18th International Conference on Information Fusion
Washington, DC - July 6-9, 2015

On Threshold Optimization for Aircraft Conflict Detection

Huimin Chen
Department of Electrical Engineering
University of New Orleans
New Orleans, LA 70148, USA
Email: hchen2@uno.edu

Vesselin P. Jilkov
Department of Electrical Engineering
University of New Orleans
New Orleans, LA 70148, USA
Email: vjilkov@uno.edu

X. Rong Li
Department of Electrical Engineering
University of New Orleans
New Orleans, LA 70148, USA
Email: xli@uno.edu

***Abstract*—We consider the detection component of a conflict detection and resolution system for air traffic management. A conflict is an event of a close encounter between two aircraft in the near future. The collision alert system has to balance between declaring alerts timely to true threats and reducing the false alarms. We compare two conflict detection schemes, namely, geometric and probabilistic method, for collision alerting based on the predicted trajectory with or without the intent information from the encounter aircraft. We study the threshold selection for each scheme by maximizing the conflict detection probability under an operational cost constraint which is proportional to the false alarm rate and inversely proportional to the response time. Through simulated aircraft encounter scenarios with various types of uncertainties in trajectory prediction, we found that the geometric method performs better than the probabilistic method when the intruder's intent is unknown. However, the probabilistic method is better under relatively large uncertainty of the predicted trajectory. We discuss the implication to the centralized and distributed conflict alert system where each aircraft can optimize its own conflict detection threshold.**

I. INTRODUCTION

Conflict detection and collision avoidance are important for aircraft safety especially when more and more autonomous air vehicles will enter the unregulated air space. A conflict is usually defined as an event where two aircraft are closer to each other than a given safety distance. Timely and reliable detection of potential conflicts relies heavily on the accuracy of the predicted aircraft trajectories. Traditionally, a centralized, mostly human-operated air traffic control (ATC) center on the ground is responsible for aircraft separation by commanding specific trajectory operations to the pilots. As the modern aircraft equipped with automatic dependence surveillance-broadcast (ADS-B), global positioning system (GPS), and powerful on-board computers has the capability to detect conflict by itself, the decentralized trajectory operation to maintain aircraft separation has become an important aspect in NextGen Avionics Roadmap [12]. When the conflict detection is made by the individual aircraft, we call the collision alert system decentralized or distributed. On the other hand, when the conflict detection is made by the ATC center that communicates with both aircraft approaching to each other, we call the system centralized. In this paper, we focus on short range conflict detection in seconds to minutes where both the ground based ATC center and the flight management system on board the aircraft may be involved in the prediction of potential conflict.

Existing conflict detection methods can be largely classified as geometric, force field and probabilistic [9]. The traffic alert and collision avoidance system (TCAS) currently operating on all commercial aircraft carrying more than 30 passengers adopted a geometric method with a sophisticated logic for conflict detection [5]. Nevertheless, probabilistic methods have been the recent trend in the literature owing to the uncertainties in predicting the aircraft's future position including wind perturbation and tracking, navigation, and control errors [11], [13], [15], [16]. The majority of the probabilistic methods compute the probability of conflict at a certain time instant and compare with a threshold to issue an alert. For example, the time instant can be the prediction of the closest encounter between the two aircraft [15]. A refined conflict probability estimate assesses the probability that the closest encounter of the two aircraft exceeds the safety zone within a given time interval [10]. A lot of research effort has been made to improve the trajectory prediction accuracy and obtain efficient algorithms to compute the conflict probability in real time [6], [7], [14], [17]. However, few attempts have been made to evaluate the performance of the collision alert system and optimize the detection threshold.

The system operating characteristic (SOC) curve, i.e., the probability of successful collision alert vs. the probability of false alert was used for threshold selection in [8]. The threshold that achieves the operating point on the SOC curve with the minimum distance to point (0,1), i.e., the ideal performance, was used in [15]. The threshold that achieves the minimum error probability was recommended in [8]. Under the equal prior assumption, the conflict detection system that minimizes the error probability treats the missed detection and the false alarm with equal cost. This seems to be the optimal response by a local decision maker to distinguish itself from an intelligent attacker when reporting its decision to the fusion center [4], however, it does not take into account any practical operating cost in handling the false alarms. In practice, the TCAS used one set of thresholds for resolution advisories and then modified the thresholds to reduce the false alarm rate [8], [5]. Here we want to compare the geometric method currently implemented in TCAS and the most promising probabilistic method in conflict detection with the optimized thresholds using a realistic constraint on the operating cost. The problem is formulated as maximizing the conflict detection probability at any given time subject to a simplified operational constraint. We simulate the aircraft encounter scenario with various uncertainties regarding the intruder's current state estimate and its intent. We consider the conflict alert issued by a centralized decision maker or by the individual aircraft and compare the optimized thresholds correspondingly.

The rest of the paper is organized as follows. Section II formulates the conflict detection problem and presents the geometric and probabilistic conflict detection method. Section III provides the details of the aircraft encounter scenario used for the comparison of conflict detection schemes without or with the intent uncertainty of the intruder. Section IV shows the detection performance of the geometric and probabilistic method with the optimized detection thresholds under decentralized and centralized configuration. Concluding remarks are in Section V.

II. Problem Formulation

A. Conflict Detection Schemes

Let A and B be two aircraft with states $\mathbf{x}_A(t)$ and $\mathbf{x}_B(t)$, respectively. Let $\mathbf{p}_A(t)$ and $\mathbf{v}_A(t)$ be the position and velocity component of the state of aircraft A and $\mathbf{p}_B(t)$ and $\mathbf{v}_B(t)$ the position and velocity component of aircraft B. If the aircraft are assumed to move along straight trajectories, then one can use the relative position $\delta\mathbf{p}(t) = \mathbf{p}_A(t) - \mathbf{p}_B(t)$ and the relative velocity $\delta\mathbf{v}(t) = \mathbf{v}_A(t) - \mathbf{v}_B(t)$ to predict the possible collision. For example, in TCAS, the relative range $r(t) = ||\delta\mathbf{p}(t)||$, relative range rate $\dot{r}(t) = ||\delta\mathbf{v}(t)||$, relative altitude $h(t) = p_{zA}(t) - p_{zB}(t)$ and relative altitude rate $\dot{h}(t) = v_{zA}(t) - v_{zB}(t)$ were used to make the collision alert decision. Specifically, at any time t^*, if

$$r(t^*) < -\tau\dot{r}(t^*) + R \text{ and } h(t^*) < -\tau\dot{h}(t^*),$$

then a collision alert is declared. The parameter τ is a threshold related to the estimated time from t^* to the closest encounter. The parameter R is a guard distance around the aircraft on the horizontal plane. It was recommended to use $\tau = 30$s and $R = 1.0$nmi in the 1.0 version of the TCAS and later changed to $\tau = 22$s and $R = 0.8$nmi in the 6.04A version [8]. Such a scheme belongs to the geometric conflict detection since the relative motion within time τ is assumed straight with deterministic initial state $\mathbf{x}_A(t^*)$ and $\mathbf{x}_B(t^*)$.

Alternatively, probabilistic methods assume that $\mathbf{x}_A(t)$ and $\mathbf{x}_B(t)$ are random processes and use probability density function (pdf) of $\delta\mathbf{p}(t)$ to compute the conflict probability. Let R_{xy} and R_z be the horizontal and vertical separation thresholds, respectively. The ellipsoidal protected zone is defined as

$$\mathcal{R} = \{\mathbf{p} \in \Re^3 : ||\Lambda\mathbf{p}|| \leq 1\}$$

where $\Lambda = \mathrm{diag}\{1/R_{xy}, 1/R_{xy}, 1/R_z\}$. For convenience, we assume that the current time is 0 and we want to find the conflict probability over the prediction horizon $0 < t < T$. At any time t, the pdf $f_{\delta\mathbf{p}(t)}(\mathbf{p})$ of $\delta\mathbf{p}(t)$ is assumed to be available through trajectory prediction. The instantaneous conflict probability (CP) is

$$P_c(t) = P\{||\Lambda\delta\mathbf{p}(t)|| \leq 1\} = \int_{\mathcal{R}} f_{\delta\mathbf{p}(t)}(\mathbf{p})d\mathbf{p}.$$

The maximum CP over $[0, T]$ is

$$P_{cm} = \max_t P_c(t).$$

One can compare P_{cm} with a threshold p^* to declare a conflict. The response time t_r to the conflict is

$$t_r = \{t : \min_{t\in[0,T]} P_c(t) > p^*\}.$$

It is in general smaller than the time of the closest encounter, which can be estimated by

$$t_m = \arg\max_t P_c(t).$$

Note that P_{cm} represents the peak risk within $[0, T]$. The conflict alert decision does not consider the response time t_r for the aircraft to make collision avoidance maneuver. Alternatively, one can use the cumulated CP over $[0, T]$ given by

$$P_{cc} = \int_0^T P_c(t)dt.$$

Unfortunately, P_{cc} is not a proper probability, which makes the threshold selection a challenging task. In [10], the authors proposed a new definition of the cumulative CP over $[0, T]$ given by

$$P_{cT} = P\{\min_{t\in[0,T]} ||\Lambda\delta\mathbf{p}(t)|| \leq 1\},$$

which can be approximated by the randomized algorithm [15]. Clearly, evaluating P_{cT} requires significant computational cost compared with the geometric scheme. For $R_{xy} = 1000$m and $R_z = 150$m, [10] recommended $P_{cT} > 0.02$ to declare a conflict.

B. Threshold Optimization for Conflict Detection

Given a conflict detection scheme, one has to adjust the threshold to make the tradeoff between false alarm and missed detection. Intuitively, as the air traffic density increases in the future, the near miss scenarios for the aircraft encounter close to the protected zone will also increase. It is difficult to control the false alarm rate at any given time owing to the uncertainty in the trajectory prediction as well as in the intent of the intruder. Nevertheless, selecting an operating point on the SOC curve is equivalent to assigning different cost values to false alarm and missed detection so that one can minimize the total expected cost. Since safety is paramount in the conflict alert system design, we do not want to manually adjust the cost values. Instead, we try to maximize the conflict detection probability under the practical constraint of the operational budget. This formulation was also used in [3] to obtain the optimal decision fusion policy where detecting the rare event is of primary concern and the false alarm rate is only part of the operational constraint. In a conflict detection situation, we can assume that the number of falsely issued collision alerts is much larger than the number of true collision alerts. Thus the ratio between the total number of the collision avoidance actions and the total number of close encounter events in a particular air space within a given time period can be used as the indication of the false alarm rate P_{FA}. The operational budget constraint has a general form of

$$C_0 + f(P_{FA}, t) \leq B$$

where C_0 is a fixed cost to maintain the nominal trajectory of the aircraft, f is a cost function related to the false alarm rate P_{FA} and the response time t to the collision event, and B is the total operational budget. Here we assume that the additional cost involved to resolve the conflict is proportional to P_{FA} but inversely proportional to t in the short range encounter. Consequently, for a conflict detection scheme that involves the choice of thresholds τ_1, ..., τ_n, we want to optimize the thresholds by solving the following constrained optimization problem

$$\max_{\tau_1,...,\tau_n} P_D(\tau_1, ..., \tau_n)$$

subject to

$$P_{FA}(\tau_1, ..., \tau_n)/t^* \leq C$$

where t^* is the estimated time from the conflict detection to the closest encounter, P_{FA} is the estimated false alarm rate, and C is the normalized operational cost for conflict resolution. Note that the cost of resolving a false conflict may not be inversely proportional to the response time t^* for a mid-air collision situation. We choose this constraint for convenience in order to compare two representative conflict detection schemes using realistic aircraft encounter scenarios with various uncertainties. The solution to the above constrained optimization is scenario dependent. It does not have a closed form in general. Nevertheless, it enforces the operating point of any conflict detection scheme for performance comparison.

C. Centralized vs. Distributed Conflict Detection Schemes

Consider a close encounter scenario between aircraft A and B. Aircraft A may implement the collision alert scheme on its own and optimize the threshold without the consideration of any input from aircraft B or the air traffic control (ATC) center. On the other hand, both aircraft A and B can report the collision alert to the ATC center and the ATC will fuse the decisions from aircraft A and B with a certain optimized policy. We call the conflict detection scheme centralized when the ATC center has the full control of the threshold selection for both aircraft A and B in order to maximize the detection probability using the optimal fusion policy. We call the conflict detection scheme decentralized or distributed if each aircraft optimizes its own detection performance without taking any input from a third party. Note that if another aircraft C is equipped with a tracking and conflict alert system that can track and predict the trajectories of aircraft A and B, then the ATC center can construct the optimal fusion policy with the additional input from aircraft C using the technique developed in [3]. However, the threshold optimization becomes computationally prohibitive when one has to enumerate all possible decision trees, each of which requires finding the optimal thresholds.

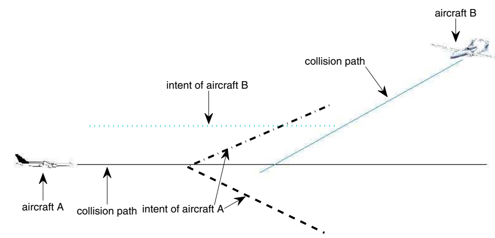

Fig. 1. Geometry of the aircraft encounter scenario.

III. Aircraft Encounter Scenarios

The performance of collision alert schemes is evaluated through Monte Carlo simulations. Two aircraft are assumed to fly at an altitude of 15,000ft above mean sea level. Let aircraft A be the own vehicle initially set at the origin. Let aircraft B be the intruder on the positive x-axis. Aircraft B has a descending path directly towards aircraft A with the relative speed 733ft/s and the descending rate 41.7ft/s. The two aircraft are modeled as cylinders 100ft in radius and 100ft in height. If the cylinders intersect, we assume that a true collision has occurred. If no avoidance action is taken, the two aircraft will have the closest encounter after approximately 33s and aircraft B is projected to pass 70ft below aircraft A. Aircraft B has a possible intent to level off and maintain the altitude to avoid the collision. Aircraft A has two possible intents to either climb up or descend before the closest encounter to aircraft B. We consider various scenarios where the intent of aircraft B is completely known, uncertain or unaware to aircraft A. In addition, we also model the knowledge about aircraft A's intents from aircraft B at different levels when comparing the centralized and distributed conflict detection schemes. The geometry of the possible collision scenarios is shown in Fig. 1.

A. Target Tracking Method

We assume that aircraft A is equipped with an EO sensor that provides angle only measurement of aircraft B with a standard deviation of 2mrad. The motion of aircraft B is assumed to be nearly constant speed using the continuous time white noise acceleration (WNA) model [2]. The initial uncertainty of the estimation error of the state $\mathbf{x}_B(0)$ is assumed to be zero mean Gaussian with the covariance of position

$$P_{\mathbf{p}_B} = \text{diag}\{(100\text{ft})^2, (100\text{ft})^2, (50\text{ft})^2\}$$

and the covariance of velocity

$$P_{\mathbf{v}_B} = \mathrm{diag}\{(5\mathrm{ft/s})^2, (5\mathrm{ft/s})^2, (2\mathrm{ft/s})^2\}.$$

The estimation error of aircraft A's own state $\mathbf{x}_A$ is assumed to be negligible. It is assumed that the measurement is available every 1s without any false alarm or missed detection. A cubature Kalman filter [1] is used to update the state estimate of aircraft B and the conflict detection scheme is based on the newly obtained state estimate using either the geometric method or the probabilistic method.

A well known issue with angle-only tracking is that the range of the target is unobservable. We consider this scenario so that the uncertainty of the range and range rate estimate will be unlikely to improve as the two aircraft approach each other. We are interested in its impact on the operating point of the conflict detection scheme.

B. Threshold Optimization

We want to compare the geometric and the probabilistic conflict detection method for the direct encounter scenario where aircraft A and B fly in straight trajectories. In the geometric method, at any time t^*, we obtain the estimated relative range $\hat{r}(t^*)$, relative range rate $\hat{\dot{r}}(t^*)$, relative altitude $\hat{h}(t^*)$, and $\hat{\dot{h}}(t^*)$ from the tracking filter. If

$$\hat{r}(t^*) < -\tau\hat{\dot{r}}(t^*) + R \text{ and } \hat{h}(t^*) < -\tau\hat{\dot{h}}(t^*),$$

then a conflict is declared. We fix $R = 150$ft and optimize τ under the operational constraint. In the probabilistic method, we consider a $T = 60$s time horizon to estimate P_{cT} using the current state estimate and the WNA model of aircraft B. The detection threshold is optimized with respect to P_{cT}.

In order to estimate the false alarm rate for each conflict detection scheme, we alter aircraft B's trajectory so that at the closest encounter, it is 170ft below aircraft A at about 33s. Under this near miss scenario, we set $\tau = 30$s and obtained the false alarm rate $P_{FA} = 0.142$ through 10,000 realizations of the geometric conflict detection scheme. If we set the cost constraint $C = 0.003$, then the resulting optimized threshold $\tau^* = 24$s, which reduces the false alarm rate to 0.07. Under the same cost constraint, we applied the probabilistic method with $R_{xy} = 100$ft and $R_z = 100$ft for the protected region. If we compare P_{cT} with the threshold $\tau_{cT} = 0.07$, then the actual false alarm rate is $P_{FA} = 0.213$. The optimal threshold in this case is $\tau^*_{cT} = 0.17$ that achieves a false alarm rate close to 0.07. Note that the test statistic P_{cT} is affected by the uncertainty of the state estimate and its distribution is time varying. Here we optimized the threshold for each conflict detection scheme at the initial time and fixed it throughout the whole state update until a conflict is declared.

C. Modeling Intended Maneuver

We assume that aircraft B has an intent to level off at 1,000ft above aircraft A when the relative range between the two aircraft is about 20,000ft. This happens about 27s prior to the closest encounter. However, aircraft A is unaware of aircraft B's intent. When a conflict is declared prior to aircraft B's intended maneuver, aircraft A will take possible collision avoidance actions to either climb up or descend. Since the projected trajectory of aircraft B is below aircraft A when there is no uncertainty in the state, the more likely action for aircraft A is to climb up. Unfortunately, with a $0.25g$ acceleration to climb up, aircraft A will collide with aircraft B that tries to level off above the nominal trajectory of aircraft A. We consider the situation where both aircraft A and B are equipped with the tracking system with angle-only sensor measurements. They both use the interacting multiple model (IMM) tracker with two models to account for possible maneuver [2]. The WNA model with a low process noise power spectrum density (PSD) of $0.05g$ is used for the nearly constant speed motion and the WNA model with a noise PSD of $0.5g$ is used for the maneuver motion. Note that the tracking accuracy improves for the angle-only tracking problem when the target maneuvers owing to the improved observability in the target range.

IV. Performance Comparison

A. Conflict Detection from Aircraft A

Once the threshold is optimized for each conflict detection scheme, we evaluate the detection performance based on 10,000 realizations of the collision trajectory where aircraft B is 70ft below aircraft A at the closest encounter. At the initial time 0, the time to the closest encounter is about 33s. Fig. 2 shows the number of conflict detections at different times when aircraft A updates the state estimate of aircraft B and applies the geometric or probabilistic conflict detection scheme with the optimized threshold. We can see that most of the detections occur at 9s by the geometric method since the threshold τ^* is set at 24s. It controls the response time allowed for aircraft A to take possible collision avoidance action. The probabilistic method declares conflict earlier and achieves a detection probability of 0.94 prior to the time window within 24s to the closest encounter, as opposed to a detection probability of 0.87 by the geometric method. However, there are a few cases that the probabilistic method declared conflict later than the geometric method. If we set a 4s response window for aircraft A and consider the conflict detection after 13s as a miss, then the geometric method yields only 1 missed detection while the probabilistic method has 45.

Next, we consider the case where aircraft B has an intent to level off at 1,000ft above aircraft A. The collision will not occur if aircraft B performs the level off maneuver at 27s prior to the closest encounter. However, aircraft A is unaware of aircraft B's intent and it will predict two possible trajectories of aircraft B. Specifically, aircraft A assumes that aircraft B will maintain its current course with probability q and will make the level off maneuver with probability $(1 - q)$. Note that the detection performance by the geometric method will not be affected by q when the threshold is set at $\tau^* = 24$s since no conflict is declared prior to aircraft B's level off maneuver. On the other hand, the probabilistic method will generate a false alarm rate of 0.32 when aircraft B does the level off maneuver at 27s to avoid the collision. Thus one has

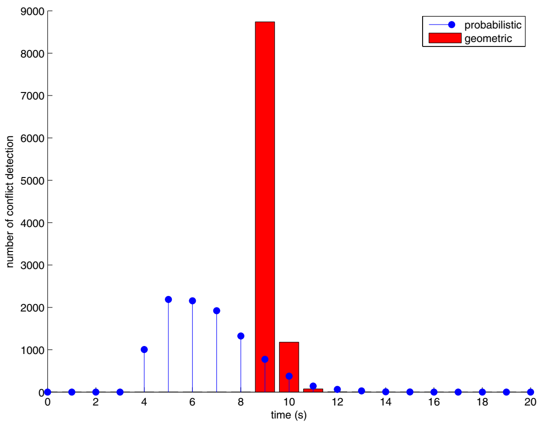

Fig. 2. Comparison of geometric and probabilistic conflict detection method on the number of declared conflicts at different times (10,000 runs).

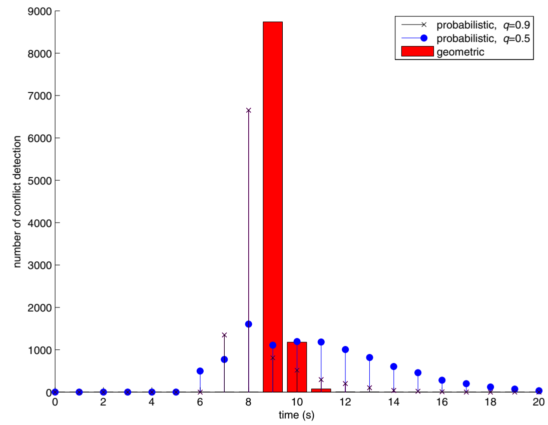

Fig. 3. Comparison of geometric and probabilistic conflict detection method in the case of uncertain intent in trajectory prediction (10,000 runs).

to optimize the threshold for the probabilistic method with the assumed q in order to meet the desired operational constraint. As q decreases, aircraft A has to defer the conflict alert decision by comparing P_{cT} with a relatively higher threshold. Fig. 3 shows the number of conflict detections at different times when aircraft B maintains the collision trajectory while the probabilistic method uses the optimized threshold based on the assumed maneuver probability $(1-q)$ by aircraft B. We can see that the detection performance of the probabilistic method degrades significantly compared with that in Fig. 2 where aircraft A knows aircraft B's intent ($q = 1$). The conflict detection probability at the time instant 24s prior to the collision is 0.80 when the assumed maneuver probability is 0.1 and it drops to 0.29 when the assumed maneuver probability is 0.5. Clearly, the increase in the optimized threshold owing to aircraft B's uncertain intent results in the conflict detection delay. In the case of $q = 0.5$, we have a probability of missed detection of 0.26 if any detection later than 20s to the collision is considered a miss. Thus under the same operational constraint related to the false alarm rate and time to the closest encounter, the probabilistic method has a better detection performance when the intent of aircraft B is known but the predicted trajectory has relatively large uncertainty. When aircraft B's intent is uncertain, the geometric method has the detection performance nearly unaffected while the probabilistic method has a significant performance degradation. This could be a justification for the TCAS to adopt the geometric method for conflict detection. Another reason is that the probabilistic method requires significant computation to estimate P_{cT} accurately while the geometric method is much more efficient computationally.

B. Conflict Detection from the Fusion Center

We consider the situation that both aircraft A and B have conflict detection systems that report the conflict to the ATC center. The ATC center fuses the decisions from both aircraft A and B and maximizes the conflict detection probability under the same operational cost constraint. We assume that aircraft B is equipped with an EO sensor that provides angle only measurement of aircraft A with a standard deviation of 2mrad. The motion of the aircraft A is assumed to be nearly constant speed using the continuous time white noise acceleration (WNA) model [2]. The initial estimation error of the state $\mathbf{x}_A(0)$ is assume to be zero mean Gaussian with the covariance of position

$$P_{\mathbf{p}_A} = \mathrm{diag}\{(100\mathrm{ft})^2, (100\mathrm{ft})^2, (50\mathrm{ft})^2\}$$

and the covariance of velocity

$$P_{\mathbf{v}_A} = \mathrm{diag}\{(5\mathrm{ft/s})^2, (5\mathrm{ft/s})^2, (2\mathrm{ft/s})^2\}.$$

The estimation error of aircraft B's own state $\mathbf{x}_B$ is assumed to be negligible. It is assumed that a measurement is available every 1s without any false alarm or missed detection. Using the technique developed in [3], we obtained the optimal policy at the fusion center given any local decision policy with the detection probability estimated using the direct encounter scenario where aircraft B is 70ft below at the closest approach to aircraft A and the false alarm probability estimated using the near miss scenario, where aircraft B is 170ft below aircraft A at the closest encounter. For the geometric method, we found the optimal threshold for aircraft A is $\tau_A^* = 31$s and the optimal threshold for aircraft B is $\tau_B^* = 30$s. The ATC center will issue an alert to both aircraft A and B when both aircraft declare the conflict. Note that the actual false alarm rate at the initial time is 0.064 since the threshold τ only takes an integer value in seconds. For the probabilistic method, the optimal threshold for aircraft A is $P_{cT,A} = 0.052$ and the optimal threshold for aircraft B is $P_{cT,B} = 0.049$. Note that both thresholds are much smaller than the threshold being optimized by the individual aircraft to maximize the conflict detection probability under the same operational cost constraint. Fig. 4 shows the detection performance at the fusion center using the optimized geometric and probabilistic methods for both aircraft A and B. We can see that the probabilistic method detects conflict earlier than the geometric

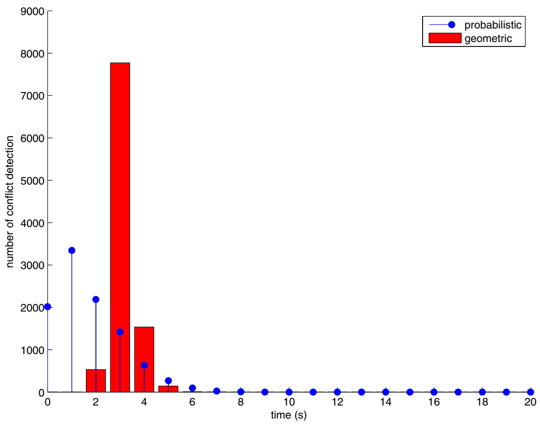

Fig. 4. Comparison of geometric and probabilistic conflict detection method with the optimal centralized policy (10,000 runs).

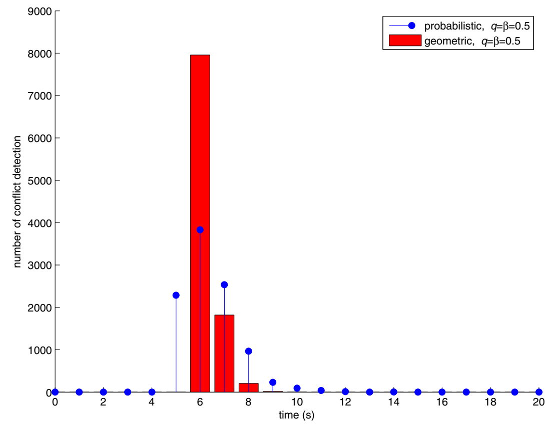

Fig. 5. Comparison of geometric and probabilistic conflict detection method with the optimal centralized policy under uncertain intent in trajectory prediction (10,000 runs).

method. Prior to 30s from the collision, the probabilistic method has a detection probability of 0.90 while the geometric method has a detection probability of 0.83. Notice that there is no missed detection by either method at the time 24s prior to the closest encounter.

In practice, no immediate action will be taken when aircraft A or B reported a conflict to the fusion center. The fusion center will confirm the collision alert when both aircraft declared the conflict. Interestingly, the optimized threshold $\tau^* = 24$s for each aircraft in a decentralized setting is close to the TCAS recommendation version 6.04A while the optimized threshold in the centralized configuration for the individual aircraft, $\tau_A^* = 31$s and $\tau_B^* = 30$s, is close to the TCAS recommendation version 1.0. Clearly, the decentralized conflict alert system has a stronger tendency to defer the collision alert than the centralized scheme with the same operational constraint.

Next, we study the effect of aircraft intent uncertainty on the conflict detection performance of the centralized system. Aircraft A assumes that aircraft B will remain in its current course with probability q and will level off at 1,000ft above aircraft A with probability $(1 - q)$. Aircraft B assumes that aircraft A will remain its current course with probability β, will climb up with probability $(1 - \beta)/2$, and will descend with probability $(1 - \beta)/2$. When $q = \beta = 0.5$, we have the optimized thresholds $\tau_A^* = 26$s and $\tau_B^* = 27$s for the geometric method. They are smaller than those used without any intent uncertainty but larger than those used in the decentralized configuration. In the probabilistic method, the optimal threshold for aircraft A is $P_{cT,A} = 0.13$ and the optimal threshold for aircraft B is $P_{cT,B} = 0.083$. Since aircraft B's possible level off maneuver is earlier than aircraft A's climb up or descend, the impact on the false alarm rate is more severe, resulting in a higher threshold at aircraft A than at aircraft B. Fig. 5 shows the detection performance of the optimized geometric and probabilistic methods with the assumed intent uncertainty $q = \beta = 0.5$ by aircraft A and B. We can see that the detection probability is 0.80 at 26s prior to the closest encounter by the geometric method while the detection probability is 0.61 at 26s prior to the closest encounter by the probabilistic method. The probabilistic method has a few early detection cases and a few late detection cases compared with the geometric method. If we consider any detection later than 22s to the closest encounter as a miss, then the geometric method has 17 missed detections while the probabilistic method has 378. It is interesting to note that the intent uncertainty has less impact on the detection performance of the centralized scheme compared with the individual aircraft using the probabilistic method.

Clearly, different conflict detection schemes have their pros and cons from various performance evaluation metrics. Their relative performance merits depend much on the specifics of the aircraft encounter scenarios. From our limited simulation study, it appears that we can make the following remarks based on the conflict detection performance with an operating cost constraint. If one can optimize the threshold for the centralized decision fusion policy, the performance will improve for both the geometric and the probabilistic method compared with the optimized conflict detection system for the individual aircraft. However, as the intent uncertainty increases, the resulting optimized threshold decreases for τ and increases for P_{cT}. The benefit of early detection by the probabilistic method becomes diminishing. The probability of a missed detection within a short response time is even larger by using the probabilistic method than using the geometric method. Taking the increase in computational cost into account, we have to be cautious in replacing the current TCAS conflict detection scheme with the probabilistic one based on the estimated conflict probability. Nevertheless, we warn the reader that these observations are not conclusive and that more thorough and comprehensive studies are needed.

V. Conclusion

We studied the detection component of the conflict detection and resolution system for air traffic management. Two representative conflict detection schemes, namely, the geometric and the probabilistic method, were compared for the collision detection based on the predicted trajectory with or without the intent information from the encounter aircraft. We applied an operational constraint and maximized the conflict detection probability for each scheme. Using the simulated aircraft encounter scenarios with various types of uncertainties in trajectory prediction, we found that the geometric method performs better than the probabilistic method when the intruder's intent is unknown. However, the probabilistic method is better under relatively large uncertainty of the predicted trajectory. We also found that the decentralized conflict alert system has a stronger tendency to defer the collision alert than the centralized scheme under the same operational constraint.

In practice, the threshold optimization for a conflict detection scheme may have to handle time varying conflict resolution cost for the conflicts involving more than two aircraft. The operating cost constraint used in this paper has obvious limitation. We hope to extend the constrained optimization framework to a more general conflict detection situation in the future.

Acknowledgment

This work is supported in part by NASA/LEQSF(2013-15)-Phase3-06 through grant NNX13AD29A.

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