Recursive Joint Track-to-Track Association and Sensor Nonlinear Bias Estimation Based on Generalized Bayes

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Abstract – Track-to-track association and sensor bias estimation are two important problems in multi-target multi-sensor tracking system. Track-to-track association becomes more complex in the presence of sensor bias and incorrect track association will lead to poor bias estimation results. Solving these two problems jointly would be attractive. This paper proposes a recursive joint track-to-track association and nonlinear bias estimation algorithm based on the generalized Bayes risk. The proposed algorithm and the conventional association-then-estimation algorithm are compared with the Monte-Carlo simulation. Simulation results show that the proposed algorithm has better track association and bias estimation performance than the conventional algorithm.

Keywords: generalized Bayes risk, recursive, track-to-track association, sensor bias estimation, joint decision and estimation.

1 Introduction

Track-to-track association and bias estimation, which are generally tightly coupled, are of great importance in multi-target multi-sensor tracking system. The coupling gives rise to the difficulties.

Much work has been carried out on solving these two problems separately. The "association-then-estimation" strategy solves the bias estimation problem and assumes the association was completely correct. The "estimation-then-association" strategy solves the association problem and assumes the bias estimation was done. These two problems may affect each other and should be considered jointly.

Several studies have been conducted on joint track-to-track association and bias estimation problem (JAE). [1] and [2] proposed a joint MAP bias estimation and data association algorithm while this algorithm describes the problem as an nonconvex mixed integer

nonlinear programming problem which is very hard to solve. A joint association, registration, and fusion approach based on expectation-maximization (EM) was proposed in [3]. However it has a drawback that expectation-maximization algorithm is a batch iterative algorithm of which the convergence speed is slow when solving complex cases. [4] proposed an extended product multi-sensor cardinalized probability hypothesis density (PM-CPHD) filter for spatial registration and data association, which leads to a more difficult problem.

The optimal Bayes joint decision and estimation (JDE) algorithm was proposed in [5] and was used to solve joint tracking and classification problem in [6]. Moreover, the optimal Bayes JDE algorithm was improved to recursive JDE (RJDE) algorithm in [7]. Optimal Bayes JDE algorithm was applied to solve JAE problem in [8]. However [8] used batch JDE rather than RJDE algorithm, and only solved linear measurement problem.

In many applications, measurements are obtained sequentially. So the computational demands of the batch JDE algorithm will increase with an increase of data. Thus RJDE algorithm would fit the problem more naturally. In this paper, we try to apply RJDE algorithm to solve JAE problem with nonlinear measurement and make it closer to reality.

This paper is organized as follows. In Section 2 we briefly describe the sensor bias model and association problem. Section 3 gives a brief introduction to the RJDE algorithm. The contribution of this paper is presented in Section 4, where we use the RJDE method to solve JAE problem with nonlinear measurement. Section 5 presents simulation results. Finally, the concluding remarks are given in Section 6.

2 **Problem formulation**

In this section, we present the association problem and the bias estimation problem.

2.1 Sensor bias model

Consider two independent sensors, s = 1, 2, tracking targets in space. Each sensor provides a set of range and azimuth measurement $\{r_{s,i}^m, \theta_{s,i}^m\}_{i=1}^{n_s}$, where n_s is the number of the targets which are detected by sensor *s* with no clutter.

Range $r_{s,i}^m$ and azimuth $\theta_{s,i}^m$, for each $i = 1, \dots, n_s$, s = 1, 2 can be modeled by

$$r_{s,i}^{m} = r_{s,i} + r_{bs} + \tilde{r}_{s} \tag{1}$$

$$\theta_{s_i}^m = \theta_{s_i} + \theta_{bs} + \tilde{\theta}_s \tag{2}$$

In the above, $r_{s,i}^m$ and $\theta_{s,i}^m$ are the measured range and azimuth; $r_{s,i}$ and $\theta_{s,i}$ are the true range and azimuth; r_{bs} and θ_{bs} are the sensor biases for the range and azimuth, which do not change over time; the measurement noises \tilde{r}_s and $\tilde{\theta}_s$ are zero-mean Gaussian random white noise with corresponding variances $\sigma_{r,s}^2$ and $\sigma_{\theta,s}^2$, and are assumed mutually independent of each other.

2.2 Definition of association

Using converted measurement Kalman filtering (CMKF) to estimate targets state individually, we get two sets of tracks $\{\tau_{si}\}_{i=1}^{n_s}$, s = 1, 2. Each track τ_{si} is composed of a set of $(\hat{x}_{si}, \hat{P}_{si})$ pairs, where \hat{x}_{si} is a target state estimate vector, and \hat{P}_{si} is a state estimation error covariance matrix.

Given the two sets of tracks, $\{\tau_{si}\}_{i=1}^{n_i}$, s = 1, 2, a track-to-track association hypothesis a could be defined as a one-to-one function Dom(a) with $\{1, \dots, n_1\}$ as it definitional domain and $\{1, \dots, n_2\}$ as it value domain. j = a(i) means that the *i*-th track from sensor 1 and the *j*-th track from sensor 2 share the same origin. $i \notin \text{Dom}(a)$ means that the *i*-th track from sensor 1 is not detected by sensor 2, and $j \notin \text{Im}(a)$ means that the *j*-th track from sensor 1. The set of all the association hypotheses is defined as $A(n_1, n_2) \triangleq \{s : D \to \{1, \dots, n_2\} | D = \text{Dom}(a) \subseteq \{1, \dots, n_1\}, \# \text{Im}((a)) = \#(D)\}$ where #(A) is the cardinality of the

3 Recursive joint decision and estimation

set A.

The conventional Bayes risk for decision is as follows.

$$\overline{R}_{D} = \sum_{i,j=1}^{N} c_{ij} P\{"H_{i}"|H_{j}\} P\{H_{j}\}$$
(3)

where $P\{H_j\}$ is the prior probability of H_j and c_{ij} is the cost of deciding on hypothesis H_i while H_j is true. In the Bayesian approach, the optimal decision is the one that minimizes \overline{R}_D . The optimal Bayes decision decides on H_i if its posterior cost $C_i(z) = \sum_j c_{ij} P\{H_j \mid z\}$ is the smallest, i.e., $C_i(z) \le C_k(z), \forall k$.

The conventional Bayes risk for estimation is $\overline{R}_E = E[C(\tilde{x})]$, where \overline{R}_E is the expectation of a cost function of the estimation error \tilde{x} . An optimal Bayes estimator is a function of measurements z that minimizes the Bayes risk, that is, $\hat{x} = \arg\min_{\hat{x}(z)} E[C(\tilde{x})]$.

For the JDE problem, [5] proposed an optimal algorithm based on the following generalized Bayes risk

$$\overline{R} = \sum_{i=1}^{M} \sum_{j=1}^{N} (\alpha_{ij} c_{ij} + \beta_{ij} E[C(x, \hat{x}) | D_i, H_j]) P\{D_i, H_j\}$$
(4)

where D_i is the *i*-th decision in other words $\{z \in D_i\}$; c_{ij} is the cost of decision D_i while the hypothesis H_j is true; $C(x, \hat{x})$ is the estimation cost function; $E[C(x, \hat{x}) | D_i, H_j]$ is the expected cost conditioned on the case that D_i is decided while H_j is true; α_{ij} and β_{ij} are nonnegative weights of decision and estimation costs, respectively, which is variable to different cases.

This new Bayes risk \overline{R} generalized conventional Bayes risk for decision and conventional Bayes risk for estimation, and expressed the inter-dependence between decision and estimation.

The generalized Bayes risk provides an approach to JDE problem. While various $C(x, \hat{x})$ is available, [7] chooses mean square error (MSE) $C(x, \hat{x}) = \tilde{x}'\tilde{x}$ as estimation cost criterion.

For given $E[C(x, \hat{x}) | D_i, H_j]$, to minimize \overline{R} of Eq.(4), the optimal decision D is

 $D = D_i \text{ if } C_i(z) \le C_k(z), \forall k$ (5) where, the posterior cost is given by

$$C_{j}(z) = \sum_{j} (\alpha_{ij}c_{ij} + \beta_{ij}E[C(x,\hat{x}) | D_{i}, H_{j}])P\{H_{j} | z\}$$
(6)

Given a set of decision regions $\{D_1, \dots, D_M\}$ as a partition of the data space, the optimal estimator for (4) with MSE criterion is the following generalized posterior mean

$$\hat{x} = \sum_{i,i} \hat{x}_{ij} \overline{P} \{ D_i, H_j \mid z \}$$
⁽⁷⁾

where, for $z \in D_i$

$$\hat{x}_{ij} = E\left[x \mid z, D_i, H_j\right] = \hat{x}_j = E\left[x \mid z, H_j\right]$$

$$\begin{split} \overline{P}\{D_{i},H_{j} \mid z\} &= \frac{\beta_{ij}P\{D_{i},H_{j} \mid z\}}{\sum_{l,k}\beta_{lk}P\{D_{l},H_{k} \mid z\}} \\ &= \frac{\beta_{ij}P\{H_{j} \mid z\}}{\sum_{l,k}1(z;D_{l})\beta_{lk}P\{H_{k} \mid z\}} \\ \mathbf{1}(z;D_{i}) &= \begin{cases} 1, & z \notin D_{i} \\ 0, & \text{else} \end{cases} \\ P\{H_{j} \mid z,D_{i}\} &= P\{H_{j} \mid z\} \end{split}$$

Moreover, these quantities are not defined if $z \notin D_i$. If $z \in D_i$ means $z \notin D_i, \forall k \neq i$, Eq.(7) can be simplified to

$$\hat{x} = \sum_{i} l(z; D_i) \breve{x}_i \tag{8}$$

where, for $z \in D_i$

$$\vec{x}_i = \vec{E} [x \mid z] = \sum_j \hat{x}_j \vec{P} \{ H_j \mid z \}$$

$$\vec{P} \{ H_j \mid z \} = \frac{\beta_{ij} P\{ H_j \mid z \}}{\sum_k \beta_{ik} P\{ H_k \mid z \}}$$

and these quantities are not defined if $z \notin D_i$.

The RJDE algorithm tries to find a JDE solution recursively based on sequential data. The batch JDE algorithm computes the decision partition of the space Z^k of all past data $\{z_1, z_2, \dots, z_k\}$. While in the recursive JDE algorithm, only the space Z_k of the current data z_k is partitioned conditioning on all previous data Z^{k-1} . And the RJDE algorithm is shown as follows, which could be proved convergent.

- 1) Initialize the parameters k = 0, \mathcal{E}_{ij}^0 and $P\{H_j\}$.
- 2) Compute the posterior cost at time k $C_{i}^{k} \left(Z^{k} \right) = \sum_{j} \left(\alpha_{ij} c_{ij}^{k} + \beta_{ij} \varepsilon_{ij}^{k} \right) P\{H_{j} \mid Z^{k}\}$ (9)
- 3) At time k+1, update $\hat{x}_{k}^{(j)} \triangleq E[x_{k} \mid Z^{k}, H_{j}] \rightarrow \hat{x}_{k+1}^{(j)}$ $P\{H_{j} \mid Z^{k}\} \rightarrow P\{H_{j} \mid Z^{k+1}\}$

These quantities are the functions of measurement z_{k+1} and could be obtained once z_{k+1} is available.

- 4) Compute the intermediate cost $C_i^k(z_{k+1}|Z^k)$ by replacing $P\{H_i | Z^k\}$ in Eq.(9) with $P\{H_i | Z^{k+1}\}$.
- 5) Based on the newly updated $C_i^k(z_{k+1}|Z^k)$, update the decision partition $\{D^{k+1}|Z^k\}$.
- 6) Based on decision partition {D^{k+1}|Z^k} calculate the conditional expected estimation cost
 c_{ij}^{k+1} ≜ mse(x̂_{k+1} | Z^k, D_i^{k+1}, H_j)
- 7) Replace \mathcal{E}_{ij}^{k} with \mathcal{E}_{ij}^{k+1} to update $C_{i}^{k}(z_{k+1}|Z^{k})$ to $C_{i}^{k+1}(z_{k+1}|Z^{k})$.

- 8) Based on the newly updated $C_i^{k+1}(z_{k+1}|Z^k)$, update the decision partition $\{D^{k+1}|Z^k\}$.
- 9) Go to step 6 until the termination conditions are satisfied. Output the RJDE solution of time k+1.
- 10) The posterior cost $C_i^{k+1}(Z^{k+1}) = C_i^{k+1}(z_{k+1} | Z^k)$ at time k+1 is obtained by taking the latest measurement z_{k+1} into account. Set k = k+1 and go to step 2.

4 Recursive joint track-to-track association and bias estimation

Consider two independent sensors, s = 1, 2, which track targets in surveillance region. The sensors provide the range and azimuth $\{r_{s,j}^m, \theta_{s,i}^m\}_{i=1}^{n_s}$ of targets, where n_s is the number of the targets which are detected by sensor *s* with no clutter.

We suppose that the dynamic equation of the targets is

$$X_{k+1} = FX_k + Gw$$

where state transition matrix $F = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$;

(10)

and noise gain matrix $G = \begin{bmatrix} \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \frac{T^2}{2} & T \end{bmatrix}^T$.

Moreover, we suppose that the set of hypotheses and the set of decisions are defined as

$$\{H_j, j = 1, \cdots, N\} = A(n_1, n_2)$$
(11)

$$\{D_j, j = 1, \cdots, M\} = A(n_1, n_2)$$
(12)

where, H_j is the *j*-th hypothesis; D_i is the *i*-th decision; M = N is the cardinality of the set $A(n_1, n_2)$ which stands for the number of track-to-track association cases, i.e., the decisions and the hypotheses are one-to-one correspondent.

We suppose that the initial probabilities of hypotheses are equal, i.e., we do not have any idea of which hypothesis is closer to the right one.

$$P\{H_j\} = \frac{1}{M}, j = 1, \cdots, M$$
 (13)

4.1 Estimate sensor bias $\hat{\zeta}_k^{(j)}$

Under the condition that hypothesis H_j is true, suppose j = a(i). Equations are given as follows

$$(r_{1,i}^{m} - r_{b1} - \tilde{r}_{1})\cos(\theta_{1,i}^{m} - \theta_{b1} - \tilde{\theta}_{1}) + x_{s1}$$

= $(r_{2,j}^{m} - r_{b2} - \tilde{r}_{2})\cos(\theta_{2,j}^{m} - \theta_{b2} - \tilde{\theta}_{2}) + x_{s2}$ (14)

$$(r_{1,i}^{m} - r_{b1} - \tilde{r}_{1})\sin(\theta_{1,i}^{m} - \theta_{b1} - \tilde{\theta}_{1}) + y_{s1}$$

= $(r_{2,j}^{m} - r_{b2} - \tilde{r}_{2})\sin(\theta_{2,j}^{m} - \theta_{b2} - \tilde{\theta}_{2}) + y_{s2}$ (15)

where, the location of sensor 1 and sensor 2 is (x_{s1}, y_{s1}) and (x_{s2}, y_{s2}) , respectively.

Using Taylor series expansion, we expand Eq.(14) and (15) at $\Delta r_{b1} = \Delta r_{b2} = 0, \Delta \theta_{b1} = \Delta \theta_{b2} = 0$, and obtain

$$\begin{bmatrix} r_{1,i}^{m} \cos \theta_{1,i}^{m} \\ r_{1,i}^{m} \sin \theta_{1,i}^{m} \end{bmatrix} + J(r_{1,i}^{m}, \theta_{1,i}^{m})\zeta_{1} + \begin{bmatrix} x_{s1} \\ y_{s1} \end{bmatrix} + v_{1}$$

$$= \begin{bmatrix} r_{2,j}^{m} \cos \theta_{2,j}^{m} \\ r_{2,j}^{m} \sin \theta_{2,j}^{m} \end{bmatrix} + J(r_{2,j}^{m}, \theta_{2,j}^{m})\zeta_{2} + \begin{bmatrix} x_{s2} \\ y_{s2} \end{bmatrix} + v_{2}$$
(16)

where, sensor bias of sensor 1 and sensor 2 is $\zeta_1 = \begin{vmatrix} r_{b_1} \\ \theta_{b_2} \end{vmatrix}$

and
$$\zeta_2 = \begin{bmatrix} r_{b_2} \\ \theta_{b_2} \end{bmatrix}$$
, respectively;

$$J(r,\theta) = \begin{bmatrix} -\cos\theta & r\sin\theta \\ -\sin\theta & -r\cos\theta \end{bmatrix} ;$$

$$v_1 = \begin{bmatrix} \tilde{x}_1^m \\ \tilde{y}_1^m \end{bmatrix} \text{ and } v_2 = \begin{bmatrix} \tilde{x}_2^m \\ \tilde{y}_2^m \end{bmatrix} \text{ are independent zero-mean}$$

random noise with corresponding covariance $R_1(r_{1,i}^m, \theta_{1,i}^m, \sigma_{r_1}, \sigma_{\theta_1})$ and $R_2(r_{2,i}^m, \theta_{2,i}^m, \sigma_{r_2}, \sigma_{\theta_2})$. Where R_1 and R_2 are covariance matrixes of convert measurement noise

$$R(r,\theta,\sigma_r,\sigma_{\theta}) = \begin{bmatrix} r^2 \sigma_{\theta}^2 \sin^2 \theta + \sigma_r^2 \cos^2 \theta & [\sigma_r^2 - r^2 \sigma_{\theta}^2] \sin \theta \cos \theta \\ [\sigma_r^2 - r^2 \sigma_{\theta}^2] \sin \theta \cos \theta & r^2 \sigma_{\theta}^2 \cos^2 \theta + \sigma_r^2 \sin^2 \theta \end{bmatrix}$$
(17)

Eq.(16) could be simplified as

$$\begin{pmatrix} \begin{bmatrix} r_{2,j}^{m} \cos \theta_{2,j}^{m} \\ r_{2,j}^{m} \sin \theta_{2,j}^{m} \end{bmatrix} + \begin{bmatrix} x_{s2} \\ y_{s2} \end{bmatrix} - \begin{pmatrix} \begin{bmatrix} r_{1,i}^{m} \cos \theta_{1,i}^{m} \\ r_{1,i}^{m} \sin \theta_{1,i}^{m} \end{bmatrix} + \begin{bmatrix} x_{s1} \\ y_{s1} \end{bmatrix} \end{pmatrix}$$
(18)
$$= H(r_{1,i}^{m}, \theta_{1,i}^{m}, r_{2,j}^{m}, \theta_{2,j}^{m})\zeta + v$$

where $\zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}$ is the parameter vector we desire to

estimate; $H(r_{1,i}^{m}, \theta_{1,i}^{m}, r_{2,j}^{m}, \theta_{2,j}^{m}) = [J(r_{1,i}^{m}, \theta_{1,i}^{m}) - J(r_{2,j}^{m}, \theta_{2,j}^{m})]$ is linearized measurement matrix; and v is zero-mean random noise with corresponding covariance $R = R_1 + R_2$.

For Eq.(18), the formulation on the left of the equal sign is pseudo measurement which is remarked as z in this paper.

We assume that the sensor bias does not change over time. Therefore the dynamic equation of sensor bias could be given as

$$\zeta_{k+1} = \zeta_k \tag{19}$$

Extended Kalman filtering (EKF) is applied to estimate $\hat{\zeta}_{k}^{(j)}$, while hypothesis H_{j} is true.

At the beginning, initialize the estimatee (i.e., the quantity to be estimated) $\hat{\zeta}_{0}^{(j)} = 0$ and estimation error covariance $\hat{P}_0^{(j)} = \sigma^2 I$, where σ is a large positive real number.

For time k,

$$\hat{\zeta}_{k+1}^{(j)} = \hat{\zeta}_{k+1|k}^{(j)} + K_{k+1}^{(j)}(z_{p,k+1} - \hat{z}_{p,k+1|k}^{(j)})$$

$$P_{k+1}^{(j)} = P_{k+1|k}^{(j)} - K_{k+1}^{(j)}S_{k+1}^{(j)}(K_{k+1}^{(j)})'$$
(20)
(21)

where,

S k

$$\hat{\zeta}_{k+1|k}^{(j)} = \hat{\zeta}_{k}^{(j)}
\hat{z}_{p,k+1|k}^{(j)} = H_{k}^{(j)} \hat{\zeta}_{k+1|k}^{(j)}
P_{k+1|k}^{(j)} = P_{k}^{(j)}
S_{k+1}^{(j)} = H_{k}^{(j)} P_{k+1|k}^{(j)} H_{k}^{(j)'} + R_{k}^{(j)}
K_{k+1}^{(j)} = P_{k+1|k}^{(j)} (H_{k}^{(j)})' (S_{k+1}^{(j)})^{-1}$$

4.2 The choice of c_{ii}

 c_{ii} is the cost of the decision D_i while the hypothesis H_i is true. When $i \neq j$, it is the cost of an incorrect decision, while c_{ii} is the cost of a correct decision. Therefore, $c_{ii} < c_{ij}, \forall i \neq j$ is needed to punish the incorrect decision. c_{ij} not only could be variable under different steps of algorithm, but also could be constant. In this paper we set $c_{ii} = 0, c_{ii} = 1, i \neq j$.

Compute $P\{H_i | Z^{k+1}\}$ 4.3

Assume that the posterior probability of hypothesis H_i at time k is $P\{H_i | Z^{k+1}\}$. Then at time k+1

$$P\{H_{j} | Z^{k+1}\} = \frac{f(z_{p,k+1} | Z^{k}, H_{j}) P\{H_{j} | Z^{k}\}}{\sum_{l} f(z_{p,k+1} | Z^{k}, H_{l}) P\{H_{l} | Z^{k}\}}$$

$$= \frac{N_{k+1}^{(j)}(z_{p,k+1}) P\{H_{j} | Z^{k}\}}{\sum_{l} N_{k+1}^{(j)}(z_{p,k+1}) P\{H_{l} | Z^{k}\}}$$
where $N_{k+1}^{(j)}(z_{p,k+1}) = N(z_{p,k+1}; \hat{z}_{p,k+1|k}^{(j)}, S_{k+1}^{(j)})$

$$(22)$$

4.4 Compute $\hat{\zeta}_k$

 $\hat{\zeta}_k$ is sensor bias estimation, which is given by

$$\hat{\zeta}_k = \sum_i \mathbb{1}(z^k; D_i^k) \breve{\zeta}_k^{(i)}$$
(23)

where, for $Z^k \in D_i^k$

$$\overline{\zeta}_{k}^{(i)} = \overline{E}[\zeta \mid Z^{k}] = \sum_{j} \widehat{\zeta}^{(j)} \overline{P}\{H_{j} \mid Z^{k}\}$$
$$\overline{P}\{H_{j} \mid Z^{k}\} = \frac{\beta_{ij} P\{H_{j} \mid Z^{k}\}}{\sum_{k} \beta_{ik} P\{H_{k} \mid Z^{k}\}}$$

and these quantities are not defined if $Z^k \in D_i^k$.

4.5 Calculate estimation cost

Based on $z^{k+1} \in D_i^{k+1}$, to calculate the conditional expected estimation cost, we have

$$\varepsilon_{ij}^{k+1} \triangleq E[C(\zeta_{k+1}, \zeta_{k+1}) | D_i^{k+1}, H_j, Z^{k+1}]$$

$$= \operatorname{mse}(\hat{\zeta}_{k+1} | D_i^{k+1}, H_j, Z^{k+1})$$

$$= E\left[\operatorname{mse}(\hat{\zeta}_{k+1}^{(ij)} | D_i^{k+1}, H_j, Z^{k+1}) | D_i^{k+1}, H_j, Z^k\right]$$

$$+ E[(\hat{\zeta}_{k+1}^{(ij)} - \hat{\zeta}_{k+1})^2 | D_i^{k+1}, H_j, Z^{k+1}]$$

$$= E\left[\operatorname{mse}(\hat{\zeta}_{k+1}^{(i)} | D_i^{k+1}, H_j, Z^{k+1}) | D_i^{k+1}, H_j, Z^k\right]$$

$$+ E[(\hat{\zeta}_{k+1}^{(i)} - \bar{\zeta}_{k+1})^2 | D_i^{k+1}, H_j, Z^{k+1}]$$

where $\operatorname{mse}(\hat{\zeta}_{k+1}^{(j)} | D_i^{k+1}, H_j, Z^{k+1}) = \operatorname{tr}(P_{k+1}^{(j)})$ is given in Eq.(21).

 $\tilde{\varepsilon}_{ij}^{k+1} \triangleq E[(\hat{\zeta}_{k+1}^{(j)} - \breve{\zeta}_{k+1}^{(i)})'(\hat{\zeta}_{k+1}^{(j)} - \breve{\zeta}_{k+1}^{(i)}) | D_i^{k+1}, H_j, Z^{k+1}]. \text{ And}$

 $\operatorname{tr}(P_{k+1}^{(j)})$ is independent from Z^{k+1} , so we could obtain

$$\varepsilon_{ij}^{k+1} = E\left[\operatorname{tr}\left(P_{k+1}^{(j)}\right) | Z^{k}, D_{i}^{k+1}, H_{j}\right]$$
$$= \operatorname{tr}\left(P_{k+1}^{(j)}\right) + \tilde{\varepsilon}_{ij}^{k+1}$$
(24)

Based on $z^{k+1} \in D_i^{k+1}$,

$$\begin{split} \hat{\zeta}_{k+1}^{(j)} - \bar{\zeta}_{k+1}^{(i)} &= \hat{\zeta}_{k+1}^{(j)} - \sum_{r} \hat{\zeta}_{k+1}^{(r)} \frac{\beta_{ir} P\{H_r \mid Z^{k+1}\}}{\sum_{l} \beta_{ll} P\{H_l \mid Z^{k+1}\}} \\ &= \frac{\sum_{r} (\hat{\zeta}_{k+1}^{(j)} - \hat{\zeta}_{k+1}^{(r)}) \beta_{ir} P\{H_r \mid Z^{k+1}\}}{\sum_{l} \beta_{ll} P\{H_l \mid Z^{k+1}\}} \end{split}$$

Therefore

$$\begin{split} \tilde{\varepsilon}_{ij}^{k+1} &\triangleq E[(\hat{\zeta}_{k+1}^{(i)} - \breve{\zeta}_{k+1}^{(i)})'(\hat{\zeta}_{k+1}^{(j)} - \breve{\zeta}_{k+1}^{(i)}) \mid D_i^{k+1}, H_j, Z^{k+1}] \\ &= \int_{z_{k+1} \in D_i^{k+1}} (\hat{\zeta}_{k+1}^{(j)} - \breve{\zeta}_{k+1}^{(i)})'(\hat{\zeta}_{k+1}^{(j)} - \breve{\zeta}_{k+1}^{(i)}) dF(z_{k+1} \mid \hat{\zeta}_k^{(j)}, H_j) \end{split}$$

We could approximate $\tilde{\mathcal{E}}_{ij}^{k+1}$ numerically by the Monte Carlo method $\tilde{\tilde{\mathcal{E}}}_{ij}^{k+1}$, since $\tilde{\mathcal{E}}_{ij}^{k+1}$ is very hard to calculate

$$\tilde{\tilde{\varepsilon}}_{ij}^{k+1} \approx \frac{1}{L_i} \sum_{l_i=1}^{L_i} \left[\hat{\zeta}_{k+1}^{(j)} \left(z_{k+1,l_i}^{m(ij)} \right) - \breve{\zeta}_{k+1}^{(i)} \left(z_{k+1,l_i}^{m(ij)} \right) \right]^2$$
(25)

where, $z_{k+1,l_i}^{m(j)}(l_i = 1, 2, \dots, L_i)$ are the simulated measurements from the distribution $f(z_{k+1}^m | \hat{x}_k^{(j)}, H_j)$ that lie inside the decision region D_i^{k+1} , while $L \triangleq \sum_i L_i$. In addition, $\hat{x}_k^{(j)}$ are the estimated target states while hypothesis H_j is true. While the target states and the sensor measurements are independent from the association hypotheses, $f(z_{k+1}^m | \hat{x}_k^{(j)}, H_j)$ should be given as $f(z_{k+1}^m | \hat{x}_k)$ or $f(z_{k+1}^m | \hat{x}_{k+1|k})$.

$$\hat{\zeta}_{k+1}^{(j)}\left(z_{k+1,l_{i}}^{m(ij)}\right) = E[\zeta_{k+1} \mid z_{k+1,l_{i}}^{m(ij)}, Z^{k}, H_{j}]$$

$$\begin{split} & \vec{\zeta}_{k+1}^{(i)} \left(z_{k+1,l_i}^{m,(ij)} \right) = \sum_j \hat{\zeta}_{k+1}^{(i)} \left(z_{k+1,l_i}^{m,(ij)} \right) \times \overline{P} \{ H_j \mid z_{k+1,l_i}^{m,(ij)}, Z^k \} \\ = & \sum_j \hat{\zeta}_{k+1}^{(j)} \left(z_{k+1,l_i}^{m,(ij)} \right) \times \frac{\beta_{ij} P\{H_j \mid z_{k+1,l_i}^{m,(ij)}, Z^k\}}{\sum_l \beta_{il} P\{H_l \mid z_{k+1,l_i}^{m,(ij)}, Z^k\}} \end{split}$$

could be calculated by the Kalman filtering. If some D_i^{k+1} are empty, $\hat{\zeta}_{k+1}^{(j)}(z_{k+1,l_i}^{m(ij)})$ and $\breve{\zeta}_{k+1}^{(i)}(z_{k+1,l_i}^{m,(ij)})$ could be replaced by predictions.

4.6 Algorithm description

2)

The recursive joint track-to-track association and bias estimation algorithm could be described as follows:

1) Initialize k = 0, c_{ij}^0 , \mathcal{E}_{ij}^0 , $P\{H_j | Z^0\} = \frac{1}{M}$, $j = 1, \dots, M$.

For time k, calculate posterior cost

$$C_i^k (Z^k) = \sum_i (\alpha_{ij} c_{ij}^k + \beta_{ij} \varepsilon_{ij}^k) P\{H_j \mid Z^k\}$$
(26)

where, Z^k is the space of all past data.

3) For time k+1, update $\hat{\zeta}_{k}^{(j)} \triangleq E[\zeta_{k} | Z^{k}, H_{j}] \rightarrow \hat{\zeta}_{k+1}^{(j)}$ $P\{H_{i} | Z^{k}\} \rightarrow P\{H_{i} | Z^{k+1}\}$

In addition, these quantities are functions of z_{k+1} .

- 4) Based on $\hat{\zeta}_{k+1}^{(j)}$, calculate association cost c_{ij}^{k+1} . In this paper, we set $c_{ii}^k = c_{ii} = 0, c_{ij}^k = c_{ij} = 1, i \neq j$.
- 5) Replace $P\{H_j | Z^k\}$ with updated $P\{H_j | Z^{k+1}\}$ and replace c_{ij}^k with c_{ij}^{k+1} in Eq.(26) to calculate intermediate cost $C_i^k (z_{k+1} | Z^k) = \sum_j (\alpha_{ij} c_{ij}^{k+1} + \beta_{ij} \varepsilon_{ij}^k) P\{H_j | Z^{k+1}\}$ (27)

and $C_i^k(z_{k+1}|Z^k)$ are functions of z_{k+1} .

- 6) Based on Eq.(27), we could determine the decision partition of current data z_{k+1} space Z_{k+1} . $\left\{D^{k+1} \mid Z^k\right\} = \left\{D_1^{k+1}, \cdots, D_M^{k+1} \mid Z^k\right\}$ $D_i^{k+1} = \left\{z : C_i^{k+1}(z) \le C_m^{k+1}(z), \forall m\right\}$
- 7) Based on the current available partition of Z_{k+1} , compute the conditioned expected estimation cost $\varepsilon_{ij}^{k+1} \triangleq \operatorname{mse}(\hat{\zeta}_{k+1} | D_i^{k+1}, H_j, Z^{k+1})$ (28)
- 8) Replace ε_{ij}^{k} in the Eq.(26) with formulation (28), in order to update the intermediate cost $C_{i}^{k}(z_{k+1}|Z^{k})$ to the posterior cost $C_{i}^{k+1}(z_{k+1}|Z^{k})$ $C_{i}^{k+1}(z_{k+1}|Z^{k}) = \sum_{j} (\alpha_{ij}c_{ij}^{k+1} + \beta_{ij}\varepsilon_{ij}^{k+1})P\{H_{j} | Z^{k+1}\}$ (29) And $C_{i}^{k+1}(z_{k+1}|Z^{k})$ is also a function of z_{k+1} .

- 9) Recalculate the decision partition $\{D_1^{k+1}, \dots, D_M^{k+1} | Z^k\}$, based on newly updated $C_i^{k+1}(z_{k+1}|Z^k)$.
- 10) Go to step 7 until the termination conditions are satisfied. The termination conditions are: a) the decision does not change between two iterations; b) the change of the expected estimation cost is smaller than a threshold.

If termination conditions are satisfied, output track-to-track association and bias estimation result of time k+1.

$$\hat{D}^{k+1} = \{ \mathbf{D}_i : \mathbf{z}_{k+1} \in \mathbf{D}_i^{k+1} \}$$
(30)

$$\hat{\zeta}_{k+1} = \overline{\zeta}_{k+1}^{(i)} = \sum_{j} \hat{\zeta}_{k+1}^{(j)} \frac{\beta_{ij} P\{H_j \mid Z^{k+1}\}}{\sum_{l} \beta_{il} P\{H_l \mid Z^{k+1}\}}$$
(31)

11) Record
$$C_i^{k+1}(z_{k+1}|Z^k)$$
 as $C_i^{k+1}(Z^{k+1})$. Then set $k = k+1$ and so to stop 2

k = k + 1, and go to step 2.

The recursion of steps 7-10 is guaranteed to converge, which can be proved similarly as the case of RJDE.

Since measurements are coming sequentially, the proposed algorithm may determine association relationship and estimate sensor biases in real time.

5 Simulation

In order to compare the performance of our algorithm and association-then-estimation algorithm, we chose a two-dimensional two-sensor scenario.

In the association-then-estimation method, we used nearest-neighbor algorithm to obtain association relationship, and then estimated sensor bias based on the obtained association relationship.

In our simulation, three targets from different location move from left to right with a constant velocity, as shown in Figure 1. The lines represent the trajectories of targets, which move together and then apart. The overall time is 100s. Figure 2 illustrates the sensor measured target trajectories.



Figure 1. Target trajectories.



Figure 2. Sensor measurements.

The sensor location of sensor1 and sensor2 is (0m, 0m)and (6000m, 0m), respectively. The sensor bias of sensor1 and sensor2 is $(-80m, 0.9^{\circ})$ and $(-40m, 0.5^{\circ})$. The two sensors have random measurement noises whose standard deviation is (10m, 0.1°). The weights α_{ij} and β_{ij} of the algorithm were chosen by cases. While in this paper, we do not go into details and only give the results $\alpha_{ii} = 1$, $\beta_{ii} = 1, \beta_{ii} = 0.1, i \neq j$.



Figure 3. Association performance of the RJAE and the association-then-estimation algorithm.





Figure 4. Bias estimation performance of the RJAE and the association-then-estimation algorithm.

Figures 3-4 illustrate the results of 100 Monte-Carlo simulations. Figure 3 shows the probability of correct association and Figure 4 illustrates the root-mean-square error (RMSE) results of the bias estimation.

In terms of the probability of correct association (PCA), it can be seen from Figure 3 that the association-then-estimation algorithm performs better than the JAE at the beginning of the simulation. This is because that the estimation of sensor bias of all hypotheses H_j need some time to converge at the beginning of the simulation. And then at the middle of the simulation the JAE performs better than the association-then-estimation algorithm which is caused by targets moving together.

It can be seen from Figure 4(a)(b) that at the beginning of the simulation the bias estimation RMSE of both JAE and association-then-estimation algorithm is high and it is hard to say which performs better, because the bias estimation need some time to converge at the beginning. While, with the targets moving together, the PCA of the association-then-estimation algorithm does not perform well. And it can be seen that the RMSE of the JAE performs better than the association-then-estimation algorithm.

6 Conclusion

In this paper, we apply RJDE algorithm to the JAE problems which fits the dynamic JAE problems better since measurements usually come sequentially. And this method provides an approach to JAE problems based on generalized Bayes risk. Proposed algorithm outperforms the basic association-then-estimation algorithm in terms of the PCA and the bias estimation RMSE. In this study, it is assumed that the hypotheses and the decisions are one-to-one correspondent (M = N). But proposed method could be generalized to $M \neq N$ cases in order to reduce computation, which is considered as part of future work.

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