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Joint Multi-Target Detection and Tracking Using Conditional Joint Decision and Estimation with OSPA-like Cost

Wen Cao **Jian Lan**

Center for Information Engineering Science Research (CIESR)
School of Electronics and Information Engineering
Xi'an Jiaotong University, Xi'an, Shaanxi 710049, P. R. China
caowen.xjtu@gmail.com, lanjian@mail.xjtu.edu.cn

X. Rong Li

Department of Electrical Engineering
University of New Orleans
New Orleans, LA70148, U.S.A
xli@uno.edu

***Abstract*—This paper addresses multitarget tracking (MTT) in clutter, including jointly detecting targets and estimating their states. Good solutions for MTT require solving the two problems jointly. A joint decision and estimation (JDE) framework based on a generalized Bayes risk was recently proposed for solving problems involving inter-dependent decision and estimation. In the JDE framework, a conditional JDE (CJDE) approach was proposed, which is conditioned on data. However, direct application of CJDE to MTT is difficult because the estimation cost for the case of multi-targets is not defined. The key to applying CJDE to MTT is to design a reasonable and tractable estimation cost. In this paper, we propose a CJDE risk that is inspired by the optimal subpattern assignment (OSPA), which is a widely used metric for MTT performance evaluation. OSPA unifies the estimation error of tracking and the cardinality error of detection, and has many nice properties. The proposed CJDE risk with the OSPA-like cost takes advantage of both OSPA and CJDE. Furthermore, this risk is not only reasonable but also easy to optimize. Based on this risk, we derive the optimal joint decision and estimation. For MTT, simulation results show that both the proposed CJDE and the existing recursive JDE (RJDE) outperform the traditional decision then estimation strategy in OSPA, and CJDE with the OSPA-like cost is better than RJDE in many cases.**

I. Introduction

Multitarget tracking (MTT) is an old and still open problem [1]. In an MTT problem, not only do the targets states vary with time but also the number of targets also changes due to the target appearing and disappearing. Often, not all the existing targets can be detected, and sensors receive clutter not originating from the targets. In an MTT scenario, we want to infer both the number of targets and their states from observations in the presence of clutter.

MTT has been studied extensively and abundant results are available. The random finite set (RFS) approach has attracted much attention in recent years. Several filters were developed: the probability hypothesis density (PHD) filter [2], the cardinality PHD (CPHD) filter [3], and the multi-target multi-Bernoulli (MeMber) filter [4]. Sequential Monte Carlo (SMC) and Gaussian mixture (GM) implementations of these filters [5]–[7] have also been reported for MTT, and numerous applications have been made. However, these methods are in nature for multitarget densities due to their RFS basis. In this paper, we focus on the point estimation rather than density estimation. In many data association-based MTT algorithms, multiple hypothesis tracking (MHT) [8] and joint probabilistic data association (JPDA) [1] methods are adopted. These methods, however, either assume a known number of targets or determine the number of targets first and then estimate their states based on the determined number. In some other applications, for example, for radar detection and target tracking of low-observed objects, tracking-before-detection (TBD) is adopted [9].

MTT problems have two main goals: deciding on the number of targets and estimating their states, and they affect each other. A correct decision on the number can help state estimation and accurate state estimation can also help make a correct decision on the number. In essence, this is a so-called joint decision and estimation (JDE) problem [10], and good solutions require solving the two problems jointly.

The prevailing strategies for solving JDE problems can be classified into the following categories [11]. (1) Separate decision and estimation: decision and estimation are considered as two separate problems without considering their inter-dependence [12], [13]. (2) Decision then estimation (DTE): make a best decision first disregarding estimation and then do estimation as if the decision were surely correct. A serious disadvantage of this strategy is that it does not account for possible decision error in the subsequent estimation. Also, decision is done disregarding the quality of the estimation it would lead to [14], [15]. (3) Estimation then decision (ETD). It would not work well if estimation is significantly dependent on decision [16], [17]. These separate or two-step strategies all have their respective drawbacks in solving JDE problems, and an effective remedy is hard to come by within these strategies.

In the general case, decision and estimation by a joint approach would be more promising than the existing methods

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since it can take advantage of the coupling between decision and estimation. Reference [10] proposed an integrated paradigm for JDE based on a new Bayes risk, which is a generalization of the traditional Bayes decision risk and estimation risk. This approach is inherently superior in performance to the conventional two-stage strategy or separate decision and estimation, especially for problems where decision and estimation are highly correlated. [11] adapted the JDE approach to the dynamic case and proposed a recursive JDE (RJDE) algorithm. The power of the proposed JDE has been illustrated by applications to several JDE problems [11], [18]–[20]. In [19], we solved an extended object tracking and classification problem in the JDE framework and proposed a random-matrix-based multiple model RJDE method for extended objects. In [20], we applied the RJDE method to a multisensor-data based joint tracking and classification (JTC) problem, which is formulated based on homogenous sensor data. [21] applied RJDE to the MTT problem by defining a new estimation cost for multi-targets, and its superiority was also verified by comparing with the traditional DTE method.

Based on JDE, we proposed a conditional JDE (CJDE) risk in [22], which is a Bayes JDE risk conditioned on data. To minimize the CJDE risk, the optimal joint decision and estimation was derived in [22]. CJDE inherits the theoretical superiority of JDE by making full use of the coupling between decision and estimation. For calculation, CJDE has simple complexity due to its conditioning on data, which is more practical. In [23], we applied the proposed CJDE approach to a practical JTC problem using multisensor data, and the superiority of CJDE was demonstrated.

In this paper, we address the MTT problem using the CJDE method. The key to applying CJDE to MTT is to design a reasonable and tractable estimation cost, especially for the case that decision is incorrect. For example, how should we define the error of an estimator of two targets, each with a state dimension n, while there is only a single target with a state dimension n? To solve the MTT problem, in the RJDE method, [21] uses a simple and direct estimation cost, which is easy to handle. However, this definition is not well justified.

For performance evaluation of MTT, the recently proposed optimal subpattern assignment (OSPA) has been widely used in the literature addressing the MTT problem [24] [25]. As a joint performance metric, OSPA has many nice properties and is shown to eliminate most shortcomings of the Hoffman-Mahler metric and the optimal mass transfer (OMAT) metric [26]. In the context of MTT, OSPA unifies the localization error of estimation and the cardinality error of decision. This is similar to the JDE risk, in which decision risk and estimation risk are combined into a unified framework. The similarity between the OSPA metric and the JDE risk motivated our work.

To solve the MTT problem, we propose a new CJDE risk that is inspired by OSPA. It takes advantage of both the OSPA and CJDE, which are an evaluation metric and an optimization objective function, respectively. An evaluation metric aims to faithfully judge the performance and thus the key is to be objective, little distortion, and computable. For an optimization function, however, its objective is to explore solutions which have good performance, and thus it is critical to be mathematically tractable and practically acceptable [27]. Considering this, we integrate OSPA and CJDE in a promising way by several improvements. The resulting risk is not only reasonable but also tractable.

Based on the proposed risk, we derive the optimal joint decision and estimation and present the corresponding algorithm. The superiority of our method is verified through an illustrative scenario of an MTT problem. Simulation results show that both RJDE and the proposed CJDE outperform the traditional DTE method in OSPA, and CJDE with OSPA-like cost performs better than RJDE in several typical cases.

This paper is organized as follows. Section II overviews the CJDE method, the existing RJDE method for MTT problems, and the OSPA metric. Section III proposes a new CJDE risk with OSPA-like cost, which is reasonable and tractable. The optimal JDE minimizing this risk is also derived. In Section IV, an illustrative MTT example is presented. The proposed method is compared with DTE and RJDE methods. Section V concludes the paper.

II. Review of CJDE approach and OSPA metric

A. Joint Decision and Estimation (JDE)

The basic idea of the JDE approach is to minimize the following generalized Bayes risk [10]

$$\bar{R} = \sum_{i,j}(\alpha_{ij}c_{ij} + \beta_{ij}E[C(x,\hat{x})|D^i, H^j])P\{D^i, H^j\} \quad (1)$$

where D^i stands for the ith decision, which is equivalent to the event $\{z \in \mathcal{D}^i\}$ ($\mathcal{D}^i$ is the decision region for D^i in the data space); c_{ij} is the cost of deciding on D^i while the truth is H^j; $P\{D^i, H^j\}$ is the joint probability of decision and hypothesis; $C(x,\hat{x})$ is the cost of estimating x by $\hat{x}$; $E[C(x,\hat{x})|D^i, H^j]$ is the expected cost conditioned on the fact that D^i is decided but H^j is true; and α_{ij} and β_{ij} are the weight coefficients of decision and estimation costs, which provide additional flexibilities. To minimize $\bar{R}$ of (1), the optimal JDE $\{D, \hat{x}\}$ was given [10]. Here, D and $\hat{x}$ are the decision and estimation results, respectively.

Remark 1: This Bayes risk $\bar{R}$ is a generalization of the traditional Bayes risks for decision and for estimation, respectively. A JDE algorithm with guaranteed global convergence was presented in [10]. This JDE approach explicitly accounts for the inter-dependence between decision and estimation, and it is theoretically superior to the existing method of separate decision and estimation or two-stage methods.

Based on JDE, we proposed a conditional JDE (CJDE) risk in [22], which is a Bayes JDE risk conditioned on data.

B. Conditional JDE (CJDE)

The basic idea of CJDE is to minimize the CJDE risk:

$$R_C(z) = \sum_{i,j}(\alpha_{ij}c_{ij} + \beta_{ij}E[C(x,\hat{x})|D^i, H^j, z])P\{D^i, H^j|z\} \quad (2)$$

To minimize $R_C(z)$ (2), given any estimation cost $E[C(x,\hat{x})|D^i, H^j, z]$, the optimal decision D is

$$D = D^i, \text{ if } C_C^i(z) \leqslant C_C^l(z), \forall l \quad (3)$$

where the posterior cost is

$$C_C^i(z) = \sum_j (\alpha_{ij}c_{ij} + \beta_{ij}E[C(x,\hat{x})|D^i, H^j, z])P\{H^j|z\} \tag{4}$$

and given D^i, the optimal estimator with $C(x,\hat{x}) = \tilde{x}'\tilde{x}$ is

$$\hat{x} = \check{x}^{(i)} = \sum_{j=1}^{N} E[x|z, H^j]\bar{P}_i\{H^j|z\} \tag{5}$$

$$\bar{P}_i\{H^j|z\} = \frac{\beta_{ij}P\{H^j|z\}}{\sum_{k=1}^{N}\beta_{ik}P\{H^k|z\}}$$

A proof of the optimal CJDE was presented in [22]. To calculate the posterior cost $C_C^i(z)$ (4), the key is to obtain $E[C(x,\hat{x})|D^i, H^j, z]$. In CJDE, with $C(x,\hat{x}) = \tilde{x}'\tilde{x}$,

$$\begin{aligned}
&\varepsilon^{ij}(z) \triangleq E[\tilde{x}'\tilde{x}|D^i, H^j, z] \\
&= \mathrm{mse}(\hat{x}^{(ij)}|D^i, H^j, z) + E[(\hat{x}^{(ij)} - \hat{x})'(\cdot)|D^i, H^j, z] \\
&= \mathrm{mse}(\hat{x}^{(j)}|H^j, z) + E[(\hat{x}^{(j)} - \check{x}^{(i)})'(\cdot)|D^i, H^j, z], \forall z \in \mathcal{D}^i \\
&= \mathrm{mse}(\hat{x}^{(j)}|H^j, z) + (\hat{x}^{(j)} - \check{x}^{(i)})'(\cdot), \forall z \in \mathcal{D}^i
\end{aligned} \tag{6}$$

where $\hat{x}$ is the CJDE estimate, $\mathrm{mse}(\hat{x}|A)$ denotes the conditional (on A) scalar mean square error, and $(\cdot)$ denotes the same term right before it. For $z \in \mathcal{D}^i$, we have $\hat{x} = \check{x}^{(i)}$, $\hat{x}^{(ij)} = E[x|D^i, H^j, z] = E[x|H^j, z] = \hat{x}^{(j)}$, and $\mathrm{mse}(\hat{x}^{(ij)}|D^i, H^j, z) = \mathrm{mse}(\hat{x}^{(j)}|H^j, z)$. Note that in the last equation above, the expectation disappear since $\hat{x}^{(j)}$ and $\check{x}^{(i)}$ are both fixed given z and D^i.

For dynamic JDE problems, measurements are usually obtained sequentially, and the above CJDE algorithm maybe computationally inefficient due to its batch form. Considering this, we propose a recursive CJDE (RCJDE) algorithm [22] based on the following RCJDE risk:

$$R_C(Z^k) = \sum_{i,j}(\alpha_{ij}c_{ij} + \beta_{ij}E[C(x_k,\hat{x}_k)|D_k^i, H^j, Z^k]) \times P\{D_k^i, H^j|Z^k\} \tag{7}$$

where x_k is the true state at time k, $\hat{x}_k$ is its estimate, and D_k^i stands for the ith decision at time k. $Z^k = \{z_1, z_2, \cdots, z_k\}$. The optimal RCJDE algorithm is given in [22].

Remark 2: The main difference between JDE and CJDE results from conditioning on data in the latter's risk, as discussed in detail in [22]. CJDE inherits the superiority of JDE by unifying decision and estimation into an integrated framework. For calculation, by conditioning on the data, CJDE is computationally simpler than the unconditional JDE. This makes CJDE more applicable in practice.

C. Recursive JDE for MTT

Extending JDE to the dynamic case, [11] proposed a recursive JDE (RJDE) method, which is a recursive implementation of JDE. The proposed RJDE was applied to MTT in [21] by defining a new estimation cost for multi-targets. Suppose H_k^j is the hypothesis that j ($1 \leqslant j \leqslant N$) targets are actually present in the surveillance region at time k and D_k^i is the decision that there are i ($1 \leqslant i \leqslant M$) targets. Conditioned on hypothesis H_k^j, X_k is a stacked vector of j target states: $X_k = [(x_k^1)', (x_k^2)', \cdots, (x_k^j)']'$, and $\hat{X}_k$ is its estimate. The expected estimation cost is defined as

$$\begin{aligned}
\varepsilon_k^{ij} &= E[C(X_k,\hat{X}_k)|Z^{k-1}, D_k^i, H_k^j] \\
&\approx \begin{cases} \frac{\tau(i)}{i}\mathrm{mse}(\hat{X}_k|D_k^i, H_k^j), & \text{if } i = j \\ \eta, & \text{if } i \neq j \end{cases}
\end{aligned} \tag{8}$$

where η is a cost parameter and $\tau(i)$ is a non-increasing positive function of i with $\tau(1) = 1$. $\hat{X}_k$ as the estimate of X_k under decision D_k^i is a stacked vector of i target state estimates. Note that the expected cost ε_k^{ij} is defined this way because X_k and $\hat{X}_k$ have different dimensions if $i \neq j$, and thus the estimation error cannot be defined in the usual way as $\tilde{X}_k = X_k - \hat{X}_k$. In (8), this phenomenon is reflected by a cost parameter η, which may be either a constant or a function of D_k^i and H_k^j. For the case that $i = j$, the normalized mean square error is adopted together with an adjustment function $\tau(i)$. For more details, see [21].

D. Optimal Subpattern Assignment (OSPA) Metric

In evaluating the performance of an MTT algorithm, the goal is to measure the distance between two sets of tracks: the set of ground truth and the set of estimated tracks output by the MTT algorithm. For measuring the distance between any two sets, there are a few known metrics starting from the Hausdorff metric. Hoffman and Mahler [28] proposed a new metric based on the Wasserstein distance, which partially fix the undesirable cardinality behavior of the Haudorff metric. Schuhmacher *et al.* [26] subsequently demonstrated a number of shortcomings of the Hoffman-Mahler metric and proposed a new consistent metric for sets, referred to as the optimal subpattern assignment (OSPA) metric. OSPA eliminates most shortcomings of the Hoffman-Mahler metric, as shown in [24].

Denote by $d^{(c)}(x,y) = \min(c, d(x,y))$ the distance between $x, y \in W$ cut off at $c > 0$, where W is a closed and bounded observation window and $W \in \mathbb{R}^N$. Π_k is the set of permutations on $\{1, 2, \cdots, k\}$ for any $k \in \mathbb{N} = \{1, 2, \cdots\}$. For $1 \leqslant p < \infty, c > 0$, and arbitrary finite subsets $X = \{x_1, \cdots, x_m\}$ and $Y = \{y_1, \cdots, y_n\}$ of W, where $m, n \in \mathbb{N}_0 = \{0, 1, 2, \cdots\}$, define

$$\bar{d}_p^{(c)}(X,Y) \triangleq \Big(\frac{1}{n}\big[\min_{\pi\in\Pi_n}\sum_{i=1}^{m} d^{(c)}(x_i, y_{\pi(i)})^p + c^p(n-m)\big]\Big)^{1/p} \tag{9}$$

if $m \leqslant n$, and $\bar{d}_p^{(c)}(X,Y) = \bar{d}_p^{(c)}(Y,X)$ if $m > n$. We call the function $\bar{d}_p^{(c)}$ the OSPA metric of order p with cut-off c.

In (9), p determines the sensitivity of $\bar{d}_p^{(c)}$ to outlier estimates, and c determines the relative weighting of how the metric penalizes the cardinality error as opposed to the localization error. For MTT performance evaluation, OSPA is widely used due to its nice properties, given in detail in [26]. OSPA considers both the cardinality error and the localization error, and its superiority is demonstrated in [26].

III. Joint Multi-Target Detection and Tracking Using CJDE with OSPA-like Cost

A. Motivation

As mentioned in Introduction, MTT is a JDE problem, and thus good solutions require deciding the number of targets and estimating their states jointly.

(a) CJDE is a promising JDE approach due to its performance optimality and simplicity. However, when applying it to the MTT problem, difficulty arises because the estimation cost $E[C(x,\hat{x})|D^i,H^j,z]$ for multi-targets is not defined. To illustrate, suppose H^i stands for the truth that there are i targets [10]. Then the target state x will have different dimensions under H^1 and H^2. But the estimator $\hat{x}$ must be the same for both cases since it does not know the truth. As a result, the estimation error $\tilde{x}=x-\hat{x}$ and the corresponding $E[C(\tilde{x})|D^i,H^j,z]$ cannot be defined for both H^1 and H^2 in a unified way directly: what is the error of an estimator $(\hat{x}_1,\hat{x}_2)$ assuming two targets, each with a state dimension of n, while there is only a single target with an n-dimensional state x?

For applying RJDE to MTT, [21] defined an estimation cost ε_k^{ij} (8), in which a cost parameter η was adopted when the decision D^i is incorrect (i.e., $i \neq j$). This is simple and direct for MTT problems. However, it is not well justified. In this paper, we consider a new and better estimation cost.

(b) In the context of MTT performance evaluation, OSPA distance is comprised of two components accounting for the localization error and the cardinality error, respectively [26]. Precisely, for $p<\infty$ and $m \leqslant n$, they are given by

$$\bar{e}^{(c)}_{p,loc}(X,Y)=\frac{1}{n}\min_{\pi\in\Pi_n}\sum_{i=1}^{m}d^{(c)}(x_i,y_{\pi(i)})^p$$
$$\bar{e}^{(c)}_{p,card}(X,Y)=\frac{c^p(n-m)}{n}$$

Taking $\bar{e}^{(c)}_{p,loc}(X,Y)$ as estimation error and $\bar{e}^{(c)}_{p,card}(X,Y)$ as decision error, the OSPA metric (9) is similar to the JDE risk (1). Specifically, the JDE risk combines the decision cost c_{ij} and the estimation cost $E[C(x,\hat{x})|D^i,H^j,z]$ to form a unified cost through the parameters α_{ij} and β_{ij}. In the OSPA metric, the decision error $\bar{e}^{(c)}_{p,card}(X,Y)$ and the estimation error $\bar{e}^{(c)}_{p,loc}(X,Y)$ are also combined in a unified way. So, they are somewhat similar in spirit.

Based on the above analysis, we consider solving the MTT problem by introducing the idea of OSPA to the CJDE risk. This is promising since it can take advantage of both the OSPA metric and the CJDE risk.

B. CJDE with OSPA-like cost

Although it is theoretically promising, direct introduction of OSPA to the CJDE risk is challenging. When using OSPA to evaluate the performance of MTT, all the required quantities, i.e., both the ground truth (the number of targets and their states) and the decision and estimation results, are already known, and we just need to plug them into (9). Note that in the OSPA metric (9), a key step is to find the optimal permutation, which results in the minimum distance between the two sets to be evaluated.

In the CJDE risk (2), we do not know the decision and estimation results beforehand since they are our goals. As a result, when introducing OSPA to the CJDE risk, the explicit form of the estimation cost is not available. This is because the optimal permutation needed by OSPA is undefined since the ground truth and the CJDE estimate are unknown. Note that even if D^i and H^j are fixed, we can not define the CJDE estimation cost, not to mention that in (2), given D^i we need to consider all possible $H^j (j=1,\cdots,N)$, and the optimal permutation under different H^j maybe different.

1) Analysis and illustration: To begin with, suppose H^j denotes that there are j targets, and then conditioning on H^j, $X=\{x^1,\cdots,x^j\}$ is the set of the true states of j targets. Suppose decision is to "choose" one hypothesis, i.e,. $D^i=$ "H^i". Conditioning on D^i, $\check{X}=\{\check{x}^1,\cdots,\check{x}^i\}$ is the set of the estimated states of i targets.

In the CJDE risk (2), the multi-targets estimation cost which measures the difference between X and $\check{X}$ needs to be defined. To to this, we need to know the correspondence relationship between elements in X and $\check{X}$ according to the OSPA metric (9). This correspondence can be described by a permutation, as detailed in [26]. For convenience, when H^j and D^i are given, denote π_{ij} as a permutation on $\{1,2,\cdots,\max(i,j)\}$, and $\Pi_{ij}=\{\pi_{ij}\}$ is the set of all possible permutations. Denote $m=\max(i,j)$ and $n=\min(i,j)$, the total number of elements in Π_{ij} is $n_{ij}=P^{\max(i,j)}_{\min(i,j)}=\frac{m!}{(m-n)!}$, where P^m_n denotes n-permutations of m. Given D^i, there are n_{ij} possible permutations under each hypothesis H^j.

To illustrate, suppose we have three hypotheses H^j $(j=1,2,3)$. Take $D=D^2$ as an example. A and B are two targets under decision D^2. Then

1) Under H^1, the true target is T. In this case, there are two possible correspondences—(A,T) and (B,T). Here, (A,T) means that target A corresponds to the true target T and (B,T) means that target B corresponds to T. Suppose permutation π^1_{21} and π^2_{21} represent (A,T) and (B,T), respectively. Finally, we have $\Pi_{21}=\{\pi^1_{21},\pi^2_{21}\}$ under H^1.

2) Under H^2, the true targets are T_1 and T_2. In this case, there are two possible correspondences—$\{(A,T_1),(B,T_2)\}$ and $\{(A,T_2),(B,T_1)\}$. Here, $\{(A,T_1),(B,T_2)\}$ means that target A corresponds to T_1 and target B corresponds to T_2. Represent $\{(A,T_1),(B,T_2)\}$ and $\{(A,T_2),(B,T_1)\}$ by permutations π^1_{22} and π^2_{22}, respectively. Then under H^2, $\Pi_{22}=\{\pi^1_{22},\pi^2_{22}\}$.

3) Under H^3, the true targets are T_1, T_2, and T_3. In this case, there are six possible correspondences—$\{(A,T_1),(B,T_2)\}$, $\{(A,T_1),(B,T_3)\}$, $\{(A,T_2),(B,T_3)\}$, $\{(A,T_2),(B,T_1)\}$, $\{(A,T_3),(B,T_1)\}$, $\{(A,T_3),(B,T_2)\}$. For example, $\{(A,T_3),(B,T_2)\}$ means that target A and target B corresponds to T_3 and T_2, respectively. Finally, $\Pi_{23}=\{\pi^1_{23},\pi^2_{23},\cdots,\pi^6_{23}\}$ under H^3.

Given D^i, H^i, and a permutation π_{ij}, according to the CJDE method, we can get the CJDE cost $C^{ij}_C(z)=\alpha_{ij}|i-j|+\beta_{ij}E[C(X,\check{X})|D^i,H^j,z]$, where $\beta_{ij}E[C(X,\check{X})|D^i,H^j,z]$ is the tracking error and $\alpha_{ij}|i-j|$ is the detection error. This is similar to the OSPA metric (9). Note that $C(X,\check{X})$ is a function of π_{ij}, which is similar to $d^{(c)}(\cdot)^p$ in (9). For example, given $D=D^2$ and $H=H^3$ in the above example, there are totally 6 permutations, each leading to a different $C^{23}_C(z)$.

In the JDE framework, given D^i, we need to consider each $H^j (j=1,\cdots,N)$. As explained above, to use the CJDE method, we need to calculate $C_C^{ij}(z)$ for $j = 1, \cdots, N$. Thus we consider a collection of permutations $\{\pi_{ij}\}_{j=1}^N$. Given D^i, there are totally $N_i = n_{i1} \times n_{i2} \cdots \times n_{iN}$ such $\{\pi_{ij}\}_{j=1}^N$.

For example, given $D = D^2$, $\{\pi_{2j}\}_{j=1}^3$ means choosing one permutation from each of Π_{21}, Π_{22}, and Π_{23}. Then all the selected permutations over $H^j(j = 1, \cdots, N)$ form a permutation collection. Suppose $\{\pi_{2j}\}_{j=1}^3 = \langle \pi_{21}^1, \pi_{22}^2, \pi_{23}^4 \rangle$, this means that the 1st permutation in Π_{21}, the 2nd in Π_{22}, and the 4th in Π_{23} are chosen.

According to OSPA [26], to obtain the OSPA distance, we need to find the optimal permutation that minimizes (9). Similarly, in the CJDE risk, given D^i, we need to find the optimal permutation collection, i.e., the one under which the CJDE cost $C_C^i(z)$ is smaller than that under any other permutation collection.

2) CJDE risk with OSPA-like cost: Based on the above, we propose the following CJDE risk

$$R^o(z) = \sum_{i=1}^M \min_{\{\pi_{ij}\}_{j=1}^N} (\sum_{j=1}^N (\beta'_{ij} E[\sum_{l=1}^{\min(i,j)} [d_l(X, \check{X})]^p | D^i, H^j, z] + \gamma_{ij}|i-j|) \times P\{D^i, H^j|z\}) \qquad (10)$$

where the estimation error $[d_l(X, \check{X})]^p$ is defined as follows:

$$[d_l(X, \check{X})]^p = \begin{cases} d(x^l, \check{x}^l_{\{\pi_{ij}\}_{j=1}^N})^p, \ j \le i \\ d(x^l_{\{\pi_{ij}\}_{j=1}^N}, \check{x}^l)^p, \ j > i \end{cases}$$

where the superscript l denotes the lth element and the subscript $\{\pi_{ij}\}_{j=1}^N$ means that this state set is under the permutation collection $\{\pi_{ij}\}_{j=1}^N$. Two parameters γ_{ij} and $\beta'_{ij} = \frac{\beta_{ij}}{\max(i,j)}$ are nonnegtive relative weights for decision and estimation costs, respectively. Given D^i, $\min_{\{\pi_{ij}\}_{j=1}^N}$ denotes the optimal permutation collection that minimize the posterior CJDE cost $C_C^i(z)$(4). Note that $[d_l(X, \check{X})]^p$ in (10) is not cut off while $d^{(c)}(\cdot)^p$ in (9) is. Other symbols are presented in the previous part.

3) Properties:

a) Relationship with OSPA and CJDE: In (10), given D^i and H^j, we have

$$R^o(z) = \min_{\pi_{ij} \in \Pi_{ij}} (\beta'_{ij} E[\sum_{l=1}^{\min(i,j)} [d_l(X, \check{X})]^p | z] + \gamma_{ij}|i-j|) \qquad (11)$$

which can be considered as a variant of the OSPA (9). Specifically, if γ_{ij}/β_{ij} in (11) is replaced by c^p (note that $\beta'_{ij} = \beta_{ij}/\max(i,j)$), and the estimation error $d_l(X, \check{X})^p$ is cut off at c, then $(\bar{R}^o(z))^{1/p}$ is exactly the OSPA (9).

In (10), if the operator for seeking the optimal permutation collection (i.e., $\min_{\{\pi_{ij}\}_{j=1}^N}$) is taken out, then $R^o(z) =$

$$\sum_{i=1}^M \sum_{j=1}^N (\beta'_{ij} E[\sum_l [d_l(X, \check{X})]^p | D^i, H^j, z] + \gamma_{ij}|i-j|) P\{D^i, H^j|z\}$$

which is just the CJDE risk (2) with estimation cost $C(x, \hat{x}) \triangleq \sum_l [d_l(X, \check{X})]^p$.

b) Rationality and Tractability: The proposed $R^o(z)$ (10) integrates the OSPA metric and the CJDE risk, and takes advantage of both. $R^o(z)$ combines the decision cost and the estimation cost into a unified framework. Besides, $R^o(z)$ is also tractable and is easy to optimize. Tractability is an important property for an objective function.

4) Optimal Solution: We get the optimal JDE $(D, \check{X})$ for $R^o(z)$ following the CJDE [22], as follows.

a) Estimation: Denote by $\hat{X}^j = \{\hat{x}_j^1, \hat{x}_j^2, \cdots, \hat{x}_j^j\}$ the state estimate conditioned on H^j and by $\check{X}^i = \{\check{x}_i^1, \check{x}_i^2, \cdots, \check{x}_i^i\}$ the CJDE estimate under D^i. For convenience, we define an $i \times j$ matrix m_{ij} to represent the permutation π_{ij}. Given D^i and H^j, all the possible matrices form a set M_{ij}. Then the permutation collection $\{\pi_{ij}\}_{j=1}^N$ is replaced by the matrix collection $\{m_{ij}\}_{j=1}^N$, and they have a one-to-one correspondence.

Given D^i and H^j, all entries $m_{ij}(s,q)$ of m_{ij} are binary numbers, defined as

$$\begin{cases} m_{ij}(s,q) = 1, \text{ if } \check{x}_i^s \leftrightarrow \hat{x}_j^q \text{ in } \pi_{ij} \\ m_{ij}(s,q) = 0, \qquad\qquad \text{else} \end{cases}$$

where $\check{x}_i^s$ is the sth $(s = 1, \cdots, i)$ element in $\check{X}^i$; $\hat{x}_j^q$ is the qth $(q = 1, \cdots, j)$ element in $\hat{X}^j$. Here, "$\check{x}_i^s \leftrightarrow \hat{x}_j^q$" means that in permutation π_{ij}, there is a correspondence between $\hat{x}_j^q$ and $\check{x}_i^s$. Following the CJDE estimator (5), each element in $\check{X}^i$ is a weighted sum of elements in different $\hat{X}^j(j = 1, 2, \cdots, N)$. Then if $\hat{x}_j^q$ corresponds to $\check{x}_i^s$, $m_{ij}(s,q) = 1$, otherwise $m_{ij}(s,q) = 0$.

In (10), p can be any value $p \geqslant 1$. For simplicity, we consider $p = 2$ in the following. With $p = 2$, we have $d(x, \check{x})^p = \tilde{x}'\tilde{x}$. This leads to the most widely used Bayes estimation cost $E[\tilde{x}'\tilde{x}]$, that is, the mean square error. It is well known that the posterior mean $\hat{x} = E[x|z]$ is the optimal estimator for this cost.

For convenience of description, under decision D^i, we define a vector $e \triangleq [e_1, e_2, \cdots, e_N]'$, where e_j denotes the order index of m_{ij} in the matrix set M_{ij}, that is, the e_jth matrix m_{ij} is chosen. For example, given D^i, $e = [2, 3, 1]'$ means that the 2nd element in M_{i1}, the 3rd element in M_{i2}, and the 1st element in M_{i3} are chosen.

Based on the above, given D^i and e, the CJDE estimate $\check{X}^i = \{\check{x}_i^t\}_{t=1}^i$ is

$$\check{x}_i^t = \bar{E}[x|z] = \sum_{j=1}^N \sum_{r=1}^j \hat{x}_j^r \bar{P}_i^{t,r,e_j} \{H^j|z\} \qquad (12)$$

where

$$\bar{P}_i^{t,r,e_j} \{H^j|z\} = \frac{m_{ij}^{e_j}(t,r)\beta'_{ij} P\{H^j|z\}}{\sum_j m_{ij}^{e_j}(t,r)\beta'_{ij} P\{H^j|z\}}$$

is the generalized posterior probability of H^j. Here, $\hat{x}_j^r$ denotes the rth component of $\hat{X}^j$. Given e, all values of e_j $(j = 1, \cdots, N)$ are available, and the specific matrix $m_{ij}^{e_j}$ is also known correspondingly. In the matrix $m_{ij}^{e_j}$, the element $m_{ij}^{e_j}(t,r)$ is 1 if $\hat{x}_j^r$ corresponds to $\check{x}_i^t$, and 0 otherwise.

b) Decision: Following CJDE, the optimal decision for $R^o(z)$ (10) is

$$D = D^i, \text{ if } C_i(z) \leqslant C_l(z), \forall l \tag{13}$$

where the posterior cost $C_i(z)$ for decision D^i is

$$C_i(z) = \sum\nolimits_j \beta_{ij} E[C_{ij}(X, \check{X}) | D^i, H^j, z] P\{H^j | z\}$$

Here

$$C_{ij}(X, \check{X}) = \min_{\{\pi_{ij}\}_{j=1}^N} \{\sum_{j=1}^{N} (\beta'_{ij} E[\sum_{l=1}^{\min(i,j)} [d_l(X, \check{X})]^p | D^i, H^j, z] + \gamma_{ij}|i-j|)\} P\{H^j|z\}$$

Note that under each D^i, the posterior cost $C_i(z)$ is calculated based on the optimal permutation collection

$$\{\pi_{ij}^{\text{opt}}\}_{j=1}^N = \arg\min_{\{\pi_{ij}\}_{j=1}^N} \{\sum_{j=1}^{N} (\beta'_{ij} E[\sum_{l=1}^{\min(i,j)} [d_l(X, \check{X})]^p | D^i, H^j, z] + \gamma_{ij}|i-j|)\} P\{H^j|z\} \tag{14}$$

Optimal CJDE Algorithm

a) Given hypothesis set $\{H^1, \cdots, H^N\}$ and decision set $\{D^1, \cdots, D^M\}$, get the collection matrix $\mathbb{M}$.

b) Given each decision candidate D^i ($i = 1, \cdots, M$), under each matrix collection $\{m_{ij}\}_{j=1}^N$ (the total number $N_i = n_{i1} \times n_{i2} \cdots \times n_{iN}$), calculate the optimal CJDE estimate $\check{X}^i$ by (12).

c) For each candidate D^i, based on $\check{X}^i$, obtain the optimal matrix collection $\{m_{ij}^{\text{opt}}\}_{j=1}^N$ by (14) and the corresponding posterior decision cost $C_i(z)$. Then obtain the optimal CJDE decision D^i by (13).

d) Based D^i and $\{m_{ij}^{\text{opt}}\}_{j=1}^N$, output the optimal CJDE estimate $\check{X}^i$, as calculated in step b).

Remark 3: With this algorithm, the number of targets and their states are inferred jointly. In the above algorithm, however, the conditional estimate $\hat{X}^j$ and its MSE P^j are assumed to be already obtained, and only the final CJDE decision and estimation results are addressed. Actually, under each hypothesis H^j, calculation of $\hat{X}^j$ and P^j is an MTT problem with a known number of targets, which can be solved by many algorithms, such as the MHT and JPDA filters.

IV. Simulation and analysis

A. Basic Assumptions

To illustrate our proposed method, we apply it to a simple yet representative joint multitarget detection and tracking problem. For simplicity, we have the following assumptions [21]:

(a) The number of targets $m_k^t \leqslant N$ is unknown but constant over time k (N is known).

(b) A target can generate at most one measurement—no multipath; a measurement can have originated from at most one target—no unresolved measurements.

(c) The number m_f of false measurements is Poisson distributed. The false measurements are i.i.d and uniformly distributed in the surveillance region of a volume V.

(d) All targets follow CV models with a linear measurement equation. To save space, the dynamic and measurement models are omitted.

To obtain the conditional estimate $\hat{X}_k^j$ and its MSE P_k^j, the JPDA filter is adopted due to its popularity and simplicity. The fundamental idea of JPDA is to compute the probabilities of all feasible measurement-to-target association events θ_k^i jointly. Then the marginal (individual measurement-to-target) association probabilities are obtained from the joint association probabilities. The target states are estimated by separate PDA (probability data association) filters using these marginal probabilities.

In the JDE framework, the posterior probability $P\{H^j|z\}$ is needed. It can be calculated by the JPDA filter without difficulty. At each time k, it is updated by

$$P\{H^j|Z^k\} = \frac{1}{c} f(z_k|H^j, Z^{k-1}) P\{H^j|Z^{k-1}\}$$

where

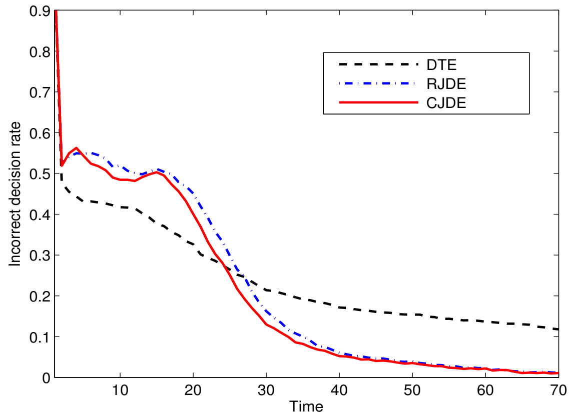
$$f(z_k|H^j, Z^{k-1}) = \sum_l f(z_k|H^j, \theta_k^l, Z^{k-1}) P\{\theta_k^l|H^j\}$$

is the likelihood of H^j, and the summation is over all possible θ_k^l, which denotes the lth measurement-to-target association event. $f(z_k|H^j, \theta_k^l, Z^{k-1})$ and $P\{\theta_k^l|H^j\}$ are obtained by JPDA. More details can be found in [21].

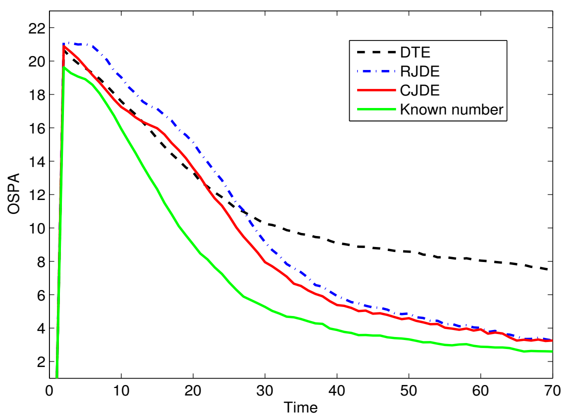
In our simulation, our proposed method is compared with DTE and RJDE in terms of OSPA and the incorrect decision rate. In DTE, first the optimal Bayes decision is made on the number of targets, which minimizes the Bayes decision risk. Then target state are estimated based on this decision. Also presented is the ideal case with a known number of targets, which sets a lower bound on the OSPA metric for all algorithms.

To jointly evaluate the performance of MTT, the OSPA metric is used since it has been widely used for MTT performance evaluation [24] [25]. Although our proposed CJDE risk uses an OSPA-like estimation cost, this risk is different from OSPA, as analyzed in Section IIIB, and the decision parameter γ_{ij} in our CJDE risk differs from the cut off value c in the OSPA metric.

The maximum number of targets is $N = 2$, and on each run, the number of targets is uniformly sampled from 1 to N and then remains constant over time. The surveillance region is $[2000\ 2000]'m \times [2000\ 2000]'m$. For simplicity, the target under H^1 has the same initial true state as the first target under H^2, which are set as $[-100m, 1m/s, -100m, 1m/s]'$, and the second target under H^2 is $[-150m, 1.5m/s, -150m, 1.5m/s]'$. The covariances of the process noise and measurement noise are $Q = \text{diag}[m^2, 0.01(m/s)^2, m^2, 0.01(m/s)^2]$ and $R = \text{diag}[100m^2, 100m^2]$, respectively. In this simulation, the detection probability $P_d = 0.75$ and there is at least one clutter measurement in the surveillance region.

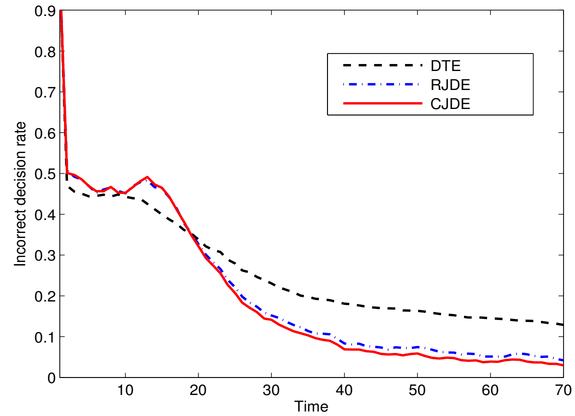


(a) Decision error rate

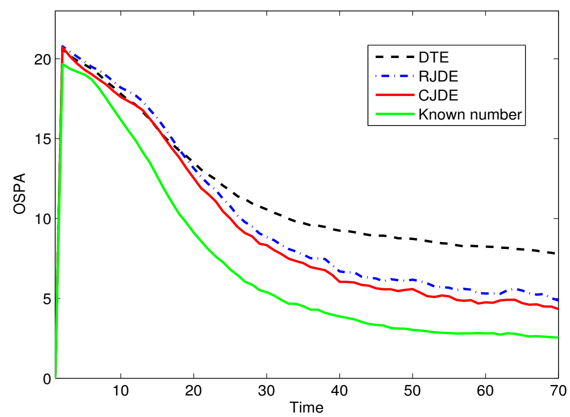


(b) OSPA

Fig. 1. Example 1: $c_{ij} = 150|i-j|, \gamma_{ij} = 400$



(a) Decision error rate



(b) OSPA

Fig. 2. Example 2:$c_{ij} = 10000|i-j|, \gamma_{ij} = 10000$

The parameters in RJDE were chosen as $\alpha_{ij} = \beta_{ij} = 1, c_{ij} = 150|i-j|$ (Example 1), $c_{ij} = 10000|i-j|$ (Example 2), and in our proposed CJDE $\beta'_{ij} = 1, \gamma_{ij} = 400$ (Example 1), $\gamma_{ij} = 10000$ (Example 2). The decision costs c_{ij} in DTE are the same as those in RJDE. In the OSPA metric, we chose $p = 2$ and the cut-off value $c = 600$. All results were obtained from 2000 Monte Carlo (MC) runs.

B. Simulation Results

In Example 1, the decision cost is $c_{ij} = 150|i-j|$, which is the same as in [21]. Fig. 1 shows that both RJDE and the proposed CJDE outperform the traditional DTE method in the decision error rate. In terms of OSPA, both RJDE and CJDE beat DTE, and CJDE outperforms RJDE.

In Example 2 $c_{ij} = 10000|i-j|$ and $\gamma_{ij} = 10000$ are both large. Fig.2 shows that the proposed CJDE still outperforms RJDE and DTE methods in terms of OSPA. Note that with large c_{ij} and γ_{ij}, the difference between the JDE methods (RJDE and CJDE) and DTE becomes smaller because the total cost is somewhat dominated by the decision cost and thus the outperformance of the JDE methods over the DTE method decreases.

Remark 4: This example verifies the effectiveness of the proposed CJDE for the MTT problem. This approach utilizes the coupling between detection and tracking, and beats the traditional DTE method in joint performance. By taking advantage of OSPA and CJDE, the proposed CJDE outperforms the existing RJDE.

It is known that for Bayes decision, if the correct decision cost is 0 and incorrect decision cost is 1 (i.e., $c_{ii} = 0, c_{ij} = 1, i \neq j$), the decision will become the Bayes optimal decision in the MAP (maximum a posteriori) sense, and it coincides with the minimum error rate decision. In this example, since JPDA is used for data association, approximations are made in calculating the posterior probability of each hypothesis. As a result, the decision in DTE method cannot be guaranteed to have the minimum decision error rate. This simulation verifies that in JDE methods, estimation helps decision and thus leads to a better decision performance.

V. CONCLUSIONS

This paper proposes a CJDE method with an OSPA-like cost for joint multitarget detection and tracking problems. For such a JDE problem, the recently proposed conditional JDE (CJDE) provides an integrated solution, which is superior in performance and simple in calculation. However, the original CJDE method cannot be applied to MTT directly since the estimation cost for the multi-target case in general is not well defined. In this paper, we propose a reasonable and tractable estimation cost, which is the key to applying CJDE to MTT.

As a widely used performance metric for MTT, OSPA considers both the localization error for tracking and the cardinality error for detection. We propose a new CJDE risk with an OSPA-like cost for MTT problems by using the similarity between the OSPA metric and the CJDE risk. The proposed risk is not only reasonable but also tractable. It takes advantage of both OSPA and CJDE. To minimize this CJDE risk, we derived the joint decision and estimation.

The effectiveness of the proposed CJDE method is demonstrated by its application to an illustrative MTT problem. Simulation results show that it outperforms the traditional DTE method in terms of OSPA and is better than a version of the recursive JDE in many cases. This illustration also demonstrates the power, the flexibility, and the simplicity of CJDE. More difficult MTT scenarios (e.g., tracking with a varying number of targets, heavy clutter density, etc.) are also under investigation.

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