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The Multiple Model Labeled Multi-Bernoulli Filter

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***Abstract*—In many applications, multi-object tracking algorithms are either required to handle different types of objects or rapidly maneuvering objects. In both cases, the usage of multiple motion models is essential to obtain excellent tracking results. In the field of random finite set based tracking algorithms, the Multiple Model Probability Hypothesis Density (MM-PHD) filter has recently been applied to tackle this problem. However, the MM-PHD filter requires error-prone post-processing to obtain target tracks and its cardinality estimate is fluctuating. The Labeled Multi-Bernoulli (LMB) filter is an accurate and computationally efficient approximation of the multi-object Bayes filter which provides target tracks. In applications using only a single motion model, LMB filter has been shown to significantly outperform the PHD filter. In this contribution, the Multiple Model Labeled Multi-Bernoulli (MM-LMB) filter is proposed. The MM-LMB filter is applied to scenarios with rapidly maneuvering objects and its performance is compared to the single model LMB filter using simulated data.**

I. Introduction

Multi-object tracking algorithms tackle the problem of jointly estimating the number of objects and their individual states using a sequence of noisy measurements. The presence of object deaths and births as well as the ambiguities in track-to-measurement association due to missed detections and false alarms render multi-object tracking significantly more challenging than single-object tracking. Starting in the 1970's, Joint Probabilistic Data Association (JPDA) [1] and Multiple Hypotheses Tracking (MHT) [2] have been popular algorithms to handle the challenges in multi-object tracking. Recently, Mahler proposed the multi-object Bayes filter [3] which uses random finite sets (RFSs) to represent the multi-object state. Thus, it naturally captures the individual objects' state uncertainty as well as the uncertainty about the number of objects in the scene.

During the last decade several moment and parameter approximations of the multi-object Bayes filter have been introduced to reduce computational complexity. The approximation of the multi-object posterior by its first moment results in the Probability Hypothesis Density (PHD) filter [4] which may be implemented using Gaussian Mixtures (GM) [5] or sequential Monte Carlo (SMC) methods [6]–[8]. The Cardinalized PHD (CPHD) filter [9], [10] approximates the multi-object Bayes filter by propagating the first moment and the cardinality distribution over time which results in a more stable cardinality estimate compared to the PHD filter. In contrast, multi-Bernoulli filters such as the Cardinality Balanced Multi-Target Multi-Bernoulli (CB-MeMBer) filter [11] approximate the multi-object posterior distribution by a multi-Bernoulli distribution and propagate its parameters over time.

In [12], Vo and Vo introduced the class of labeled RFS which augments the state vector of each object by a track label. Further, it is shown that two specific classes of labeled RFS, the Generalized Labeled Multi-Bernoulli (GLMB) and the δ-GLMB RFS, enable an analytic implementation of the multi-object Bayes filter. The δ-GLMB filter outperforms the CPHD filter due to a more accurate update step and the absence of the error-prone track extraction which is required in sequential Monte-Carlo (SMC) implementations of the CPHD filter. However, the improved performance is obtained at the cost of a higher computational complexity. The labeled Multi-Bernoulli (LMB) filter proposed in [13], [14] is an efficient approximation of the δ-GLMB filter which uses the δ-GLMB update step in each iteration. Yet, it approximates the resulting δ-GLMB distribution by an LMB distribution to reduce the computational complexity. Due to the identical update, the LMB filter delivers comparable results to the δ-GLMB filter in a wide range of applications and significantly outperforms the PHD, CPHD, and multi-Bernoulli filters [14]. In [**?**], LMB filter is used to realize the multi-sensor environment perception system of the autonomous car of Ulm University [15], [16] which demonstrates the real-time capability as well as the robustness of the LMB filter.

In vehicle environment perception, the tracking algorithm is required to track all relevant objects (e.g. cars and pedestrians) in the vehicle's surrounding which typically exhibit different motion characteristics [17]–[19]. Thus, the multi-object tracking algorithm is required to use multiple models (MM) to obtain decent estimates for the object's positions. Further, MM approaches are also required in applications with rapidly maneuvering objects. Multiple model filters are typically based on Jump-Markov models. In [20], Mahler compares several MM approaches for the PHD filter. The Jump-Markov-System (JMS) PHD filter proposed in [21] turns out to be the only mathematically sound MM-PHD filter. In [22], the MM-PHD filter is applied to time-lapse cell microscopy sequences. Further, Meissner *et al.* applied the MM-PHD filter to road user tracking at a public intersection [17]–[19] and proposed a Classifying MM-PHD filter incorporating the estimation of the objects' class into the filter.

In this contribution, a Multiple Model Labeled Multi-Bernoulli (MM-LMB) filter based on a Jump-Markov-System is proposed. The prediction of the individual tracks in the

MM-LMB filter is shown to be identical to the prediction of single-object MM filters. Further, the update equations for the MM-LMB RFS are derived. The performance of the proposed MM-LMB filter is evaluated using two scenarios. The first scenario only contains a single object which facilitates the evaluation of the switching behavior for the motion models. Further, this scenario is used to compare the performance to a standard LMB filter. The second scenario contains multiple maneuvering object as well as appearance and disappearance of objects. Hence, this scenario is used to show the multi-object tracking performance of the proposed MM-LMB filter.

This paper is organized as follows: First, the class of labeled random finite sets is reviewed. Section III outlines the Labeled Multi-Bernoulli filter and the Multiple Model Labeled Multi-Bernoulli filter is proposed in Section IV. Finally, tracking results using simulated data are presented in Section V.

II. BACKGROUND

This section summarizes briefly the class of labeled random finite sets introduced in [12] and provides the required background for the labeled multi-Bernoulli (LMB) RFS. For additional details and other labeled multi-object distributions (e.g. labeled Poisson RFS or δ-Generalized Labeled Multi-Bernoulli RFS), the reader is referred to [12], [23] and [24].

A. Notation

Throughout this contribution, the following notation is used: Single-object states are denoted by small letters (e.g. x), multi-object states by capital letters (e.g. X), and labeled distributions and states by bold face letters (e.g. $\boldsymbol{\pi}$, $\mathbf{x}$, $\mathbf{X}$). Blackboard bold letters represent spaces (e.g. the state space $\mathbb{X}$ and the measurement space $\mathbb{Z}$). Finite subsets of spaces are denoted by $\mathcal{F}(\cdot)$ and subsets comprising exactly n elements are represented by $\mathcal{F}_n(\cdot)$. For two functions $f(x)$ and $g(x)$, the inner product is abbreviated by

$$\langle f, g\rangle \triangleq \int f(x)g(x)dx.$$

Furthermore,

$$h^X \triangleq \prod_{x\in X} h(x)$$

denotes the multi-object exponential notation for real-valued functions h where $h^{\emptyset} = 1$ by definition. The generalized Kronecker delta function and the inclusion function supporting sets, vectors and integers as input arguments are given by

$$\delta_Y(X) \triangleq \begin{cases} 1, & \text{if } X = Y \\ 0, & \text{otherwise,} \end{cases}$$

$$1_Y(X) \triangleq \begin{cases} 1, & \text{if } X \subseteq Y \\ 0, & \text{otherwise.} \end{cases}$$

B. Labeled Random Finite Sets

In [12], the class of labeled RFS was introduced to enable the estimation of the objects' states and their individual trajectories within the RFS framework. To facilitate the estimation of an object's trajectory, the state vectors $x \in \mathbb{X}$ are augmented by a label $\ell \in \mathbb{L}$ where $\mathbb{L}$ is a discrete space. Hence, a labeled single-object state is given by $\mathbf{x} = (x, \ell)$ and a labeled multi-object state $\mathbf{X} = \{\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}\}$ is an RFS on the space $\mathbb{X}\times\mathbb{L}$. In multi-object tracking applications, the labels in a multi-object state are required to be distinct, i.e. there must not be two state vectors with the same label in a realization of the labeled multi-object state $\mathbf{X}$. The class of labeled RFS [12] ensures distinct labels of the tracks using the distinct label indicator

$$\Delta(\mathbf{X}) = \delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|) \tag{1}$$

which simply requires the cardinality of the RFS $|\mathbf{X}|$ to be identical to the number of track labels within this set. Using the projection $\mathcal{L}((x, \ell)) = \ell$, the set of track labels of the labeled RFS $\mathbf{X}$ is obtained by $\mathcal{L}(\mathbf{X}) = \{\mathcal{L}(\mathbf{x}) : \mathbf{x} \in \mathbf{X}\}$.

C. Labeled Multi-Bernoulli RFS

A Bernoulli RFS represents the uncertainty about the existence of a single object in an intuitive way. With probability $1-r$, the Bernoulli RFS X is given by the empty set and with probability r it is given by a singleton. Hence, the parameter r is commonly called the existence probability of an object and the probability density of a Bernoulli RFS is given by [3]:

$$\pi(X) = \begin{cases} 1-r, & \text{if } X = \emptyset, \\ r\cdot p(x), & \text{if } X = \{x\}. \end{cases} \tag{2}$$

A multi-Bernoulli RFS X is the union of M independent Bernoulli RFSs $X^{(i)}$, i.e. $X = \bigcup_{i=1}^{M} X^{(i)}$. Thus, a multi-Bernoulli RFS is completely defined by the parameter set $\{(r^{(i)}, p^{(i)})\}_{i=1}^{M}$.

A labeled multi-Bernoulli (LMB) RFS with state space $\mathbb{X}$ and label space $\mathbb{L}$ is given by the parameter set $\boldsymbol{\pi} = \{r^{(\ell)}, p^{(\ell)}\}_{\ell\in\mathbb{L}}$. Equivalently, the LMB RFS may also be represented by

$$\boldsymbol{\pi}(\mathbf{X}) = \Delta(\mathbf{X})w(\mathcal{L}(\mathbf{X}))p^{\mathbf{X}} \tag{3}$$

where the weights and the spatial distributions are given by

$$w(L) = \prod_{i\in\mathbb{L}}\left(1-r^{(i)}\right)\prod_{\ell\in L}\frac{1_{\mathbb{L}}(\ell)r^{(\ell)}}{1-r^{(\ell)}}, \tag{4}$$

$$p(x,\ell) = p^{(\ell)}(x). \tag{5}$$

III. THE LABELED MULTI-BERNOULLI FILTER

The labeled multi-Bernoulli (LMB) filter [13], [14] is an approximation of the multi-object Bayes filter [3] based on labeled random finite sets. Compared to PHD, CPHD, and CB-MeMBer filter, the LMB filter delivers significantly more accurate estimates in addition to the integrated estimation of the object's trajectories [14]. Compared to the δ-GLMB filter [12] and the marginalized δ-GLMB filter [25], the LMB filter provides almost identical results in a wide range of scenarios at a significantly lower computational complexity due to the possibility to separate the tracking problem into several statistically independent sub-problems. Further, the use of the δ-GLMB filter's update equations ensures accurate state estimates.

In the following, the prediction and update steps of the LMB filter are outlined. For additional details and the derivation of the filter, refer to [13], [14]. For notational convenience, time indices of posterior quantities are omitted and the ones of of predicted quantities are abbreviated by a "+" (e.g. $x_+ \triangleq x_{k+1|k}$).

A. Prediction

The prediction step assumes the prior density as well as the birth density to be an LMB RFS. The prior density with state space $\mathbb{X}$ and label space $\mathbb{L}$ is given by the parameter set $\boldsymbol{\pi} = \{(r^{(\ell)}, p^{(\ell)})\}_{\ell \in \mathbb{L}}$, i.e.

$$\boldsymbol{\pi}(\mathbf{X}) = \Delta(\mathbf{X}) w(\mathcal{L}(\mathbf{X})) p^{\mathbf{X}} \tag{6}$$

$$w(L) = \prod_{i \in \mathbb{L}} \left(1 - r^{(i)}\right) \prod_{\ell \in L} \frac{1_{\mathbb{L}}(\ell) r^{(\ell)}}{1 - r^{(\ell)}}, \tag{7}$$

$$p(x, \ell) = p^{(\ell)}(x). \tag{8}$$

The birth density also follows an LMB RFS with state space $\mathbb{X}$ and label space $\mathbb{B}$. It is given by the parameter set $\boldsymbol{\pi}_B = \{(r_B^{(\ell)}, p_B^{(\ell)})\}_{\ell \in \mathbb{B}}$ or equivalently by

$$\boldsymbol{\pi}_B(\mathbf{X}) = \Delta(\mathbf{X}) w_B(\mathcal{L}(\mathbf{X})) \left[p_B\right]^{\mathbf{X}}, \tag{9}$$

$$w_B(L) = \prod_{i \in \mathbb{B}} \left(1 - r_B^{(i)}\right) \prod_{\ell \in L} \frac{1_{\mathbb{B}}(\ell) r_B^{(\ell)}}{1 - r_B^{(\ell)}}, \tag{10}$$

$$p_B(x, \ell) = p_B^{(\ell)}(x). \tag{11}$$

Observe that the label space of the new born objects and the one of existing objects have to be distinct, i.e. $\mathbb{L} \cap \mathbb{B} = \emptyset$.

If the prior density and the birth density are given by (6) and (9), respectively, the predicted density is an LMB RFS with state space $\mathbb{X}$, label space $\mathbb{L}_+ = \mathbb{B} \cup \mathbb{L}$, and parameter set

$$\boldsymbol{\pi}_+ = \left\{\left(r_{+,S}^{(\ell)}, p_{+,S}^{(\ell)}\right)\right\}_{\ell \in \mathbb{L}} \cup \left\{\left(r_B^{(\ell)}, p_B^{(\ell)}\right)\right\}_{\ell \in \mathbb{B}}, \tag{12}$$

where the existence probabilities and spatial distributions of the surviving objects are determined by

$$r_{+,S}^{(\ell)} = \eta_S(\ell) r^{(\ell)}, \tag{13}$$

$$p_{+,S}^{(\ell)} = \langle p_S(\cdot, \ell) f(x|\cdot, \ell), p(\cdot, \ell)\rangle / \eta_S(\ell), \tag{14}$$

$$\eta_S(\ell) = \langle p_S(\cdot, \ell), p(\cdot, \ell)\rangle. \tag{15}$$

Here, the state dependent persistence probability of the track with label ℓ is represented by $p_S(\cdot, \ell)$ and $f(x|\cdot, \ell)$ denotes the single-object Markov transition density.

B. Update

The predicted LMB RFS on $\mathbb{X} \times \mathbb{L}_+$ is given by the parameter set

$$\boldsymbol{\pi}_+ = \left\{\left(r_+^{(\ell)}, p_+^{(\ell)}\right)\right\}_{\ell \in \mathbb{L}_+}. \tag{16}$$

Following [13], the predicted LMB RFS is converted into an equivalent δ-GLMB representation to facilitate the exact measurement update of the δ-GLMB filter. Afterwards, the multi-object posterior is approximated by the LMB RFS on $\mathbb{X} \times \mathbb{L}_+$ with parameter set

$$\boldsymbol{\pi}(\cdot|Z) = \left\{\left(r^{(\ell)}, p^{(\ell)}(\cdot)\right)\right\}_{\ell \in \mathbb{L}_+}. \tag{17}$$

The updated existence probabilities and spatial distributions of the individual tracks are calculated using

$$r^{(\ell)} = \sum_{(I_+, \theta) \in \mathcal{F}(\mathbb{L}_+) \times \Theta_{I_+}} w^{(I_+, \theta)}(Z) 1_{I_+}(\ell), \tag{18}$$

$$p^{(\ell)}(x) = \frac{1}{r^{(\ell)}} \sum_{(I_+, \theta) \in \mathcal{F}(\mathbb{L}_+) \times \Theta_{I_+}} w^{(I_+, \theta)}(Z) 1_{I_+}(\ell) p^{(\theta)}(x, \ell), \tag{19}$$

where

$$w^{(I_+, \theta)}(Z) \propto w_+(I_+) [\eta_Z^{(\theta)}]^{I_+}, \tag{20}$$

$$p^{(\theta)}(x, \ell|Z) = \frac{p_+(x, \ell) \psi_Z(x, \ell; \theta)}{\eta_Z^{(\theta)}(\ell)}, \tag{21}$$

$$\eta_Z^{(\theta)}(\ell) = \langle p_+(\cdot, \ell), \psi_Z(\cdot, \ell; \theta)\rangle, \tag{22}$$

$$\psi_Z(x, \ell; \theta) = \begin{cases} \frac{p_D(x,\ell) g(z_{\theta(\ell)}|x,\ell)}{\kappa(z_{\theta(\ell)})}, & \text{if } \theta(\ell) > 0 \\ q_D(x, \ell), & \text{if } \theta(\ell) = 0 \end{cases}. \tag{23}$$

Here, $p_D(x, \ell)$ is the state dependent detection probability of track ℓ, the missed detection probability is abbreviated by $q_D(x, \ell) = 1 - p_D(x, \ell)$, the single-object measurement likelihood is denoted by $g(z|x, \ell)$, and $\kappa(\cdot)$ is the intensity of the clutter process. The number of clutter measurements follows a Poisson distribution with an expected number of λ_c measurements. Further, Θ_{I_+} is the space of the track label to measurement assignments $\theta : I_+ \to \{0, 1, ..., |Z|\}$, where unique assignments are ensured by the property $\theta(i) = \theta(i') > 0 \implies i = i'$, As shown in [13], the posterior LMB RFS (17) with existence probabilities and spatial distributions given by (18) and (19), respectively, matches the first moment (or the PHD) of the unlabeled multi-object posterior.

IV. The Multiple-Model LMB Filter

In [20], Mahler discusses several approaches for the usage of jump-Markov systems (JMS) to track multiple maneuvering objects using the multi-object Bayes filter. In single-object JMS filters, augmented states $\tilde{x} = (x, o)$ are typically used to represent the object's state and the according motion model $o \in \mathbb{O}$ where $\mathbb{O}$ denotes the discrete space of all possible motion models. Following [20], the multi-object state has to be a finite set of augmented states, i.e.

$$\tilde{X} = \{\tilde{x}_1, \ldots, \tilde{x}_n\} = \{(x_1, o_1), \ldots, (x_n, o_n)\}. \tag{24}$$

In the context of labeled random finite sets, the corresponding multi-object state is consequently given by

$$\tilde{\mathbf{X}} = \{\tilde{\mathbf{x}}_1, \ldots, \tilde{\mathbf{x}}_n\} = \{(x_1, \ell_1, o_1), \ldots, (x_n, \ell_n, o_n)\}, \tag{25}$$

where the state vector is augmented by the track label ℓ and the mode o. The state transition matrix $\{t_{o,o'}\}_{o,o'}$ models the transition of the jump variables o and $t_{o,o'}$ denotes the probability that a track switches from mode o' to mode o.

A. Prediction

The spatial distribution of each track ℓ is given by the joint distribution

$$p^{(\ell)}(x,o) = p^{(\ell)}(x|o)p^{(\ell)}(o) \quad \forall\ o \in \mathbb{O}, \tag{26}$$

where $p^{(\ell)}(o)$ denotes the probability that track ℓ is currently in mode o and $p^{(\ell)}(x|o)$ is the spatial distribution of track ℓ conditioned on mode o.

Using (26), the prior density is assumed to be an LMB RFS on the augmented space which is given by the parameter set

$$\boldsymbol{\pi} = \left\{ \left(r^{(\ell)}, p^{(\ell)}(o)p^{(\ell)}(\cdot|o) \right) \right\}_{\ell \in \mathbb{L}}, \tag{27}$$

i.e. each track is represented by its existence probability $r^{(\ell)}$ and its spatial distribution is a joint probability density on $\mathbb{X} \times \mathbb{O}$. Consequently, the Bernoulli component of each track ℓ naturally captures the uncertainty about the current motion model.

Similar to the LMB filter in Section III, the birth distribution follows an LMB RFS. However, the spatial distributions of the tracks are required to be defined on the augmented state space $\mathbb{X} \times \mathbb{O}$, i.e.

$$\boldsymbol{\pi}_B = \left\{ \left(r_B^{(\ell)}, p_B^{(\ell)}(o)p_B^{(\ell)}(\cdot|o) \right) \right\}_{\ell \in \mathbb{B}}. \tag{28}$$

Again, the label space $\mathbb{B}$ of the birth distribution and the label space $\mathbb{L}$ of already existing objects are required to be distinct, i.e. $\mathbb{L} \cap \mathbb{B} = \emptyset$.

For the prediction step, it is assumed that the survival probability of the tracks is independent of the current motion model which is a reasonable assumption in most applications:

$$p_S^{(\ell)}(x',o') = p_S^{(\ell)}(x'). \tag{29}$$

The prediction of the joint distribution is given by

$$\begin{aligned} p_{+,S}^{(\ell)}(x,o) &= \frac{\int p_S^{(\ell)}(x')f(x,o|x',o')p^{(\ell)}(x',o')d(x',o')}{\eta_S(\ell)} \\ &= \frac{\sum_{o'\in\mathbb{O}} \int p_S^{(\ell)}(x')t_{o,o'}f(x|x',o')p^{(\ell)}(x'|o')p^{(\ell)}(o')dx'}{\eta_S(\ell)} \\ &= \sum_{o'\in\mathbb{O}} t_{o,o'}p^{(\ell)}(o')\frac{\int p_S^{(\ell)}(x')f(x|x',o')p^{(\ell)}(x'|o')dx'}{\eta_S(\ell)} \end{aligned} \tag{30}$$

where the mode transition is assumed to be independent of the state transition, i.e.

$$f(x,o|x',o') = t_{o,o'}f(x|x',o'). \tag{31}$$

Further, the normalizing constant follows

$$\eta_S(\ell) = \int p_S^{(\ell)}(x')p^{(\ell)}(x',o')d(x',o') \tag{32}$$

$$= \sum_{o'\in\mathbb{O}} p^{(\ell)}(o') \int p_S(x')p^{(\ell)}(x'|o')dx'. \tag{33}$$

The prediction of the joint distribution (30) facilitates the following factorization into the probability

$$p_{+,S}^{(\ell)}(o) = \sum_{o'\in\mathbb{O}} f(o|o')p^{(\ell)}(o') \tag{34}$$

that track ℓ is in mode o after prediction and the according spatial distribution

$$p_{+,S}^{(\ell)}(x|o) = \frac{\int p_S^{(\ell)}(x')f_o(x|x')p^{(\ell)}(x'|o')dx'}{\eta_S(\ell)} \tag{35}$$

which is conditioned on the mode o. Using (13), the predicted existence probability of track ℓ follows

$$r_{+,S}^{(\ell)} = \eta_S(\ell)r^{(\ell)}. \tag{36}$$

Hence, the predicted LMB RFS of the multiple model LMB filter is given by the parameter set

$$\boldsymbol{\pi}_+ = \left\{ \left(r_{+,S}^{(\ell)}, p_{+,S}^{(\ell)}(o)p_{+,S}^{(\ell)}(\cdot|o) \right) \right\}_{\ell \in \mathbb{L}} \tag{37}$$

$$\cup \left\{ \left(r_B^{(\ell)}, p_B^{(\ell)}(o)p_B^{(\ell)}(\cdot|o) \right) \right\}_{\ell \in \mathbb{B}} \tag{38}$$

with label space $\mathbb{L}_+ = \mathbb{B} \cup \mathbb{L}$.

B. Update

Assume that the predicted LMB RFS is given by the parameter set

$$\boldsymbol{\pi}_+ = \left\{ \left(r_+^{(\ell)}, p_+^{(\ell)}(o)p_+^{(\ell)}(\cdot|o) \right) \right\}_{\ell \in \mathbb{L}_+}. \tag{39}$$

Using the same approximation as in Section III, the multi-object posterior is obtained by the LMB RFS with parameter set

$$\boldsymbol{\pi}(\cdot|Z) = \left\{ \left(r^{(\ell)}, p^{(\ell)}(o)p^{(\ell)}(\cdot|o) \right) \right\}_{\ell \in \mathbb{L}_+}, \tag{40}$$

where the updated parameters are given by

$$r^{(\ell)} = \sum_{(I_+,\theta)\in\mathcal{F}(\mathbb{L}_+)\times\Theta_{I_+}} w^{(I_+,\theta)}(Z)1_{I_+}(\ell), \tag{41}$$

$$p^{(\ell)}(x|o) = \frac{1}{r^{(\ell)}} \sum_{(I_+,\theta)\in\mathcal{F}(\mathbb{L}_+)\times\Theta_{I_+}} w^{(I_+,\theta)}(Z)1_{I_+}(\ell)p^{(\ell,\theta)}(x|o), \tag{42}$$

$$p^{(\ell)}(o) = \frac{1}{r^{(\ell)}} \sum_{(I_+,\theta)\in\mathcal{F}(\mathbb{L}_+)\times\Theta_{I_+}} w^{(I_+,\theta)}(Z)1_{I_+}(\ell)p^{(\ell,\theta)}(o). \tag{43}$$

The measurement update of the spatial distributions conditioned on mode o are obtained by

$$p^{(\ell,\theta)}(x|o) = \frac{\psi_Z(x,\ell;\theta)p_+^{(\ell)}(x|o)}{\eta_Z^{(\theta)}(\ell|o)} \tag{44}$$

$$\eta_Z^{(\theta)}(\ell|o) = \int \psi_Z(x,\ell;\theta)p_+^{(\ell)}(x|o)dx \tag{45}$$

resembling equations (21) and (22) of the standard LMB filter update. Observe that the likelihood function

$$\psi_Z(x,\ell;\theta) = \begin{cases} \frac{p_D(x,\ell)g(z_{\theta(\ell)}|x,\ell)}{\kappa(z_{\theta(\ell)})}, & \text{if } \theta(\ell) > 0 \\ q_D(x,\ell), & \text{if } \theta(\ell) = 0 \end{cases} \tag{46}$$

is identical to (23).

The updated probability for track ℓ being in mode o is calculated by marginalizing the state x out of the updated joint probability density $p^{(\ell,\theta)}(x, o)$:

$$p^{(\ell,\theta)}(o) = \frac{\int p^{(\ell,\theta)}(x, o)dx}{\eta_Z^{(\theta)}(\ell)} \quad (47)$$

$$= \frac{\int \psi_Z(x, \ell; \theta)p_+^{(\ell)}(x|o)p_+^{(\ell)}(o)dx}{\eta_Z^{(\theta)}(\ell)} \quad (48)$$

$$= \frac{p_+^{(\ell)}(o)\eta_Z^{(\theta)}(\ell|o)}{\eta_Z^{(\theta)}(\ell)}. \quad (49)$$

Here, the normalizing constant

$$\eta_Z^{(\theta)}(\ell) = \sum_{o\in\mathbb{O}} \eta_Z^{(\theta)}(\ell|o)p_+^{(\ell)}(o) \quad (50)$$

ensures that the probabilities of the individual modes sum up to one. Further, (50) represents the likelihood of measurement $z_{\theta(\ell)}$ for track ℓ averaged over all modes $o \in \mathbb{O}$. The updated component weights are obtained by utilizing (50):

$$w^{(I_+,\theta)}(Z) \propto w_+(I_+)[\eta_Z^{(\theta)}]^{I_+}. \quad (51)$$

V. Results

The performance of the proposed MM-LMB filter is evaluated using a setup with two motion models, constant velocity (CV) and constant acceleration (CA). The results are divided into two parts: First, a single object in clutter is tracked investigated to illustrate the switching behavior of the MM-LMB filter and to compare the accuracy of the MM-LMB to an LMB filter with a single model. Additionally, a scenario containing multiple maneuvering objects is used to evaluate the performance of the MM-LMB filter.

The MM-LMB filter and the LMB filter are implemented using Gaussian Mixtures (GM). The implementation uses a partitioning of tracks and measurements into approximately independent groupings to reduce computational complexity. Further, static birth locations next to possible birth locations are used. For further implementation details, refer to [13] and [14]. In all simulations, the probability of remaining in the current motion model is $t_{o,o} = 0.98$. Further, the survival probability is $p_S = 0.99$. The sensor is assumed to deliver Cartesian measurements of the objects' x and y position with standard deviation $\sigma_x = \sigma_y = 1$. The state independent detection probability is given by $p_D = 0.98$ and clutter follows a Poisson distribution a mean number of $\lambda_c = 60$ measurements which are uniformly distributed over the measurement space $[-1000, 1000] \times [-1000, 1000]$. For the CV model, the standard deviation of the process noise is $\sigma_v = 0.3$ m/s. The CA model uses a standard deviation of $\sigma_a = 0.5$ m/s^2.

A. Single Object Tracking

Fig. 1 shows the absolute values of the velocity of the tracked object. The object is not moving for the first 10 time steps, accelerates between for $10 < k < 30$, keeps a constant velocity for $30 \leq k \leq 50$, decelerates between $50 < k < 60$, and moves with constant velocity until $k = 100$.

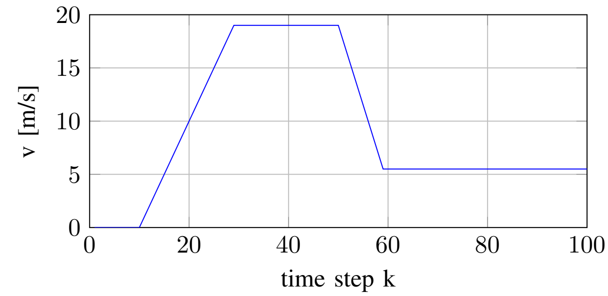

Fig. 1. Ground truth velocity of the object.

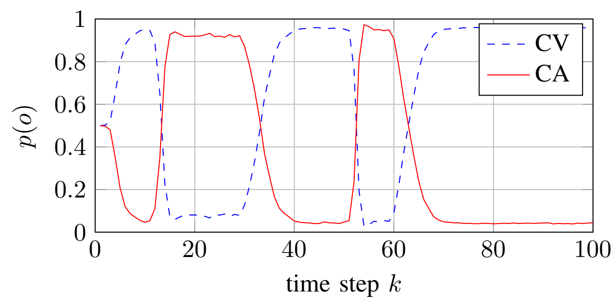

Fig. 2. Probability of the CV and CA motion models for each time step (averaged over 100 Monte Carlo runs).

The probability of the two motion models for each time step k is depicted by Fig. 2 where the result is averaged over 100 Monte Carlo runs. As expected, the MM-LMB filter switches to the CA model while the object is accelerating or decelerating. Due to the relatively small accelerations, the MM-LMB filter requires approximately 3 time steps to switch to the correct model. Further, the trajectories of the model probabilities are very smooth, i.e. the estimated motion model is very robust while the object is not maneuvering.

In Fig. 3, the performance of the MM-LMB filter is compared to an LMB filter using the OSPA distance [26]. The LMB filter uses a constant velocity model where the standard deviation of the process noise is set to $\sigma_v = 1.0$ m/s to be able to track the maneuvering object. The MM-LMB filter outperforms the LMB filter for most time steps which is expected to the usage of suitable models matching the actual behavior of the object. The MM-LMB filter shows a slightly higher OSPA distance at $k = 11$ and $k = 51$ when the object starts to accelerate and decelerate. The reason for this is the slight delay for model switching, i.e. the weight for the CV model in the MM-LMB is higher than the one for the CA model although the object already starts to accelerate. The delay is mainly due to the fact, that the switching probabilities are quite low and that the process noise of the CV model approximately covers the acceleration of the object.

B. Multi Object Tracking

The performance of the MM-LMB filter for multi-object scenarios is evaluated using the scenario depicted by Fig. 4. All

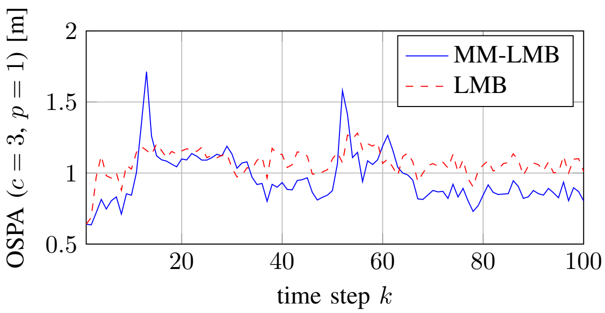

Fig. 3. Comparison of the OSPA distances of the LMB and the MM-LMB filter for the single-object scenario (averaged over 100 Monte Carlo runs).

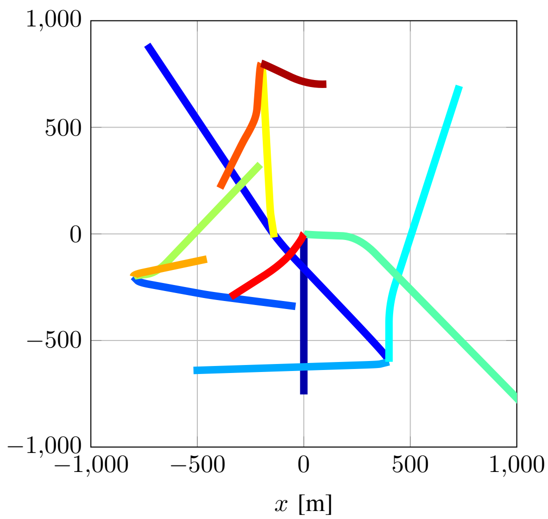

Fig. 4. Ground truth trajectories of the objects in the multi-object scenario.

objects are appearing with a low velocity and start accelerating at arbitrary time steps. The parameters of the simulation are identical to the ones used in the single object scenario.

The cardinality estimate and OSPA distance of the MM-LMB filter for the scenario depicted by Fig. 4 is shown in Fig. 5 and 6, respectively. The MM-LMB filter estimates the cardinality very precisely due to the relatively small measurement noise. Further, the OSPA distance indicates that the filter provides very accurate state estimates. The peaks of the OSPA distance are related to cardinality changes or, e.g. at $k \approx 10$, due to changes of the objects' acceleration.

VI. Conclusion

In this contribution, the Multiple Model Labeled Multi-Bernoulli (MM-LMB) filter is proposed which is capable to track maneuvering objects by using multiple motion models. The prediction of the individual tracks resembles the prediction of the multiple model single-object filters since all tracks are assumed to be statistically independent during prediction. The update step of the MM-LMB filter is similar to the one of the LMB filter by updating each component of the tracks

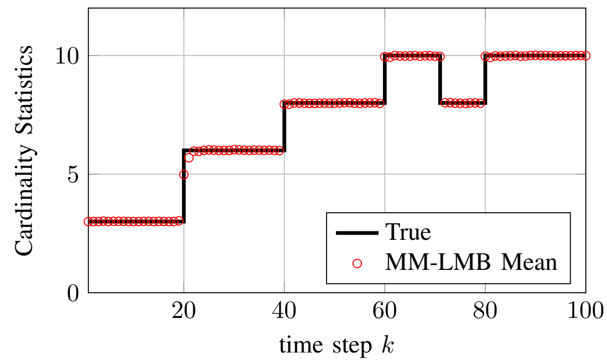

Fig. 5. Cardinality estimate and standard deviation of the cardinality estimte of the MM-LMB filter for the scenario in Fig. 4 (averaged over 100 Monte Carlo runs).

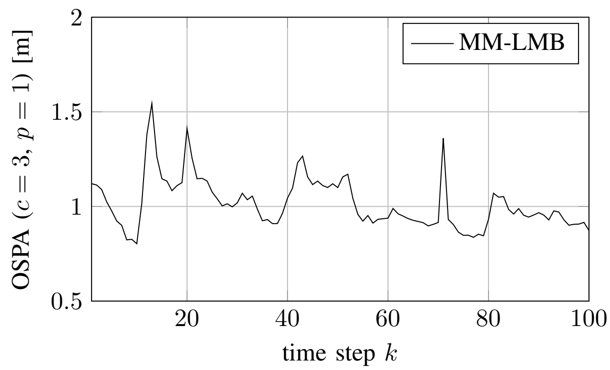

Fig. 6. OSPA distance of the MM-LMB filter for the scenario in Fig. 4 (averaged over 100 Monte Carlo runs).

spatial distribution with the assigned measurement. Here, the likelihood for assigning a measurement has to be averaged over all motion model. Further, the filter update implicitly adapts the weight of the motion models. The simulation results illustrate that the MM-LMB filter successfully adapts to the current motion model of the objects. Further, the MM-LMB filter outperforms a single model LMB filter with respect to the OSPA distance due to using more accurate process models.

In the future, the environment perception system will be realized using the proposed MM-LMB filter. Compared to the current system setup including two independent LMB filters for vehicle and pedestrian tracking, the MM-LMB filter is expected to deliver superior tracking performance since classification errors during pre-processing do not lead to missed detections in one of the trackers and false alarms in the other one.

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