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Cooperative Terrain Based Navigation and Coverage Identification Using Consensus

André R. Braga and Marcelo G.S. Bruno
Division of Electronics Engineering
Aeronautics Institute of Technology
São José dos Campos SP 12228-900, Brazil

Emre Özkan, Carsten Fritsche, Fredrik Gustafsson
Department of Electrical Engineering
Linköping University
SE-581 83 Linköping, Sweden

***Abstract*—This paper presents a distributed online method for joint state and parameter estimation in a Jump Markov NonLinear System based on a distributed recursive Expectation Maximization algorithm. State inference is enabled via the use of Rao-Blackwellized Particle Filter and, for the parameter estimation, the E-step is performed independently at each sensor with the calculation of local sufficient statistics. An average consensus algorithm is used to diffuse local sufficient statistics to neighbors and approximate the global sufficient statistics throughout the network. The evaluation of the proposed algorithm is carried out on a Terrain Based Navigation problem where the unknown parameters of the observation noise model contain relevant information about the terrain properties.**

I. Introduction

In a sensor network, several physically dispersed agents operate in a way they iterate cooperatively with a common goal, e.g., the estimation of certain unknown information. Each network node has direct access to the measurements (observations) of its own sensors and, using its communication capacity, the global estimation performance can be improved. In the ideal case, the optimum performance can be reached when a fusion center (or leader node) has access to all the observations (centralized architecture) [1]. More flexible systems do not even need a controller data fusion node because they can configure themselves. This provides additional robustness, since a failure in the fusion center would collapse the whole system [1]. This flexibility can be reached when the nodes communicate with the neighbors, sending local data in such a way that the information is spread over the network [2].

Furthermore, when the nodes are not fully connected, which means that an information needs multiple hops to be spread over the entire set of nodes or to a fusion center, the centralized solution for the estimation problem is very costly from the communication point of view. In order to design such systems, there is a need of techniques and methods that use only local communication between neighbors in a cooperative way.

Among possible solutions, consensus algorithm are promising where all the nodes in the network aim on reaching an agreement on some certain quantity. The use of the consensus averaging technique [3] has already been studied in linear distributed estimation problems using distributed Kalman Filter (KF) [4] and also in nonlinear problems using Particle Filter (PF) [1], [5]–[7]. It uses the advantage of asymptotically reaching the optimal centralized solution.

One of the drawbacks of this approach is the potentially prohibitive communication overhead due to the multi-iterative consensus step [8]. The aim of this work is to investigate how well the consensus algorithm performs in the problem of sequential estimation of mixtures with constrained communication (consensus iterations).

The evaluation scenario consists of the problem of Terrain Based Navigation (TBN), whose concept is basically the use of terrain height variations along the aircraft flight path together with information from the Inertial Navigation System (INS) to provide high performance position estimates in an autonomous manner without any support information sent to the aircraft [9]. It is based on the altitude over sea-level measured by a barometric sensor and provided by the Air Data Computer (ADC) and the ground clearance measured with a Radar Altimeter (RALT) pointed downward. These measurements are compared with a reference database map [10]. The challenge with TBN is how to deal with its highly nonlinear, non-analytical nature [11]. Because of that, it has been a technical driver for real-time applications of the PF in both the signal processing and robotics communities that basically concern on positioning using Geographical Information System (GIS) containing database with features of surrounding landscape [12].

When measuring the ground clearance, the RALT will sometimes react on echoes from tree tops or buildings. It results in different bias and variance on the observations depending on the terrain category beneath the aircraft which can be modeled as a Gaussian Mixture Model (GMM) [10], [11].

The main objective with the estimation of static parameters is the learning of complex noise models due to reflections from the overflown terrain. This enables the positioning task to work properly without a priori knowledge of the altimeter bias behavior and also provides the terrain coverage characteristics as a useful information.

This paper is organized as follows. In Section II, the system model and notations are presented. In Section III, the Expectation Maximization (EM) algorithm is described in the context of sensor networks. In Section IV the solution for the joint state and parameter estimation based on Sequential Monte Carlo (SMC) is presented. In Section V, the proposal of a system identification solution for sensor networks is provided. In Section VI, the evaluation of the proposed solution is addressed in the problem of joint TBN and coverage identification. Finally, in Section VII, some conclusions and perspectives for future extensions are discussed.

II. System Model

The topology of the sensor network is modeled as $\mathcal{G} = (\nu, \varepsilon)$, which is an undirected graphical model where ν is the set of N_S sensor nodes and ε are the set of edges, each as an unordered pair of distinct nodes. The neighborhood of a node $s \in \nu$ is defined as $\Gamma(s) \triangleq \{r|(s,r) \in \varepsilon\}$.

The Jump Markov NonLinear System (JMNLS) at each node s has the following form,

$$r_{s,t} \sim \Pi(r_{s,t}|r_{s,t-1}), \tag{1a}$$
$$x_{s,t} \sim f(x_{s,t}|x_{s,t-1}), \tag{1b}$$
$$y_{s,t} \sim g(y_{s,t}|x_{s,t}, \theta_{r_{s,t}}). \tag{1c}$$

This hybrid system presents a continuous state variable $x_{s,t} \in \mathbb{R}^{n_x}$ with transition density (1b) and a discrete mode variable $r_{s,t} \in \{1, ..., K\}$ where K is the number of modes following a Markov model with transition probabilities

$$\pi_{k\ell} = \Pi(\ell|k) = \mathbb{P}(r_t = \ell|r_{t-1} = k). \tag{2}$$

The complete system state $z_{s,t} \triangleq \{x_{s,t}, r_{s,t}\}$ is observed indirectly through the measurements $y_{s,t} \in \mathbb{R}^{n_y}$ whose observation model is parameterized by a set of mode based parameters $\theta_{r_{s,t}}$ as in (1c).

The transition probabilities, that form the $K \times K$ Transition Probability Matrix (TPM), Π, as well as the set of Gaussian component parameters for each mode, $\theta_k = \{\mu_k, \sigma_k\}$, are considered the unknown parameters of the model that must be identified,

$$\theta = (\{\theta_k\}_{k=1}^{K}, \Pi). \tag{3}$$

It must be noticed that, although each node presents its own independent sequence of discrete states, they all share the common set of parameters. The main idea of using cooperation between nodes is to take advantage of this property in the parameters identification.

Although not directly, the local state estimation can also take advantage of cooperation since it relies on the estimated parameters for the observation model. It can be seen then as a cooperative calibration task.

III. Expectation Maximization

The main idea of this Maximum Likelihood (ML)-based solution [13] is the approximation of the joint log-likelihood of the set of observations, $y_{1:n}$, and the so called missing data, $z_{1:n}$, by a function which is a projection, $\mathcal{Q}(\theta, \theta')$, onto space defined by available observations and a current estimate θ' of the likelihood maximizer [14],

$$\begin{aligned}\mathcal{Q}(\theta, \theta') &= \mathbb{E}_{\theta'}[\log p(y_{1:n}, z_{1:n}|\theta)|y_{1:n}] \\ &= \int \log p(y_{1:n}, z_{1:n}|\theta) p(z_{1:n}|y_{1:n}, \theta') dz_{1:n}.\end{aligned} \tag{4}$$

This strategy is based on the assumption that maximizing the complete likelihood, $p(y_{1:n}, z_{1:n}|\theta)$, is easier than $p(y_{1:n}|\theta)$ [15]. In that $y_{1:n} = \{y_{s,1:n}\}_{s=1}^{N_S}$ and the same applies to $z_{1:n}$.

It is assumed here that the nonlinear dynamical system corresponding to each mode belongs to the curved exponential family composed of $\mathcal{S}$, ψ and $\mathcal{A}$; respectively, the sufficient statistics, natural parameter and log-partition function [16].

The complete data likelihood can then be factorized as in (5) when considering a JMNLS and making use of the indicator function $\mathbb{1}(.)$ [17] as well as with the assumption that y, x and r are Independent and Identically Distributed (i.i.d.) between nodes.

$$\begin{aligned}&\log p(x_{1:n}, r_{1:n}, y_{1:n}|\theta) \\ &= \sum_{s=1}^{N_S}\sum_{t=1}^{n} \log p(x_{s,t}, r_{s,t}, y_{s,t}|x_{s,t-1}, r_{s,t-1}, \theta) \\ &= \sum_{s=1}^{N_S}\sum_{t=1}^{n} \log \Pi(r_{s,t}|r_{s,t-1}) \\ &+ \sum_{s=1}^{N_S}\sum_{t=1}^{n} \log(g(y_{s,t}|x_{s,t}, \theta_{r_{s,t}}) f(x_{s,t}|x_{s,t-1})) \\ &= \sum_{s=1}^{N_S}\sum_{k=1}^{K}\sum_{\ell=1}^{K}\sum_{t=1}^{n} \log(\pi_{k\ell}) \mathbb{1}(r_{s,t} = \ell, r_{s,t-1} = k) \\ &+ \sum_{s=1}^{N_S}\sum_{k=1}^{K}\sum_{t=1}^{n} \mathbb{1}(r_{s,t} = k) \\ &\times (\langle \psi_k(\theta_k), s_{k,t}(y_{s,t}, x_{s,t}, x_{s,t-1})\rangle - \mathcal{A}_k(\theta_k)).\end{aligned} \tag{5}$$

Based on (5), the auxiliary quantity, or projection function, can be written as

$$\begin{aligned}\mathcal{Q}(\theta, \theta') = \sum_{s=1}^{N_S}\Bigg(&\sum_{k=1}^{K}\sum_{\ell=1}^{K} \mathcal{S}^{(1)}_{s,k\ell,n} \log(\pi_{k\ell}) \\ &+ \sum_{k=1}^{K}\left(\left\langle \psi_k(\theta_k), \mathcal{S}^{(3)}_{s,k,n}\right\rangle - \mathcal{A}_k(\theta_k)\mathcal{S}^{(2)}_{s,k,n}\right)\Bigg),\end{aligned} \tag{6}$$

with the introduction of local sufficient statistics, whose computation is detailed in [18].

In order to obtain $\mathcal{S}_{s,n} \triangleq [\mathcal{S}^{(1)}_{s,k\ell,n}\ \mathcal{S}^{(2)}_{s,k,n}\ \mathcal{S}^{(3)}_{s,k,n}]^T$, the EM algorithm requires the computation of densities associated with a smoothing problem. And, for an online implementation, the case of additive functionals is possible by means of the forward-only smoothing techniques [16].

Let then an intermediate quantity be defined as $T_{s,t}(z_{s,t}) \triangleq \mathbb{E}_{\theta'}\left[\sum_{t=1}^{n} s(z_{s,t}, z_{s,t-1})|z_{s,t}, y_{s,1:t}\right]$, the sufficient statistics can be obtained from $T_{s,t}(z_{s,t})$ by

$$\mathcal{S}_{s,t} = \mathbb{E}_{\theta'}\left[T_{s,t}(z_{s,t})|y_{s,1:t}\right] = \int T_{s,t}(z_{s,t}) p(z_{s,t}|y_{s,1:n},\theta') dz_{s,t}. \quad (7)$$

It means the additive smoother output can be computed by the filtered estimate of $T_{s,t}(z_{s,t})$ in (7). Furthermore, the additive structure enables the recursive calculation, being the basis for the online EM algorithm [19], where the intermediate quantity is updated at each time step and the new parameter estimate $\hat{\theta}^t$ is computed according to (8), where the mapping $\Lambda_k(.)$ is described in [17].

$$\hat{\theta}_k = \Lambda_k\left(\frac{\mathcal{S}^{(3)}_{k,n}}{\mathcal{S}^{(2)}_{k,n}}\right) \qquad \hat{\pi}_{k\ell} = \frac{\mathcal{S}^{(1)}_{k\ell,n}}{\sum_{j=1}^{K}\mathcal{S}^{(1)}_{kj,n}}. \quad (8)$$

Using the additive property for the sufficient statistics [18] and a stochastic approximation type of forgetting, the recursive update for the intermediate quantity can be performed as

$$T_{s,t}(z_{s,t}) \leftarrow \int [(1-\eta_t)T_{s,t-1}(z_{s,t-1}) + \eta_t s_t(z_{s,t-1}, z_{s,t})] \times p(z_{s,t-1}|z_{s,t}, y_{s,1:t-1},\theta') dz_{s,t-1}, \quad (9)$$

where $\{\eta_t\}_{t\geq 1}$ is a decreasing sequence of step-sizes, which satisfy the usual stochastic approximation requirement that $\sum_{t\geq 1}\eta_t = \infty$ and $\sum_{t\geq 1}\eta_t^2 \leq \infty$ [17].

IV. Sequential Monte Carlo Online EM for JMNLS

A. Filtering

This work uses the strategy proposed in [18] that utilizes Rao-Blackwellization for integrating out the discrete state variable using conditional Hidden Markov Model (HMM) filters. The idea is to decompose the complete state-space density,

$$p(x_{s,1:t}, r_{s,t}|y_{s,1:t}) = p(r_{s,t}|x_{s,1:t}, y_{s,1:t}) p(x_{s,1:t}|y_{s,1:t}). \quad (10)$$

Here, the second factor can be approximated using a PF represented by the set of N_P weighted particles $\{x^{(i)}_{s,1:t}, w^{(i)}_{s,t}\}_{i=1}^{N_P}$. The first factor of (10) can be approximated with a conditional HMM filter.

By defining $\alpha^{(i)}_{s,t|t-1}(\ell) \triangleq \mathbb{P}(r_{s,t}=\ell|x^{(i)}_{s,1:t-1}, y_{s,1:t-1})$, the computation of the mode prediction probabilities is performed by marginalization over $r_{s,t-1}$, yielding

$$\alpha^{(i)}_{s,t|t-1}(\ell) = \sum_{k=1}^{K}\pi_{k\ell}\alpha^{(i)}_{s,t-1|t-1}(k). \quad (11)$$

In Rao-Blackwellized filtering, each node s can define the following quantity,

$$\gamma^{(i)}_{s,t}(r_{s,t}) \triangleq p(y_{s,t}, x^{(i)}_{s,t}, r_{s,t}|x^{(i)}_{s,1:t-1}, y_{s,1:t-1}) = g(y_{s,t}|x^{(i)}_{s,t},\theta_{r_{s,t}}) f(x^{(i)}_{s,t}|x^{(i)}_{s,t-1})\alpha^{(i)}_{s,t|t-1}(r_{s,t}). \quad (12)$$

The update on the mode probabilities can now be calculated by

$$\alpha^{(i)}_{s,t|t}(\ell) = \frac{\gamma^{(i)}_{s,t}(\ell)}{\sum_{k=1}^{K}\gamma^{(i)}_{s,t}(k)}, \quad (13)$$

and the unnormalized particle weights can be calculated as

$$\bar{w}^{(i)}_{s,t} \propto w^{(i)}_{s,t-1}\frac{p(x^{(i)}_{s,t}, y_{s,t}|x^{(i)}_{s,1:t-1}, y_{s,1:t-1})}{q(x^{(i)}_{s,t}|x^{(i)}_{s,1:t-1}, y_{s,1:t-1})} = w^{(i)}_{s,t-1}\frac{\sum_k^K\gamma^{(i)}_{s,t}(k)}{q(x^{(i)}_{s,t}|x^{(i)}_{s,1:t-1}, y_{s,1:t-1})}. \quad (14)$$

The filtering computation can then be summarized by Algorithm 1, considering the use of $q(x_{s,t}|x^{(i)}_{s,1:t-1}, y_{s,1:t-1}) = f(x_{s,t}|x^{(i)}_{s,t-1})$ as the proposal distribution.

Algorithm 1 Rao-Blackwellized Particle Filter (RBPF) for JMNLS at node s [18]

Input: $\{x^{(i)}_{s,1:t-1}, w^{(i)}_{s,t-1}, \{\alpha^{(i)}_{s,t-1|t-1}(\ell)\}_{\ell=1}^{K}\}_{i=1}^{N_P}$
Input: $\hat{\theta}^{t-1}$
1: Resampling if necessary. Result denoted as $\{\tilde{x}^{(i)}_{s,1:t-1}, \tilde{w}^{(i)}_{s,t-1}, \{\tilde{\alpha}^{(i)}_{s,t-1|t-1}(\ell)\}_{\ell=1}^{K}\}_{i=1}^{N_P}$.
2: **for** $i = 1 \rightarrow N_P$ **do**
3: Compute $\{\tilde{\alpha}^{(i)}_{s,t|t-1}(\ell)\}_{\ell=1}^{K}$ as in (11)
4: Draw $x^{(i)}_{s,t} \sim f(x_{s,t}|\tilde{x}^{(i)}_{s,t-1})$
5: Compute $\{\gamma^{(i)}_{s,t}(\ell)\}_{\ell=1}^{K}$ as in (12)
6: Compute $\{\tilde{\alpha}^{(i)}_{s,t|t}(\ell)\}_{\ell=1}^{K}$ as in (13)
7: Compute unnormalized particle weights $\bar{w}^{(i)}_{s,t}$ as in (14)
8: **end for**
9: Normalize particle weights obtaining $\{w^{(i)}_{s,t}\}_{i=1}^{N_P}$
Output: $\{x^{(i)}_{s,1:t}, w^{(i)}_{s,t}, \{\alpha^{(i)}_{s,t|t}(\ell)\}_{\ell=1}^{K}\}_{i=1}^{N_P}$

B. Online E-Step

For the computation of the intermediate quantity, from where the sufficient statistics are obtained in (7), the backward density can then be extended to

$$p(x_{s,1:t-1}, r_{s,t-1}|z_{s,t}, y_{s,1:t-1},\theta') \propto f(x_{s,t}|x_{s,t-1})\Pi(r_{s,t}|r_{s,t-1}) \times p(r_{s,t-1}|x_{s,1:t-1}, y_{s,1:t-1},\theta') p(x_{s,1:t-1}|y_{s,1:t-1},\theta'), \quad (15)$$

by plugging in the filter approximations

$$p(x_{s,1:t-1}, r_{s,t-1} = k | x_{s,t}^{(i)}, r_{s,t} = \ell, y_{s,1:t-1}, \theta')$$
$$\approx \sum_{j=1}^{N_P} \frac{\tilde{w}_{s,t}^{(i,j)}(\ell,k)}{\sum_{u=1}^{N} \sum_{m=1}^{K} \tilde{w}_{s,t}^{(i,u)}(\ell,m)} \delta(x_{s,1:t-1} - x_{s,1:t-1}^{(j)}), \tag{16}$$

with

$$\tilde{w}_{s,t}^{(i,j)}(\ell,k) = f(x_{s,t}^{(i)} | x_{s,t-1}^{(j)}) \pi_{k\ell} \alpha_{s,t-1|t-1}^{(j)}(k) w_{s,t-1}^{(j)}. \tag{17}$$

Finally, the computation of the intermediate quantity, $\hat{T}_{s,t}^{(i)}(\ell) \approx T_{s,t}(x_{s,t}^{(i)}, r_{s,t} = \ell)$, can be done by

$$\hat{T}_{s,t}^{(i)}(\ell) = \sum_{j=1}^{N_P} \sum_{k=1}^{K} \Bigg(\frac{\tilde{w}_{s,t}^{(i,j)}(\ell,k)}{\sum_{u=1}^{N} \sum_{m=1}^{K} \tilde{w}_{s,t}^{(i,u)}(\ell,m)}$$
$$\left[(1-\eta_t) \hat{T}_{s,t-1}^{(j)}(k) + \eta_t s_t(x_{s,t-1}^{(j)}, r_{s,t-1} = k, x_{s,t}^{(i)}, r_{s,t} = \ell) \right] \Bigg). \tag{18}$$

The Online E-Step can be summarized by Algorithm 2.

Algorithm 2 Online E-Step for JMNLS at node s [18]

Input: $\{x_{s,1:t}^{(i)}, w_{s,t}^{(i)}, \{\alpha_{s,t|t}^{(i)}(\ell)\}_{\ell=1}^{K}\}_{i=1}^{N_P}$
Input: $\{\{\hat{T}_{s,t-1}^{(i)}(\ell)\}_{\ell=1}^{K}\}_{i=1}^{N_P}$
Input: $\hat{\theta}^{t-1}$
1: Compute $\{\{\hat{T}_{s,t}^{(i)}(\ell)\}_{\ell=1}^{K}\}_{i=1}^{N_P}$ as in (18)
2: Compute $\hat{\mathcal{S}}_{s,t} = \sum_{i=1}^{N_P} \sum_{\ell=1}^{K} w_{s,t}^{(i)} \alpha_{s,t|t}^{(i)} \hat{T}_{s,t}^{(i)}(\ell)$
Output: $\hat{\mathcal{S}}_{s,t}$

The computational complexity of this algorithm is $O(K^2 N_P^2)$, which is basically the effort for the computation of (18). In [18], a way to reduce its complexity was proposed and relies on path-based smoothing.

The M-Step will be subject to the type of cooperation considered between nodes.

V. Networked System Identification

A lot of focus has been given in the research community to the problem of distributed parameter estimation in sensor networks using EM. Some efforts have been concentrated on estimation of GMM density parameters.

The work on [20] takes the advantage of the fact that E-steps can be computed at each node and then properly accumulated by means of some message passing but all the observations are collected and the EM is performed offline.

The solution proposed in [21] also operates in batches, but considers a consensus filter in order to make the network reach an agreement on the global sufficient statistics.

In [22], an algorithm that process sequentially the observations in a distributed manner is proposed. It relies on a simplified information exchange protocol by which only one or few nodes share their own sufficient statistics and presents an evaluation on a scalar GMM density estimation.

Since, in the problem of navigation for multiple nodes, each one has its own space state, it was decided to let each one to run its own independent RBPF. It leads to a more flexible network and not to very large state space.

A. Centralized Online EM for JMNLS

Within the centralized approach, it is assumed that a fusion center is available and is able to concentrate all available information in the network at each time step. The result of Algorithm 2 is here considered as the output information provided by each node.

Strictly speaking, the global sufficient statistics is simply the sum of all local quantities [21], but considering the M-step in (8), the average of the local sufficient statistics will lead to the same result because the term $1/N_S$ will be cancelled. This assumption enable the use of average consensus on the network without the need for each node to have knowledge on the total number of nodes in the network.

Then, for the parameter calculation point of view, the global summary quantities can be viewed as averages of local summary quantities from all nodes, the global sufficient statistics $\mathcal{S}_t$ can be calculated as an average like

$$\hat{\mathcal{S}}_t = \frac{1}{N_S} \sum_{s=1}^{N_S} \hat{\mathcal{S}}_{s,t}. \tag{19}$$

In the centralized approach, each node can calculate the local quantities based on its observations and the current parameter set. Then, each node sends its local sufficient statistics to a centralized unit which calculates the estimated parameters based on both (8) and (19). Finally, the update on the estimated parameter set is sent back to every node connected to the graph $\mathcal{G}$, introduced in Section II.

B. Consensus Based Distributed Online EM for JMNLS

In the consensus strategy, a linear iterative method is used in order to compute, or approximate, the average of the sufficient statistics in a distributed manner. It requires the system to operate in two time scales. One is the estimation step, where both the filtering and identification are performed, i.e. Algorithms 1 and 2 are executed. The other is the inner iterative consensus, in that the nodes exchange information over the network $\mathcal{G}$ aiming the approximation of the global sufficient statistics.

This enables the use of consensus in order to approximate asymptotically the global summary quantities through information diffusion over the network [23]. Between estimation time steps, each inner consensus iteration q at node s updates its local state, $\zeta_s^{(q)}$, with a linear combination of its own state and the states of its neighbors

$$\zeta_s^{(q)} = a_{ss} \zeta_s^{(q-1)} + \sum_{r \in \Gamma(s)} a_{sr} \zeta_r^{(q-1)}. \tag{20}$$

The local state is initialized as the output of Algorithm 2, i.e. $\zeta_s^{(0)} = \hat{\mathcal{S}}_{s,t}$, and a_{sr} is the linear weight on $\zeta_r^{(q-1)}$ at node s. Setting $a_{sr} = 0$ means that $r \notin \Gamma(s)$. Notice that the weight matrix $A \in \mathbb{R}^{N_S \times N_S}$ has the sparsity defined by the communication graph $\mathcal{G}$ [23].

The choice for the weights should be made in such a way that

$$\lim_{q \to \infty} \zeta_s^{(q)} = \frac{1}{N_S} \sum_{r=1}^{N_S} \zeta_r^{(0)}. \tag{21}$$

This work proposes to utilize the consensus strategy in order to make the nodes in the network to agree in the global sufficient statistics, $\hat{\mathcal{S}}_t$, of the network.

In order to comply with the convergence criteria, the following conditions are necessary and sufficient for the symmetric and (doubly) stochastic weight matrix A

$$\mathbf{1}^T A = \mathbf{1}^T, \tag{22a}$$
$$A\mathbf{1} = \mathbf{1}, \tag{22b}$$
$$\rho(A - \mathbf{1}\mathbf{1}^T / N_S) < 1, \tag{22c}$$

where $\rho(.)$ denotes the spectral radius of a matrix and $\mathbf{1}$ is a vector of ones [3].

The operation performed at each consensus iteration is given by Algorithm 3. The parameter q_{max} provides a tradeoff between accuracy on the global average calculation and resources allocation, such as time and communication load, in the cooperative task.

Algorithm 3 Average Consensus at node s

Input: Initial internal state at node s: $\zeta_s^{(0)} = \hat{\mathcal{S}}_{s,t}$
1: $q \leftarrow 0$
2: **while** $q < q_{max}$ **do**
3: Increment iteration: $q \leftarrow q + 1$
4: Broadcast $\zeta_s^{(q-1)}$ to all neighbors $r \in \Gamma(s)$
5: Receive $\zeta_r^{(q-1)}$ from all neighbors $r \in \Gamma(s)$
6: Compute $\zeta_s^{(q)} = a_{ss}\zeta_s^{(q-1)} + \sum_{r \in \Gamma(s)} a_{sr}\zeta_r^{(q-1)}$
7: **end while**
Output: Global $\hat{\mathcal{S}}_t$ approximation as s

VI. SIMULATIONS

The simplified model used for solving the TBN is the 2D position and velocity estimation for an aircraft flying over a terrain. The only available measurement is the height above terrain and it is also assumed that altitude above sea level is known. For this kind of mixed estimation problem it is appropriate to use joint inference and learning algorithms, i.e. applying a PF due to the nonlinear nature of the measurement together with a learning strategy for calibration.

The dynamics of the aircraft movement is based on a constant velocity model [24] and is presented in (23), where the pairs $\{p_t^X, p_t^Y\}$ and $\{v_t^X, v_t^Y\}$ represent the 2D position and velocity at time instant t, respectively

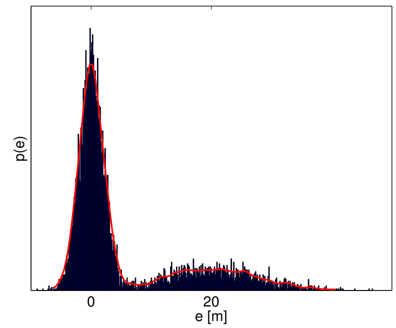


Figure 1. Distribution for terrain elevation error due to radar altimeter model.

$$x_{t+1} = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_t^X \\ p_t^Y \\ v_t^X \\ v_t^Y \end{bmatrix} + \begin{bmatrix} \frac{T^2}{2} & 0 \\ 0 & \frac{T^2}{2} \\ T & 0 \\ 0 & T \end{bmatrix} \begin{bmatrix} w_t^X \\ w_t^Y \end{bmatrix}, \tag{23}$$

where

$$\begin{bmatrix} w_t^X \\ w_t^Y \end{bmatrix} \sim \mathcal{N}(0, Q) \quad , \quad Q = \begin{bmatrix} Q_{XX} & 0 \\ 0 & Q_{YY}. \end{bmatrix}. \tag{24}$$

Further, the measurement is the terrain elevation based on the radar height, modeled as

$$y_t = h(p_t^X, p_t^Y) + e_{r_t}, \tag{25}$$

where $h(.,.)$ is a non-analytical and nonlinear lookup table that represents the terrain elevation database, according to position $\{p_t^X, p_t^Y\}$.

The observation noise, e_{r_t}, is primarily modeled to consider the effect of multiple reflections of the RALT echo signal on the open terrain model [25]. The echo reflections are modeled by a 2-state Markov chain [26] with unknown mode-dependent mean, standard deviation (θ_{r_t}) and TPM (Π). Figure 1 presents a histogram of the terrain elevation error due to the measurement model considered in the simulated scenario.

The network consists of four aircraft flying on a finger four formation [27] and, for the centralized solution, the flight leader (node 1) is the fusion center. Figure 2 presents the graphical model, $\mathcal{G}$, where the dashed white lines indicate the flight path and the ground clearance is highlighted with the black dashed ones towards the ground. The bidirectional communication links (edges ε) are also displayed as gray lines connecting the nodes.

The performance is assessed via 100 independent Monte Carlo runs evaluating a 90 seconds (1800 measurements) of stabilized flight over a simulated terrain created by a 2D Gaussian lowpass filter over uniform noise [12].

At time $t = 0$, $x_{s,0}^{(i)} = x_{s,0}$, $w_{s,0}^{(i)} = 1/N_P$ and $\alpha_{s,0|0}^{(i)}(\ell) = \mathbb{1}(r_{s,0} = \ell)$ for $i = 1, ..., N_P$. The parameter estimation starts

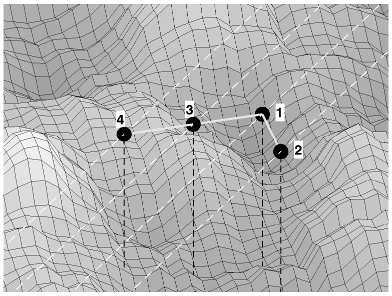


Figure 2. Networked flying platforms over terrain.

Table I
SIMULATION PARAMETERS

Parameter	Value
Number of Particles (N_P)	300
Number of Modes (K)	2
$[\mu_1\ \mu_2]\ [m]$	[0 20]
$[\sigma_1\ \sigma_2]\ [m]$	[1 4]
$[\pi_{11}\ \pi_{22}]$	[0.85 0.6]
$Q_{XX}\ ,\ Q_{YY}\ [m/s^2]$	100
$v_0^X\ ,\ v_0^Y\ [m/s]$	170
$T\ [ms]$	50
η_t	$t^{-0.7}$

after 2.5 seconds (50 iterations) in order to guarantee that the M-step update is numerically well-behaved [17]. The initial system parameters are: $[\mu_1\ \mu_2] = [-5\ 25]$, $[\sigma_1\ \sigma_2] = [5\ 5]$ and $[\pi_{11}\ \pi_{22}] = [0.5\ 0.5]$.

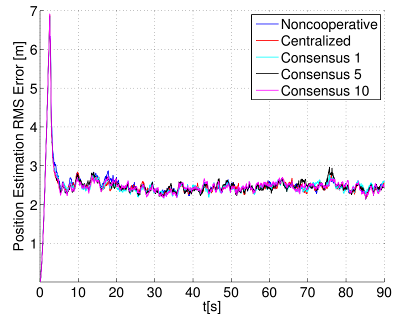
For the consensus strategy, a fixed matrix, A, of consensus iteration weights based on the maximum-degree rule [23] was assumed and is given as

$$A = \begin{bmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 1/4 & 3/4 & 0 & 0 \\ 1/4 & 0 & 1/2 & 1/4 \\ 0 & 0 & 1/4 & 3/4 \end{bmatrix}. \tag{26}$$

Also, the distributed strategy was evaluated considering three different iterative average consensus configurations in that the maximum number of consensus iterations, q_{max}, was limited to 1, 5 and 10 iterations. The main simulation parameters are presented in Table I.

A. Results Analysis

When the position estimation Root Mean Square (RMS) error is addressed, the different strategies do not differ much on performance in the steady state as presented in the upper part of Figure 3. From the navigation performance point of view, it means that the network does not take significant advantage of cooperation between nodes in terms of position accuracy. However, when looking with more details in the initial instants of the simulation, in the lower part of Figure 3, the estimation error increases up to the starting point of



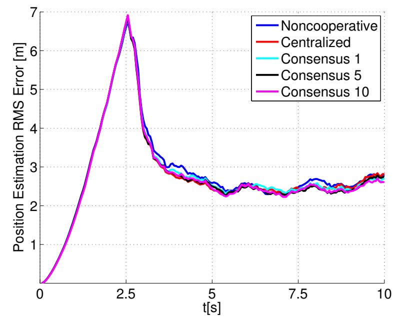


Figure 3. Position RMS error.

the parameter estimation (2.5 seconds). The strategies that rely on some sort of cooperation also lead to a slightly faster convergence towards the steady state positioning error.

In Figure 4, the dynamic behavior of parameter estimation is presented during the simulation time. It can be perceived that the centralized solution presents the lowest overall variance on parameter estimation, while the noncooperative approach the highest. The variance of the consensus solution is in-between the noncooperative and centralized, providing a good trade off between complexity and accuracy.

In Figures 5 and 6, the RMS error for the parameter estimation presents a performance improvement when using cooperation between nodes. As expected, the centralized approach leads to the best results. The consensus strategy with 10 iteration performs almost as good as the centralized approach. When the number of consensus iterations is reduced, the performance degrades but still outperforms the noncooperative solution.

B. Communication Analysis

In Table II, the network throughput requirements are presented for each strategy. The network throughput is the total communication bandwidth required by the entire network in order to accomplish each strategy. It is based on the assumption of a four-Byte representation for a floating point value and that the set of sufficient statistics and parameters demand 10 and 6 floats (40 and 24 Bytes), respectively.

The consensus with only one iteration has similar communication requirements as the centralized approach and provides substantially performance increase as was shown in previous section.

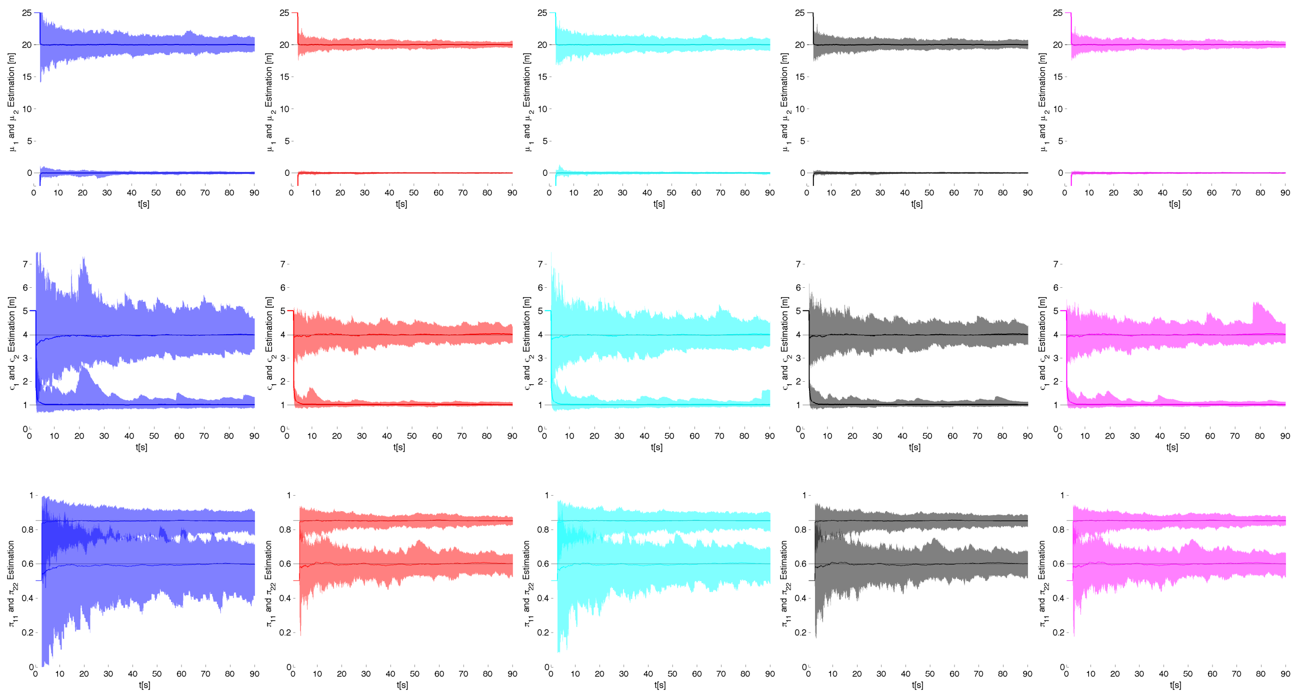


Figure 4. Parameter identification results. From left to right: Noncooperative, Centralized, Consensus 1, Consensus 5, and Consensus 10. From top to bottom: (μ_1, μ_2),(σ_1, σ_2), and (π_{11}, π_{22}). The lines show the averages and the shaded areas show the upper and lower bounds.

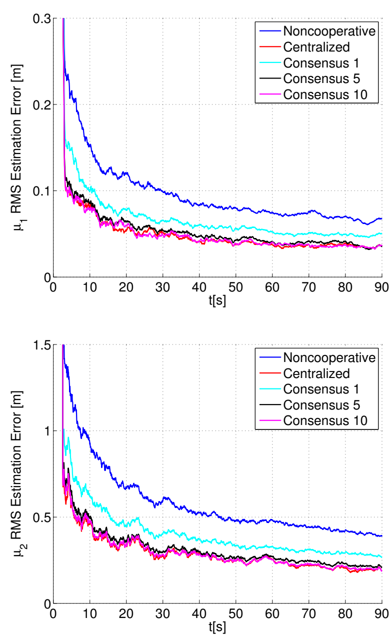


Figure 5. Noise mean RMS estimation error.

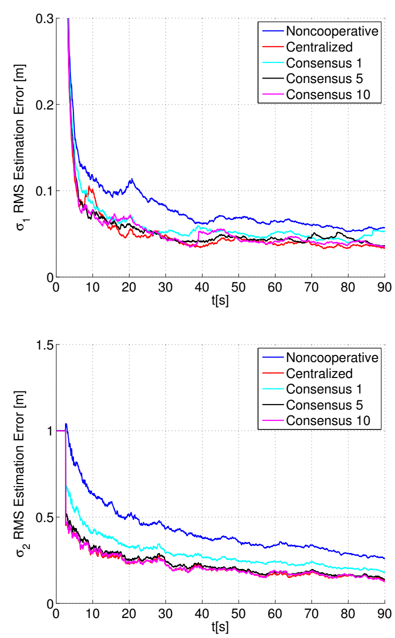


Figure 6. Noise standard deviation RMS estimation error.

Table II
NETWORK THROUGHPUT

Strategy	Throughput [KB/s]
Noncooperative	0
Centralized	4.5
Consensus 1	4.7
Consensus 5	23.5
Consensus 10	46.9

It must be highlighted that the centralized approach is very dependent on the network topology and, the larger the network, the more costly it is, since it demands more hops for the information to be concentrated and then spread from one single node. Another advantage of the consensus approach is that it is more robust and flexible, since it does not rely on a single point on the network and also does not require routing tables for the information flow.

VII. CONCLUSION

We have proposed a method of networked distributed system identification in JMNLS jointly with state estimation using the online EM and Rao-Blackwellization. The method was evaluated in a TBN in that there is also the interest for identifying observation noise parameters, that characterize the overflown terrain coverage.

The results on a network of flying platforms in formation flight show that cooperation using consensus on the sufficient statistics provides an improvement on the parameters identification, approximating the results obtained by a centralized approach. The state estimation takes advantage of cooperation between node in terms of convergence during the calibration process (parameters identification).

As possible future extensions for this work, the increase on the accuracy for the navigation function can be exploited by adding inter-node distance measurements. Real maps and measurements can also be used in order to test the effectiveness of this approach in realistic scenarios.

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