^k\right\} = \frac{\Lambda_k^{\tau} P\left\{\chi_k^{\tau} | \mathbf{Z}^{k-1}\right\}}{1 - (1 - \Lambda_k^{\tau}) P\left\{\chi_k^{\tau} | \mathbf{Z}^{k-1}\right\}}, \tag{4}$$

where χ^{τ} denotes the hypothesis of target existence for track τ.

The status of each track is determined by its target existence probability in track management part: confirmation, tentativeness, or termination. With the incorrect measurement likelihood ratio, false tracks which are initialized by the clutter measurements may be confirmed and true tracks which follows interesting targets may be terminated.

III. Spatial Clutter Measurement Density Estimator in homogeneous measurement spaces

The early version of the SCMDE procedure published in [10] is represented in this section and the SCMDE is denoted as "the standard SCMDE" in this paper. The inverse of CMD, sparsity, at the point of interest $\mathbf{z}_{k,i}$ given the measurement set $\mathbf{Z}_k$ at time k is evaluated.

If the order of sparsity is N, we first look up the distance between $\mathbf{z}_{k,i}$ and the N-th nearest measurement from $\mathbf{z}_{k,i}$ among $\mathbf{Z}_k$.

$$r_{\min}(\mathbf{z}_{k,i}; N) = \min_{\substack{N\text{th} \\ j \neq i}} \|\mathbf{z}_{k,j} - \mathbf{z}_{k,i}\| \tag{5}$$

where $\|\mathbf{x}\|$ is 2-norm distance of residual vector $\mathbf{x}$.

Denote by $V^*(\mathbf{z}_{k,i}; N)$ the volume of a hypersphere centered at $\mathbf{z}_{k,i}$ which "touches" the N-th nearest measurement.

$$V^*(\mathbf{z}_{k,i}; N) = C_M \left(r_{\min}(\mathbf{z}_{k,i}; N)\right)^M \tag{6}$$

where M denotes the dimension of measurement space, and C_M denotes the unit sphere volume in the measurement space. C_M is described with Gamma function $\Gamma(\cdot)$ and measurement dimension such as

$$C_M = \frac{\pi^{M/2}}{\Gamma\left(1+M/2\right)} = \frac{2}{M}\frac{\pi^{M/2}}{\Gamma\left(M/2\right)} \tag{7}$$

which results in constant $C_1 = 2$, $C_2 = \pi$ and $C_3 = 4\pi/3$.

Since the N numbers of measurements are positioned in the volume of (6), the estimated sparsity (inverse of CMD) equals

$$\hat{\gamma}^*_{k,i} = \frac{V^*\left(\mathbf{z}_{k,i};N\right)}{N} \tag{8}$$

Figure 1 shows the illustration of the SCMDE in the homogeneous measurement space.

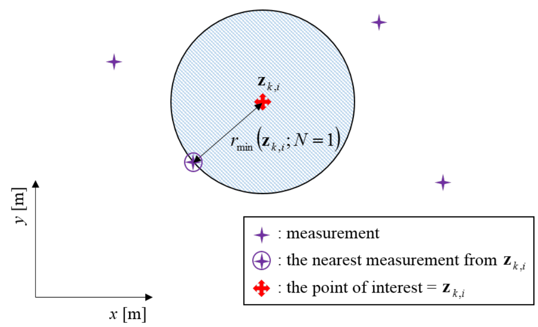


Fig. 1. SCMDE for $N = 1$ in the homogeneous measurement space

While most CMDEs provide the estimated CMD $\hat{\rho}$ to a data association filter, the SCMDE produces the estimated inverse of CMDE, $\hat{\rho^{-1}}$, for the filter. Because the SCMDE uses the hypersphere volume to measure the sparsity of clutter measurements, the algorithm works only in homogeneous measurement spaces. We propose this procedure for nonhomogeneous measurement spaces in the next section.

IV. Nonhomogeneous Spatial Clutter Measurement Density Estimator

The procedure described in (5)-(8) is valid for uniform clutter in the absence of target measurements, and is also recommended [10] for the homogeneous measurement space of which the coordinates have the same units. When used in nonhomogeneous spaces, the SCMDE produces in accurate estimates. The SCMDE used in a 2-dimensional nonhomogeneous space needs to obtain the area of hyperplane as depicted as an example in Figure 2: two measurements A, B and the point of interest $\mathbf{z}_{k,i}$. Even the 2-norm distances from the point to measurement A and B are equal to d, we cannot distinguish which one is the nearest measurement from the point of interest because the coordinate units are different.

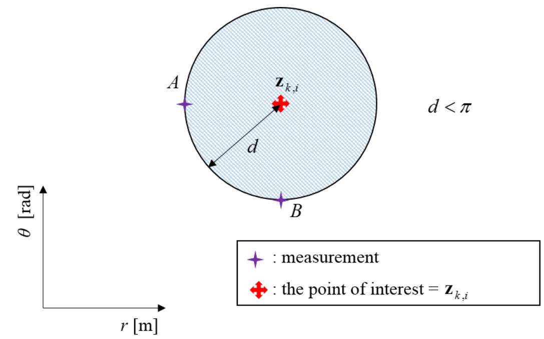


Fig. 2. Limitation of the standard SCMDE in the nonhomogeneous measurement space: which one is the nearest measurement of $\mathbf{z}_{k,i}$?

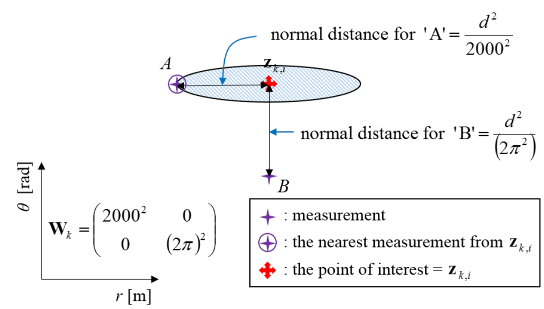


Fig. 3. Minimal hyperellipsoid in the nonhomogeneous measurement space

To generalize the SCMDE, we use a weighting matrix $\mathbf{W}_k$ to alter the shape of hypersphere centered at $\mathbf{z}_{k,i}$, and touches the N-th closest measurement.

First, define the normalized distance between $\mathbf{z}_{k,i}$ and $\mathbf{z}_{k,j}$ as

$$\gamma\left(\mathbf{z}_{k,i},\mathbf{z}_{k,j}\right) = \left(\mathbf{z}_{k,i}-\mathbf{z}_{k,j}\right)^{\mathrm{T}}\mathbf{W}_k^{-1}\left(\mathbf{z}_{k,i}-\mathbf{z}_{k,j}\right) \tag{9}$$

where $\mathbf{x}^{\mathrm{T}}$ denotes the transpose of $\mathbf{x}$. If $\mathbf{z}_{k,j}$ is designated as the N-th closest measurement of $\mathbf{z}_{k,i}$, then

$$\gamma_{min}\left(\mathbf{z}_{k,i};N\right) = \min_{\substack{N\text{th}\\ j\neq i}} \gamma\left(\mathbf{z}_{k,i},\mathbf{z}_{k,j}\right) \tag{10}$$

Denote by $V\left(\mathbf{z}_{k,i};N\right)$ the volume of the $\mathbf{W}_k$-shaped hyperellipsoid centered at $\mathbf{z}_{k,i}$ which "touches" the N-th nearest measurement. The volume of the hyperellipsoid is obtained by multiplying the weight matrix to the normal distance such as

$$V\left(\mathbf{z}_{k,i};N\right)=C_M\sqrt{\left|\gamma_{min}\left(\mathbf{z}_{k,i};N\right)\mathbf{W}_k\right|} \tag{11}$$

where $|\mathbf{A}|$ denotes the determinant of $\mathbf{A}$. The elements of matrix $\mathbf{W}_k$ should be chosen with the consideration of the nonhomogeneous relation between the measurement coordinates to form a reasonable normalized distance. These elements are set using the range of interest of each measurement coordinates. When the detection ranges of sensor in Figure 2 are $0 \sim 2,000m$ for range and $-\pi \sim \pi$ for bearing, the weighting matrix $\mathbf{W}_k$ becomes $diag[(2000)^2,(2\pi)^2]$. Also we can discriminate that the nearest measurement of the point of interest is A by (9). The hyperellipsoid of the example is depicted in Figure 3.

The estimated sparsity equals

$$\hat{\gamma}_{k,i}=\frac{V\left(\mathbf{z}_{k,i};N\right)}{N} \tag{12}$$

This procedure is suitable for nonhomogeneous measurement spaces. Note that, if $\mathbf{W}_k=\sigma\mathbf{I}_M$, where $\mathbf{I}_M$ denotes the M-dimensional identity matrix, (5)-(8) and (9)-(12) yield the same result. Thus the proposed procedure (9)-(12) is a generalization of (5)-(8).

V. Simulation Studies

Performance of the proposed method is evaluated by three different simulation studies. The simulation environments considered in this paper are listed below.

- Homogeneous measurement space: 2-dimensional Cartesian (xy) coordinates
- Nonhomogeneous measurement space I: polar $(r\theta)$ coordinates
- Nonhomogeneous measurement space II: inconvertible case

No target exists in the scenarios so that there are only clutter measurements in the measurement spaces. We compared the proposed SCMDE, called the generalized SCMDE described in Section IV, with the standard SCMDE in Section III. The sampling time (scan) equals $1s$ and a single run consists of 50 scans, The mean values of estimated sparsity of the two SCMDEs along the line of interest are used for performance evaluation through 500 Monte Carlo trials per scenario.

A. Homogeneous measurement space

The measurement space of the first scenario consists of 2-dimensional Cartesian coordinates and is shown in Figure 4. The relation between the measurement variables in this scenario is homogeneous that the unit of each variable is meter. The distribution of clutter measurements is uniform and the true CMD equals $5\times10^{-5}/m^2$ and the number of the clutter measurements follows Poisson distribution with the average number of 220.5 per scan. The edge points of the line of interest are $(x=0,y=750)$ and $(x=1500,y=750)$.

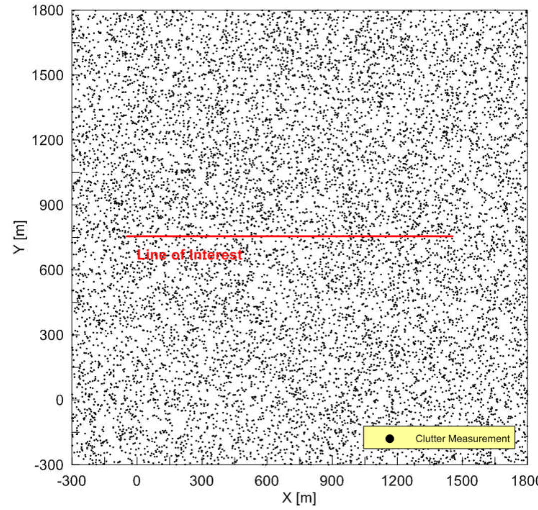


Fig. 4. Homogeneous measurement space and line of interest

Figure 5 shows the inverse of the mean estimated sparsity along the line of interest compared with the exact value. The notations of 'SCMDE' and 'gSCMDE' indicate the standard SCMDE and the proposed SCMDE respectively. The two SCMDEs produce identical results close to the true CMD as expected.

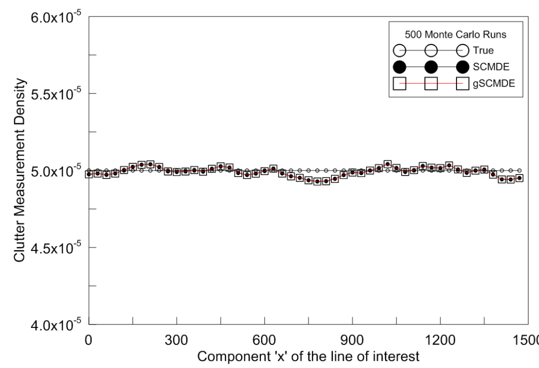
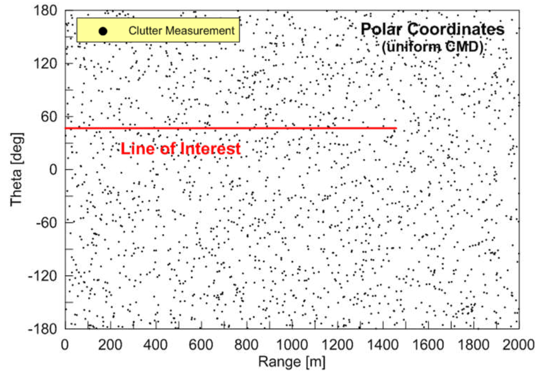


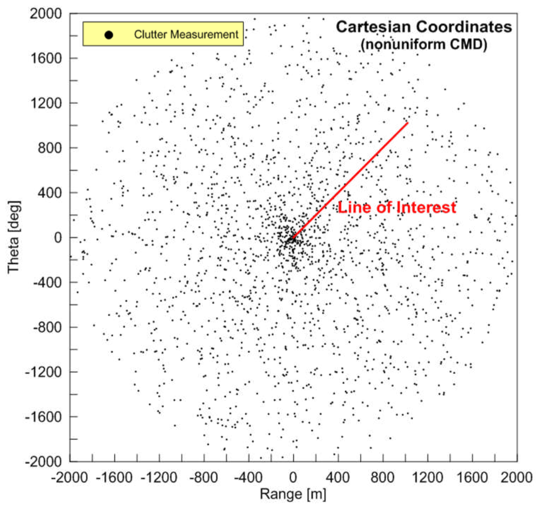
Fig. 5. The estimated CMD statistics (inverse of the mean estimated sparsity) in Cartesian (xy) coordinates

B. Nonhomogeneous measurement space I

The measurement space in the second scenario is a polar coordinate system which is used for most of active sensor systems. The space is nonhomogeneous as measurement variables are range and bearing with different units. The detectable range and bearing of the sensor are $0m \sim 2000m$ and $-\pi \sim \pi$ respectively. However, the measurement in polar coordinates can be represented in Cartesian coordinates by measurement conversion [14]. While the standard SCMDE can be applied to homogeneous measurement space only, the proposed method can be employed for all measurement spaces. True CMD equals $3.18\times10^{-3}/(m\cdot rad)$ and the average number of clutter measurements in the space is 40 per scan.



a) (Original) Measurement space



b) Converted measurement space

Fig. 6. Nonhomogeneous measurement space I and line of interest

As described in Figure 6, the distribution of clutter measurement is uniform in polar coordinates but is nonuniform in Cartesian coordinates. When the measurements are converted into Cartesian coordinates, the clutter measurements are densely populated near the origin. The line of interest starts from $(r = 0, \theta = \pi/4)$ and ends at $(r = 1500, \theta = \pi/4)$. Figure 7 and Figure 8 show the Monte Carlo simulation results using converted measurements in Cartesian coordinates and measurements in polar coordinates respectively. The true CMD is only representable in the original measurement space, which is illustrated in Figure 8. Although the results in Figure 7 cannot be compared with the exact CMD, the proposed method has reasonable estimation performance for Cartesian coordinates as depicted in Figure 7 which shows that CMD is high near the origin and it becomes low as far from the origin. The proposed method also works well in polar coordinates since the estimated result is close to the true CMD as shown in Figure 8.

C. Nonhomogeneous measurement space II

Next, we consider the target motion analysis with passive information which is one of the challenging areas in target tracking. The measurement space in the third scenario consists of bearing(θ) and Doppler frequency(f) of which the detectable range of bearing is $-\pi \sim \pi$ and that of Doppler frequency is $200Hz \sim 300Hz$.

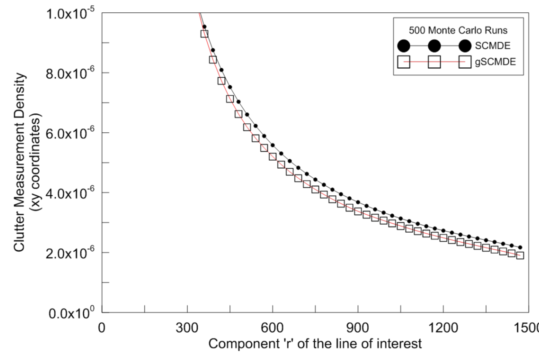


Fig. 7. The estimated CMD statistics in Cartesian (xy) coordinates

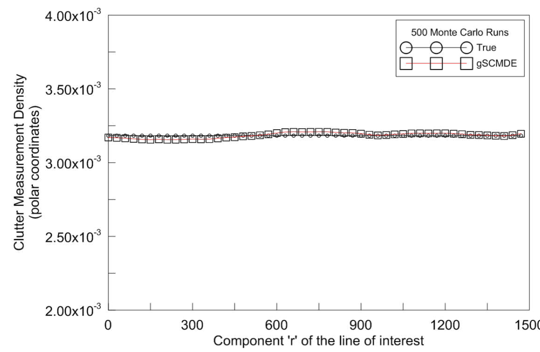


Fig. 8. The estimated CMD statistics in polar $(r\theta)$ coordinates

In contrast to the second scenario, the measurement space is not convertible to the homogeneous measurement space. In addition, the spatial distribution of the measurements is nonuniform. Figure 9 describes the measurement space in the third scenario. The base CMD is $6.36 \times 10^{-2}/(rad \cdot Hz)$, increasing to $1.27 \times 10^{-1}/(rad \cdot Hz)$ within the shaded patch.

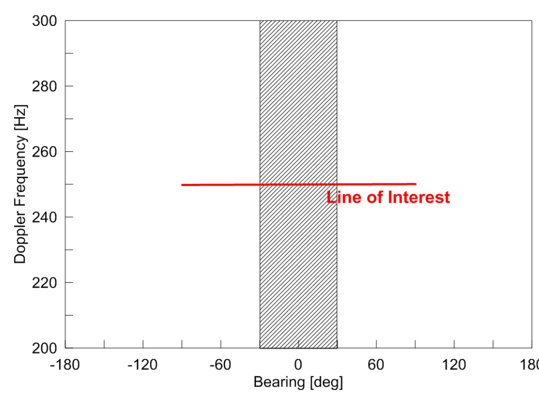


Fig. 9. Nonhomogeneous measurement space II and line of interest

Since it is impossible to utilize the standard SCMDE in this scenario, only the proposed method is compared with the true CMD. The line of interest starts from ($\theta = -\pi/2, f = 250$) and ends at ($\theta = \pi/2, f = 250$). The inverse of the mean estimated sparsity along the line of interest is shown in Figure 10.

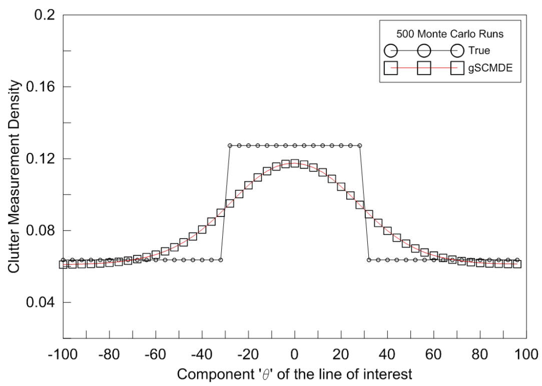


Fig. 10. The estimated CMD statistics in θf coordinates

The proposed method shows adaptive estimation performance in the spatially nonuniform and nonhomogeneous measurement space. The high CMD area is so narrow that the estimated CMD cannot achieve the true value of high CMD. Due to smudge effect (effect of neighboring regions with different CMD on CMD estimation) generated near the boundary of high and low density areas, the estimates show biased results around the border lines.

VI. Conclusions

This paper presents the generalized version of SCMDE for application in nonhomogeneous measurement spaces. The proposed method is identical to the standard SCMDE in homogeneous measurement spaces and the estimation of the proposed method is adaptive to the environments and accurate compared to the true CMD as shown in the simulation results. As the proposed method does not take into account of the target existence, performance degradation in CMD estimation is expected for such environments. Discriminating target measurements from the clutter measurements for CMDEs is reserved for future studies.

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