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Probabilistic GNSS Signal Tracking for Safety Relevant Automotive Applications

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***Abstract*— Signal processing based on software defined radio (SDR) carries out innovative potential for advanced data fusion algorithms. In the field of satellite navigation, SDR is a key enabling technology for new integration approaches based on GNSS base band processing. As multipath and non-line-of-sight (NLOS) errors are the main challenge for GNSS in automotive applications, much research for the mitigation of those influences is still ongoing. Existing localization approaches are suitable to a limited extent for meeting the requirements of services and applications, especially in the context of driver assistance and autonomous driving where a proper computation of confidence levels is of mayor importance. If unhandled, multipath and NLOS errors introduce a bias to the GNSS observations resulting in an overestimation of the position confidence which violates the integrity of subsequent applications. This paper proposes an integrated probabilistic approach for a continuous implementation of the Bayesian framework within the signal processing of GNSS receivers. Special attention will be on modeling ambiguities on signal level that in traditional GNSS receivers ends up in multipath or NLOS errors. The idea is based on the theory of multiple object tracking to resolve ambiguities within the measurement space. The outcome of this paper is a model which condenses local error sources as generic system properties and proposes a Bayesian filter which improves the integrity and accuracy of the position estimate in urban areas without additional environmental knowledge. This approach is a step towards the applicability of low cost GNSS for safety relevant applications in the automotive area.**

I. Introduction

Localization is of steadily increasing importance for nowadays and next generation ADAS and ITS applications. Especially in the context of autonomous driving with demanding requirements for integrity and availability, current mass market solutions are not able to provide an appropriate level of quality. Nevertheless, from mass-market point of view (service providers and car manufacturers), GNSS is still the most appropriate technology as it provides a precise and available service for low cost applications.

Unfortunately, GNSS was intentionally designed for aviation and maritime applications and the protection level definitions which have been adopted and extended by EGNOS do not apply for automotive applications [1]. The mayor reason for that is the lack of a proper modeling of local error sources like Non-Line-of-Sight (NLOS) or multipath effects.

The offset, which is introduced to the GNSS raw measurements by multipath, leads to a degraded performance of the estimated uncertainty in the final position estimate. Probabilistic data fusion implementations often tend to violate their estimated confidence interval, as an unobservable offset in the pseudorange measurements cannot be handled by the Bayesian filter framework and therefore results in an unobservable position bias in the state estimation. These influences cause false alarms in subsequent applications and become more relevant, as future applications will advance into inner cities and urban areas where buildings and urban vegetation prevent the GNSS signal from directly arriving at the receiver antenna. Well known data fusion approaches try to improve the localization quality by the extension of the receiver with additional sensors like yaw rate, acceleration or velocity information. These approaches are still subject to multipath and just extend the time until the position estimate violates the confidence interval. The second group of algorithms adds a dedicated multipath model, which helps to identify the corrupted satellite measurements. In literature there are different approaches to reduce the influence of multipath to a position solution. An overview of these so called Multipath Mitigation algorithms is given in [2]. A straightforward approach is identifying multipath by considering digital maps with modeled 3D buildings in order to validate the direct line of sight to each satellite. In the Bayesian framework this approach is used to predict multipath affected GNSS observations [3]. Another algorithm for determining NLOS with the help of environmental knowledge is described in [4][5]. A Non-Line-of-Sight signal detection respecting the satellite shadows in an urban scenario is presented in [6].

Another group is based on additional hardware like a multi-antenna approach in [7] for checking the consistency of the GNSS signal reception. An infrared camera to identify NLOS measurements is proposed in [8]. As these approaches cause additional costs to implement and do not rigorously implement the Bayesian framework (which is assumed to be required for safety relevant applications) the next group is of significant interest. It uses statistic tests and probabilistic filtering for the identification and mitigation of multipath. A known representative in this category is Receiver Autonomous Integrity Monitoring algorithm (RAIM) or an extension called Probabilistic Multipath Mitigation (PMM)[9].

This approach was implemented within the European research project GAIN [10] and was able to show the benefits regarding the improvement of the integrity in a safety relevant automotive use case. The localization framework was based on the integration of GALILEO together with the multipath mitigation algorithm and lane level accurate digital maps. Within this project, the vehicle dynamics like velocity, gear shift and acceleration have been adjusted based on the current position on the map in order to improve the efficiency and fuel

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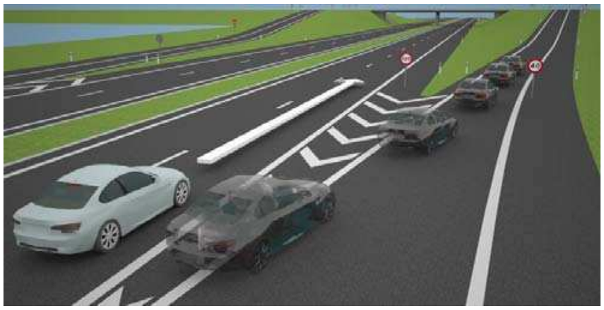

Fig. 1. Safety relevant use case from the European Research project GAIN — Due to wrong confidence estimation the vehicle is supposed to drive on the exit ramp and is reducing the velocity autonomously. This is a dangerous false alarm.

consumption of vehicles. Image 1 shows a typical scenario of a false alarm, based on a wrong position integrity. Obviously the current position estimate of the solid vehicle is wrong in terms of mean value and covariance. Due to the violated integrity interval the vehicle is supposed to drive on the exit ramp and is reducing the velocity autonomously. As in reality the vehicle is still on the highway, this false alarm causes a dangerous situation.

This paper evaluates the possible extension of multipath mitigation algorithms for software defined GNSS under special consideration of the improvement of confidence levels in urban areas. As all approaches mentioned before are implemented on observation level, they suffer from the GNSS receivers internal signal processing, which is performed to provided the observation data. This signal preprocessing on radio level is, especially in ambiguous situations falsifying the error distribution of the measures which are provided. That means subsequent systems tend to underestimate the influence of multipath and NLOS influences. The core idea is to extend the existing signal preprocessing of GNSS receivers and to include the Bayesian framework at the earliest possible receiver stage. An approach on this level is also described in [11] with special consideration of tracking time-delays, amplitudes, phases and a Bayesian multisensor navigation incorporating pseudorange measurements and a multipath model is presented in [12]. The basic assumption of this approach is not to identify corrupted signals and weighting them down within the filtering process — it is to avoid, that a GNSS receiver provides those faulty measurements at all. If an ambiguous situation appears, this information is shared to the subsequent application, in order to prevent false alarm decisions. This philosophy is an integral part for leveraging GNSS technology for safety critical ADAS functions. The focus of this approach is to reduce restrictions to a minimum in order to guarantee that no information, especially multimodalities in the measurement space are discarded. One mayor problem from multipath mitigation perspective is the early evaluation of the auto correlation function for each satellite and the irreversible assignment of a dedicated correlation maximum to the pseudorange measurement. The proposed approach uses the idea of well-known tracking implementations, as derivatives of probabilistic data association (PDA) in order to deal with the multimodal representation of all correlation maxima within the correlation function and to resolve ambiguities within the Bayesian framework.

A. Software defined GNSS

The idea of signal manipulation on radio level is the the so called software defined radio (SDR). In general the idea is to substitute signal processing hardware on radio level by a software implementation. This approach is very demanding in terms of computing power and data throughput but is very interesting from scientific point of view as it allows a flexible implementation and validation of complex signal processing algorithms. In the field of satellite based navigation it is called Software Defined GNSS and it compensates hardware evaluation kits. A famous implementation of a GNSS software receiver is provided by the GNSS SDR project [13]. Complex algorithms like the proposed GNSS pseudorange error density tracking using a Dirichlet Process Mixture in [14] or the Bayesian approach to multipath mitigation in GNSS receivers in [11] where implemented based on this technology. In the course of this chapter the high level idea of the proposed algorithm is shown and where it is logically set in the context of the receiver architecture.

A classical GNSS receiver can be split up into two algorithmic blocks — the *signal acquisition* and the *signal tracking* stage 2. The signal acquisition is in charge to evaluate the signal in space for the absolute presence of satellites. Once the acquisition unit has detected the presence of a given satellite and estimated the rough offset on the residual carrier and the code phase delay with respect to the local replicas, the fine synchronization stage, named *signal tracking*, is activated to refine these values, keep track and demodulate the navigation data from the specific satellite. This fine synchronization is fundamental for measuring the pseudorange, based on code phase measurements, or also the carrier phase measurements. The whole signal tracking process is a two-dimensional (code and carrier) signal replication process. In classical implementations it consists of two interoperating feedback loops, a Delay Lock Loop (DLL) for code tracking and a Phase Lock Loop (PLL) for carrier tracking [15].

These classical strategies cannot deal well with multiple received signals as they keep track of each satellite signal under the assumption that the signal received is unimodal. As a result of this assumption the tracking loops do irreversibly assign the tracking parameters for each cycle to the pseudorange and Doppler measures. If the tracking loop is locked on a reflected signal, this influence and the resulting inaccuracies cannot be provided with these measurements. Thus, a statistical filter in a subsequent signal processing suffers from the GNSS receivers internal signal preprocessing. The logical approach is to perform the signal tracking within a statistical filter that is able to deal with measurement ambiguities.

The proposed algorithm is based on the assumption that a satellite presence originating from the signal acquisition

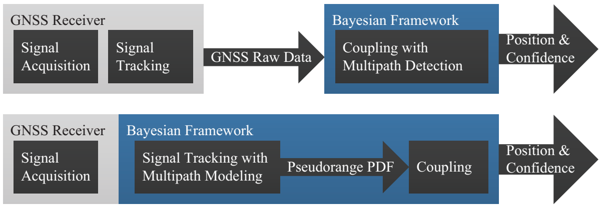

Fig. 2. Implementation of the Bayesian framework within the software receiver.

stage is true, as this observation is a long term evolution and very robust against short term varying influences. Given this indication, the proposed architecture will keep the existing stage of signal acquisition and start from the signal tracking.

The signal received at the input of a GNSS receiver in a additive Gaussian noise environment can be represented by

$$y_{RF}(t) = \sum_{i=1}^{N_s} r_{RF}(t) + \eta_{RF}(t) \quad (1)$$

It is assumed that each of the signals N_s is useful and that s is corresponding to the satellites visible [15]. For each satellite signal in space i, which is received the following composition can be assumed:

$$r_{RFi} = A_i c_i(t-\tau_i) d_i(t-\tau_i) cos(2\pi(f_{RF}-f_d)t+\omega_{RFi}) \quad (2)$$

where

- A_i is the amplitude of the i^{th}) useful signal,
- τ_i is the code phase delay introduced by the transmission channel,
- $c_i(t-\tau_i)$ is the spreading sequence which is given by the product of several terms and it is assumed to take values in the set $/-1, 1/$,
- $d_i(t-\tau_i)$ is the navigation message,
- ω_{RFi} is the initial carrier phase offset,
- f_{RF} is the carrier frequency, depending on the GNSS signal band.

The spreading sequence $c_i(t)$ can be expressed as $c_i(t) = c_{1,i}(t)c_{2,i}(t)s_{b,i}(t)$, where $c_{1,i}$ is the periodic repetition of the primary spreading code, $c_{2,i}$ the secondary code and $s_{b,i}(t)$ is the subcarrier signal. In multipath environment, the number of received satellite signals s increases and N_s splits into N_x and N_{mp} where x indicates the valid signals and mp the signals received via multipath. Equation 1 extends to

$$y_{RF}(t) = \sum_{i=1}^{N_x} r_{RF}(t) + \eta_{RF}(t) + \sum_{i=1}^{N_{mp}} r_{RF}(t) + \eta_{RF}(t). \quad (3)$$

The main focus of this work is to prospect the decomposition of equation 3 from the theoretical perspective and how it can be integrated with the Bayesian filter framework. Most approaches in literature neglect either NLOS or multipath effects and — most likely — there are many other local influences not covered at all by any high level multipath modeling. In order to incorporate all local signal disturbances, the approach is to observe all influences from the earliest stage in the GNSS signal processing. Multipath errors are said to be hard to observe, as they are caused by a rapidly changing local environment. In a multipath scenario the GNSS receiver always provides one of the multiple received satellite signals while the internal signal processing is trying to estimate the most likely solution. Unfortunately, the solution provided, is not always true. This approach is working on the result of the autocorrelation function in which all of these cases are reflected. The idea is to not explicitly differentiate between the signal influences like NLOS, multipath or further disturbances as they all can be found as special severity within the autocorrelation function. Multimodalities in this representation caused by those influences will be statistically evaluated and respected. The lower part of image 2 represents the general idea more clearly. The Bayesian framework is incorporated within the signal tracking block using the result of the autocorrelation as the basis for generating multiple measurements.

II. Bayesian approach on RF level

In this section the theoretical background of the proposed system is discussed. The general idea is based on multiple object tracking algorithms [16] and was initially implemented as the probabilistic multipath mitigation algorithm (PMM) [9] on observation level. As described in I the generation of this observation data is already subject to a loss of information. Additionally, this approach can result in high computational load with a large number of satellites. In order to overcome these limitations, the receiver internal signal processing shall partly be substituted by the proposed approach.

Multiple object tracking algorithms (MOT) can be divided into two groups. The first one is the group of single instance filters, which are using one single filter instance for representing multiple objects within a scene. From a statistical point of view, this can be regarded as a multi-modal probability density function (PDF), where each mode represents one single object. Currently, the probability hypothesis density (PHD) filter family is often used for single instance filter implementations. The second group—the so called multiple instance filters—uses one distinct filter instance for each object hypothesis. For the automotive field and its limited computation resources, the latter group is of high practical relevance as its computational demand is small compared to single instance filters. Therefore, this paper is focused on an implementation based on multiple instance filters. Fortunately, the main challenge for MOTs—the solution of the data association problem—does not exist for the proposed GNSS related solution, as it is initially unequivocal by CDMA from which satellite the measurements do originate. That means there is a fixed number of filter instances regarding the number of satellites being observed, where for each time step k the measurement set $\{\mathbf{z}\}$, originating from the autocorrelation function can directly be associated to.

A Straightforward approach for assigning several observations to an object without a hard association decision is

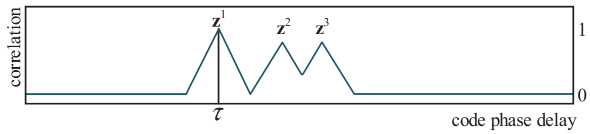

Fig. 3. Ambiguous situation with several possible code phase delays $\mathbf{z}^1, \mathbf{z}^2, \mathbf{z}^3 \in \{\mathbf{z}\}$.

the probabilistic data association (PDA) method [17]. This method also assumes that the objects already tracked do exist. The state of each object is updated by each of the observations $\{\mathbf{z}\}_k = \{\mathbf{z}_k^1, \ldots, \mathbf{z}_k^{n_z}\}$ weighted by its likelihood $p(\mathbf{z}_k^i|\mathbf{x}_k)$ resulting in a Gaussian mixture representation. The subsequent approximation of the Gaussian mixture by a single Gaussian is a remaining problem. It needs to be evaluated to which extent this assumption affects the performance under different multipath situations.

A typical situation for GNSS signal tracking under Multipath in the code domain is illustrated in 3. Several correlation maxima, each representing a possible candidate for the true code phase delay $\hat{\tau}$, will be assumed as measurements $\mathbf{z}^1, \ldots, \mathbf{z}^N \in \{\mathbf{z}\}$, where N is a system parameter defining the number of phase delays to be considered. The measurement set $\{\mathbf{z}\}$ consists of a subset of multipath measurements $\{\mathbf{z}\}^{\text{mp}}$ and one measurement $\mathbf{z}^{\text{x}}$ representing $\hat{\tau}$. The subsequent task is to identify if $\mathbf{z}^{\text{x}} \in \{\mathbf{z}\}$ and to separate $\mathbf{z}^{\text{x}}$ from $\{\mathbf{z}\}^{\text{mp}}$. In order to reach this goal a set of all possible association hypotheses is defined where each hypothesis is represented by a discrete association event in $A_i^m \in A^m \subset A$. The subsets A^m contain all association events A_i^m assuming that m is the number of valid phase delay measurements.

As previously mentioned, there is either one valid phase delay $\hat{\tau}$ and additional delayed measurements - this set is defined to be A^1 (multipath scenario) or none of the phase delays is representing the true delay - which is subset A^0 and represents a NLOS scenario. From these assumptions the cardinality of A^m is derived by $\binom{N}{m} = N|m = 1$. For N phase delays, the set of association events is given by

$$A = \begin{cases} A^0 & = A^{\text{NLOS}} = \begin{cases} A_0^0 & \text{if } \{\mathbf{z}\} = \{\mathbf{z}\}^{\text{NLOS}} \end{cases} \\ A^1 & = A^{\text{Multipath}} \begin{cases} A_1^1 & \text{if } \mathbf{z}^1 \text{ is LOS} \\ \vdots & \\ A_N^1 & \text{if } \mathbf{z}^N \text{ is LOS} \end{cases} \end{cases} \quad (4)$$

A. State Update

The core idea of the PGT is to condition the posterior state (PDF) on the association events, that is

$$p(\mathbf{x}_k|\mathbf{Z_k}) = \sum_{A_i^m \in A} p(\mathbf{x}_k|\mathbf{Z_k}, A_i^m) P(A_i|\mathbf{Z}_k). \quad (5)$$

The conditioned posterior PDFs $p(\mathbf{x}_k|\mathbf{Z_k}, A_i^m)$ can be determined by performing standard filtering operations. The association probability $P(A_i^m|\mathbf{Z}_k)$ is determined by applying the Bayes rule:

$$P(A_i^m|\mathbf{Z}_k) = \eta \cdot p(\{\mathbf{z}\}_k|A_i^m, \mathbf{Z}_{k-1}) P(A_i^m|\mathbf{Z}_{k-1}). \quad (6)$$

The unconditional association probability $P(A_i|\mathbf{Z}_{k-1})$ is assumed to be independent from previous measurements $P(A_i^m|\mathbf{Z}_{k-1}) = P(A_i^m)$. The prior distribution of this probability is assumed to be uniformly distributed. Thus, $P(A_i^m)$ cancels out in equation 6.

The likelihood for the appearance of the measurement set $\{\mathbf{z}\}_k$ needs to be split into several components. As defined previously, the set of measurements consists of one true measurement and a set of clutter measurements. For the sake of notational consistency the true measurement will be assumed as set of measurements containing actually one element. Both sets are assumed to be independent where the likelihood can be written as

$$p(\{\mathbf{z}\}_k|A_i^m, \mathbf{Z}_{k-1}) = p(\{\mathbf{z}\}_k^x|A_i^m, \mathbf{Z}_{k-1}) p(\{\mathbf{z}\}_k^{mp}|A_i^m, \mathbf{Z}_{k-1})$$

and for multipath measurements, independence between two time points is assumed. That assumption derives

$$p(\{\mathbf{z}\}_k|A_i^m, \mathbf{Z}_{k-1}) = p(\{\mathbf{z}\}_k^x|A_i^m, \mathbf{Z}_{k-1}) p(\{\mathbf{z}\}_k^{mp}|A_i^m) \quad (7)$$

Each of the two likelihood expressions can be split up into a cardinal and a spatial likelihood. The cardinal likelihood defines the probability for a certain number of true or multipath measurements. It is assumed that the presence of a true measurement is independent from previous measurements and the cardinal measurement likelihood is given by $P(n_x|A_i^m)$, while the cardinal multipath likelihood is denoted as $P(n_{mp}|A_i^m)$. The sum of both cardinalities is given by $n_z = n_{mp} + n_x$. The spatial likelihood is defined by the product of the spatial likelihoods for each measurement. For true measurements it is derived from the Kalman filter equations as $p(\mathbf{z}_k^i|\mathbf{Z}_{k-1}) = \Lambda_k^i$. The spatial likelihood for the set of true measurements is given by

$$p(\{\mathbf{z}\}_k^x|A_i^m, \mathbf{Z}_{k-1}) = \prod_{j=1}^{n_z} (\Lambda_k^i)^{C_{n_z}^m[i,j]}. \quad (8)$$

For multipath measurements, the single likelihoods are assumed to follow a rayleigh distribution. Thus, the overall spatial multipath likelihood is given by

$$p(\{\mathbf{z}\}_k^{\text{mp}}|A_i^m) = V^{-(n_z - m)}. \quad (9)$$

Inserting these four likelihood terms into equation 7 gives

$$p(\{\mathbf{z}\}_k|A_i^m, \mathbf{Z}_{k-1}) = \\ P(n_x|A_i^m) P(n_{mp}|A_i^m) F^{-(n_z-m)} \prod_{j=1}^{n_z} (\Lambda_k^i)^{C_{n_z}^m[i,j]}. \quad (10)$$

B. Proposed Sensor Model

The number of true measurements n_x is supposed to be either 0 or 1. It follows a discrete uniform distribution

$$P(n_x|A_i^m) = \frac{P_D P_E}{\overline{n}} \text{ for } n_x > 0, \tag{11}$$

P_D denotes the detection probability (true positive) and P_E the existence probability. From the condition that the satellite to be tracked is given, P_E is set to 1. The multipath density is modeled using a Poisson distribution determined by the parameter λ which represents the expectation value of the number of multipath measurements within the observed correlation frame, that is $\lambda = E[n_{mp}/F]$. Deeper considerations on distribution assumptions and noise behaviour are evaluated in [14]. With this parameter the distribution of the multipath cardinality is given by

$$P(n_{\text{mp}} = k) = \frac{(\lambda \cdot F)^k}{k!} e^{-\lambda \cdot F}. \tag{12}$$

C. Filter Equations

The association probabilities

$$P(A_i^m|\mathbf{Z}_k, i = 0) = \frac{1 - P_D P_E}{1 - \delta P_E} = \frac{1 - P_D}{1 - \delta} \tag{13}$$

and

$$P(A_i^m|\mathbf{Z}_k, i > 0) = \frac{P_D n_z^{\underline{m}} \lambda^{-m}}{\overline{n}(1-\delta)} \prod_{j=1}^{n_z} (\Lambda_k^i)^{C_{n_z}^m[i,j]} \tag{14}$$

where $n^{\underline{m}}$ denotes the falling factorial $n!/(n-m)!$ and the auxiliary variable δ is defined as

$$\delta = P_D - \frac{P_D}{\overline{n}} \cdot \sum_{m=1}^{\min(\overline{n}, n_z)} (n_z)^{\underline{m}} \cdot \lambda^{-m} \sum_{i=1}^{\binom{n_z}{m}} \prod_{j=1}^{n_z} (\Lambda_k^i)^{C_{n_z}^m[i,j]}. \tag{15}$$

III. Discussion and Conclusion

In this paper, a generalized approach for an integrated multipath mitigation strategy for software defined GNSS was proposed. The idea is to improve the integrity of the final position solution by considering the ambiguities in the autocorrelation functions of the code phase delay caused by multipath and NLOS effects. This is reached by rigorously integrating the Bayesian framework until the signal level and adopting approaches from the field of multiple object tracking under clutter. The next steps are the implemenatation within the proposed GNSS software receiver from section I-A and the evaluation of the system. A particle filter will be used in order to implement the equations derived in section II.

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