# Proximity Report Triggering Threshold Optimization for Network-Based Indoor Positioning

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Abstract—Driven by the promising beaconing techniques, this paper presents a general received-signal-strength (RSS) threshold optimization procedure for proximity reports to support networkbased indoor positioning. The desired RSS threshold is found through optimizing a metric function (for instance the localization root-mean-square-error) in terms of both the deployment information and the RSS modeling in consideration of an evaluation area. The resulting RSS threshold provides a trade-off between triggering many but less informative proximity reports with a low threshold, and triggering few but very informative proximity reports with a high threshold, and thus enables enhanced performance for some low-cost and low-complex proximity based positioning algorithms. The proposed framework is also validated with real RSS measurements collected in an office area with deployed bluetooth low-energy (BLE) beacons.

# I. INTRODUCTION

Over the past few years, indoor localization and tracking using wireless networks has received considerable attention due to the ever increasing demand on location-awareness in various sectors. So far, most of the efforts have been made to improve the localization accuracy using advanced technologies, for instance statistical sensor fusion [1], dedicated to optimally fuse different types of position-related measurements. Such measurements include round-trip-timeof-arrival (RTOA), received-signal-strength (RSS), angle-ofarrival (AOA), speed, and acceleration measured from indoor wireless infrastructures, etc (for instance, cellular, wireless fidelity (Wi-Fi) and bluetooth low-energy (BLE) nodes) or by mobile device(s), or a combination. RSS reports are typically also used for radio resource management in cellular networks. Such reports may be periodic or event-triggered and may also include the RSS measurements with respect to the triggering node as well as other nodes. Since RSS is strongly correlated to the inter-distance between the node and the device, such eventtriggered reports are also referred to as proximity reports.

One way of obtaining proximity information in the network is to configure devices with an event-triggering threshold  $P_{th}$ to trigger a proximity report when RSS passes the threshold. More precisely,

$$\text{Proximity} \triangleq \begin{cases} 0, & \text{RSS} \le P_{th} \\ 1, & \text{RSS} > P_{th} \end{cases}.$$

Such proximity report indicates whether or not a target device is in the coverage area (depending on the threshold) of a *reference network node*. This is an example of cell identifier positioning, where the device is associated to the known position of the reference network node, either in a mobile centric manner or in a network centric manner [2]. We give an illustrative example in Fig. 1 for better explanation of this concept. Harness of time series of proximity reports may result in new-fashioned positioning system with lower communication bandwidth, smaller database, as well as cheaper deployment and maintenance cost. Instead of finding an accurate position estimate with unaffordable cost, the ambition of a proximity based positioning system is to promptly (possibly in real time) identify which zone the target of interest is in or about to enter or leave and further trigger events or performance/measurement report accordingly.

The focus of this paper is not on any specific proximity based positioning algorithm but about determining an appropriate RSS threshold for proximity report triggering. In the literature, RSS thresholding was once considered for cooperative localization in [3]. Therein, an RSS threshold was found to get rid of those neighboring sensors with worse RSS observations so as to optimize the network localization accuracy. In this paper, RSS thresholding is considered for non-cooperative, infrastructure-based indoor positioning. As compared to [3], our focus is the overall positioning performance in a given service area, and we give thorough treatment on the measurement campaign, RSS modeling, model fitting and parameter calibration, signaling, and performance evaluation using real data measured from a live network. Our contributions in this paper are two-folds. First, we propose a general framework for selecting a reasonable RSS threshold for proximity report based positioning. Figure 2 illustrates the proposed procedure, where the final RSS threshold is the output of an optimization process, given a preselected RSS model and a priori known sensor deployment information as the input. The resulting RSS threshold can enable subsequent design of novel proximity based positioning algorithms. For example, we have introduced in a companion paper [4] a proximity based particle filtering algorithm that requires much less communication bandwidth and memory storage for measurement reports. Second, the proposed RSS threshold optimization procedure is validated with real RSS measurements collected in a live BLE network.

The remainder of this paper is organized as follows: Section II lists the prerequisites for performing the RSS threshold optimization, including the deployment information, RSS modeling, and selection of an evaluation set of sample positions. We give three representative RSS models, namely a conventional linear log-distance model, a piece-wise linear log-distance model, as well as a more advanced nonlinear Gaussian process (GP) regression (GPR) model. This section corresponds to the first step shown in Fig. 2. Section III introduces a general RSS threshold optimization procedure. This section corresponds to the second step shown in Fig. 2.



Fig. 1. An illustrative example of proximity based indoor positioning. In this example, three transmitter nodes are deployed as reference network nodes to locate a mobile station. The arcs indicate the coverage radius of the nodes, given an RSS threshold. The proximity vectors, e.g., [1, 1, 0], indicate the proximity relative to the three reference network nodes.



Fig. 2. Key steps of the proposed RSS threshold optimization procedure. The first two steps are performed in the offline training phase, while the third step is performed in the online positioning phase.

In Section IV, we validate the proposed RSS threshold experimentally with real RSS measurements collected in an office area with deployed BLE beacons. Finally, Section V concludes the paper.

Throughout this paper, matrices are presented with uppercase letters and vectors with boldface lowercase letters. The operator  $\left[\cdot\right]^{T}$  stands for vector/matrix transpose and  $\left[\cdot\right]^{-1}$  stands for the inverse of a non-singular square matrix. The operator tr( $\cdot$ ) denotes the trace of a square matrix.  $\|\cdot\|$  stands for the Euclidean norm of a vector and  $|\cdot|$  denotes the cardinality of a set. The operator  $\mathbb{E}(\cdot)$  stands for the statistical expectation. The operator  $\ln(\cdot)$  stands for the natural logarithm and  $\log_{10}(\cdot)$ stands for the logarithm to base 10. Further,  $\nabla_{\theta} = \partial/\partial\theta$ denotes the gradient operator.  $\mathcal{N}(\mu, \sigma^2)$  denotes a Gaussian distribution with mean  $\mu$  and variance  $\sigma^2$ . Lastly,  $\operatorname{erf}(\cdot)$  stands for the standard Gaussian error function.

#### II. PREREQUISITES

#### A. Deployment

Throughout this paper, we restrict ourselves to indoor positioning scenarios where a number of N reference network nodes, such as cellular base stations, BLE beacons, Wi-Fi routers, Zigbee devices, or a combination, are deployed. The reference network nodes are often placed rather uniformly in the surveillance area and mounted either on the ceiling or high on the wall to give a panoramic view. The geographical position  $\mathbf{p}_{r}$  and the transmission power  $P_{T}$  of each of the reference network node are assumed to be known *a priori*. In addition to a transmitted reference signal, the reference network node may also broadcast information such as a sensor ID, position, network configurations, as well as a tiny amount of information about the event to be triggered.

### B. RSS Modeling

In the literature, there exist a plethora of deterministic and stochastic RSS models. The appropriateness, however, depends on the actual scenario [5]. Once an RSS model is predefined, the corresponding model parameters can be calibrated either offline given a batch of all available RSS measurements or online given RSS measurements that come in sequentially. We could also resort to expert knowledge or history records on any empirical model when a training phase cannot be carried out due to some reasons. In what follows, we focus on offline calibration of RSS model parameters, given a calibration set of M RSS measurements at different calibration locations,  $p_j, j = 1, 2, \ldots, M$ . We start with the canonical linear model, followed by a piece-wise linear log-distatnce model. These two models both represent an RSS measurement in terms of the one dimensional (1-D) distance between the transmitter and the receiver. We further introduce a nonlinear Gaussian process regression (GPR) [6] model, which represents an RSS measurement in terms of the 2-D or 3-D geographical position and is more realistic for complex indoor environments.

**Example I: Linear log-distance model.** The classical linear log-distance model is given by

$$r_j = \underbrace{A + 10B \log_{10} \left(\frac{d_j}{d_0}\right)}_{\mu_j} + e_j, \quad j = 1, 2, ..., M$$

where  $r_j$  is the *j*th RSS, *A* is a path loss factor, *B* is a path loss exponent, and  $d_j$  is a short-hand notation of the Euclidean distance between a calibration position  $\mathbf{p}_j$  and the reference network node's position  $\mathbf{p}_r$ , i.e.,  $d_j \triangleq ||\mathbf{p}_j - \mathbf{p}_r||$ . Often, the measurement term  $e_j$  is assumed to be Gaussian distributed with zero mean and variance  $\sigma^2$ , hence  $r_j \sim \mathcal{N}(\mu_j, \sigma^2)$ . Performing the conventional linear least-squares (LLS) fitting yields an optimal estimate of the unknown parameters  $[A, B]^T$ , namely,

$$[\hat{A}, \hat{B}]^T = \arg\min_{[A,B]} \sum_{j=1}^M (r_j - \mu_j)^2.$$
 (1)

We further compute an estimate of the noise variance,  $\sigma^2$ , by

$$\hat{\sigma}^2 = \frac{1}{M} \sum_{j=1}^{M} \left( r_j - \hat{A} - 10\hat{B}\log_{10}\left(\frac{d_j}{d_0}\right) \right)^2.$$
 (2)

In practice, the RSS measurements were collected subject to an intrinsic threshold,  $P_{dec}$  (in dBm), beyond which a data package cannot be decoded accurately. According to [7], more accurate fitting results can be obtained by taking into account this truncation effect. The corresponding loglikelihood function can be easily expressed as

$$ll(A, B, \sigma^2) \triangleq \sum_{j=1}^{M} \ln \left\{ \frac{\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\left(r_j - \mu_j\right)^2 / 2\sigma^2\right]}{\frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{P_{\operatorname{dec}} - \mu_j}{\sqrt{2\sigma^2}}\right)\right]} \right\}.$$

A set of parameter estimates can be found through maximizing the above log-likelihood function, namely,

$$\hat{\boldsymbol{\theta}} = [\hat{A}, \hat{B}, \hat{\sigma}^2] = \arg \max_{[A, B, \sigma^2]} ll(A, B, \sigma^2).$$

We could use the fitting results given in (1) and (2) as the starting point for the above numerical search.

**Example II: Piece-wise linear log-distance model.** The dual-slope piece-wise log-distance model is formulated as two linear log-distance models valid at different ranges from the reference network node. Concretely,

$$r_{j} = \begin{cases} A_{1} + 10B_{1} \log_{10} \left(\frac{d_{j}}{d_{0}}\right) + e_{1,j}, & d_{j} \leq d_{c} \\ A_{1} + 10B_{1} \log_{10} \left(\frac{d_{c}}{d_{0}}\right) + 10B_{2} \log_{10} \left(\frac{d_{j}}{d_{c}}\right) + e_{2,j}, & d_{j} > d_{c} \end{cases}$$
(3)

where  $d_c$  is often called critical distance in the literature. Dual-slope model is characterized by a path loss factor  $A_1$ and a path loss exponent  $B_1$  from the reference distance  $d_0$ up to the critical distance  $d_c$ . Beyond  $d_c$ , the RSS falls off in terms of another path loss exponent  $B_2$ . Calibration of the model parameters can be done as follows. For the RSS measurements with  $d_j \leq d_c$ , the conventional LLS fitting (without considering the truncation effect) is used to calculate  $\hat{A}_1$ ,  $\hat{B}_1$ , and  $\hat{\sigma}_1^2$ , i.e., in light of (1) and (2). As the truncation effect is more obvious for the RSS data with  $d_j > d_c$ , we solve  $\hat{B}_2$  and  $\hat{\sigma}_2^2$  through maximizing the following log-likelihood function with respect to  $B_2$  and  $\sigma_2^2$ ,

$$ll(B_{2},\sigma_{2}^{2}) = \sum_{j=1}^{M_{2}} \ln \left\{ \frac{\frac{1}{\sqrt{2\pi\sigma_{2}^{2}}} \exp\left[-\left(r_{j}-\mu_{2,j}\right)^{2}/2\sigma_{2}^{2}\right]}{\frac{1}{2} \left[1-\operatorname{erf}\left(\frac{P_{\operatorname{dec}}-\mu_{2,j}}{\sqrt{2\sigma_{2}^{2}}}\right)\right]}\right\}$$
(4)

where  $\mu_{2,j} \triangleq \hat{A}_1 + 10\hat{B}_1 \log_{10}(d_c/d_0) + 10B_2 \log_{10}(d_j/d_c)$ and for simplicity the data are sorted so that the first  $M_2$ elements have  $d_j > d_c$ . Finally, we obtain a complete set of calibrated parameters  $\hat{\theta} = [\hat{A}_1, \hat{B}_1, \hat{B}_2, \hat{\sigma}_1^2, \hat{\sigma}_2^2]$ .

**Example III:** Gaussian process regression (GPR) model. In the following, we adopt GPR to model RSS. Similar work can be found for instance in [8]. The motivation is that for different geographical positions but with the same Euclidean distance to a node, the corresponding radio channel conditions (line-of-sight or non-line-of-sight, richness of the multi-paths, strength of the reflections, and so on) can be different. We represent the underlying RSS as a real-valued Gaussian process,  $\tilde{r}(\mathbf{p})$ , nonlinearly in terms of 2-D or 3-D geographical position. By ignoring the subscript *j*, we give a function view of the underlying RSS as follows:

$$\tilde{r}(\mathbf{p}) = A + 10B \log_{10} \left( \frac{||\mathbf{p} - \mathbf{p}_{\mathrm{r}}||}{d_0} \right) + e(\mathbf{p})$$

where A, B follow the same meanings as given in the first example. Similarly, the error term,  $e(\mathbf{p})$ , due to the large-scale shadowing effect follows a zero-mean Gaussian distribution

$$e(\mathbf{p}) \sim \mathcal{N}(0, \sigma_s^2)$$

However, in contrast to the independence assumption made in the previous examples, the measurement errors (due to the shadowing effect) observed at two positions, say  $\mathbf{p}$  and  $\mathbf{p}'$ , relative to the same reference network node are assumed to correlate in space according to the well-established Gudmundson's model [9], namely

$$\mathbb{E}\left[e(\mathbf{p})e(\mathbf{p}')\right] = \sigma_s^2 \cdot \exp\left[\frac{-||\mathbf{p} - \mathbf{p}'||}{l_c}\right]$$

with  $l_c$  being the correlation distance [6].

The above nonlinear GPR model of the underlying RSS is completely specified by its mean function and covariance function, namely

$$\tilde{r}(\mathbf{p}) \sim \mathcal{GP}(m(\mathbf{p}), k(\mathbf{p}, \mathbf{p}'))$$

where we followed the notation of a Gaussian process  $\mathcal{GP}(\cdot, \cdot)$  defined in [6, Section 2.2] and

$$m(\mathbf{p}) \triangleq \mathbb{E}\left[\tilde{r}(\mathbf{p})\right] = A + 10B \log_{10}\left(\frac{||\mathbf{p} - \mathbf{p}_{\mathbf{r}}||}{d_0}\right),$$

 $k(\mathbf{p}, \mathbf{p}') \triangleq \mathbb{E}\left[ (\tilde{r}(\mathbf{p}) - m(\mathbf{p}))(\tilde{r}(\mathbf{p}') - m(\mathbf{p}')) \right] = \mathbb{E}\left[ e(\mathbf{p})e(\mathbf{p}') \right].$ 

Assume we have a training/calibration data set

$$\mathcal{D} = \{(\mathbf{p}_j, r(\mathbf{p}_j)) | j = 1, 2, \dots, M\}$$

collected (can be sparsely) at different calibration locations,  $\mathbf{p}_j$ , in the offline phase. To be realistic, we assume that the actually observed RSS,  $r(\mathbf{p}_j)$ , is of the form

$$r(\mathbf{p}_j) = \tilde{r}(\mathbf{p}_j) + n_j, \quad j = 1, 2, \dots, M.$$

The noise terms  $n_j$ , j = 1, 2, ..., M are assumed to be i.i.d. Gaussian with zero mean and variance  $\sigma_n^2$ , accounting for the joint influence of the interference from other devices, signal absorption from human bodies, as well as the (unsuccessfully removed) small-scale fading. We write the likelihood function of the observed RSS measurements as

$$p(\mathbf{r}(\mathbf{P}); \boldsymbol{\theta}) \sim \mathcal{N}(\mathbf{m}(\mathbf{P}), \mathbf{C}(\mathbf{P}, \mathbf{P}))$$
 (5)

where the following notations are newly introduced:

$$\boldsymbol{\theta} = [A, B, \sigma_s^2, \sigma_n^2, l_c]^T,$$
  

$$\mathbf{P} \triangleq [\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_M],$$
  

$$\mathbf{r}(\mathbf{P}) \triangleq [r(\mathbf{p}_1), r(\mathbf{p}_2), \dots, r(\mathbf{p}_M)]^T,$$
  

$$\mathbf{m}(\mathbf{P}) \triangleq [m(\mathbf{p}_1), m(\mathbf{p}_2), \dots, m(\mathbf{p}_M)]^T,$$
  

$$\mathbf{K}(\mathbf{P}, \mathbf{P}) \triangleq \begin{bmatrix} k(\mathbf{p}_1, \mathbf{p}_1) & k(\mathbf{p}_1, \mathbf{p}_2) & \dots & k(\mathbf{p}_1, \mathbf{p}_M) \\ k(\mathbf{p}_2, \mathbf{p}_1) & k(\mathbf{p}_2, \mathbf{p}_2) & \dots & k(\mathbf{p}_2, \mathbf{p}_M) \\ \vdots & \vdots & \ddots & \vdots \\ k(\mathbf{p}_M, \mathbf{p}_1) & k(\mathbf{p}_M, \mathbf{p}_2) & \dots & k(\mathbf{p}_M, \mathbf{p}_M) \end{bmatrix},$$
  

$$\mathbf{C}(\mathbf{P}, \mathbf{P}) \triangleq \mathbf{K}(\mathbf{P}, \mathbf{P}) + \sigma_n^2 \cdot \mathbf{I}_M.$$

The parameters included in  $\theta$  are usually unknown and need to be calibrated. A parameter estimate,  $\hat{\theta}$ , can be found through maximizing the likelihood function (5) numerically using for instance the limited-memory BFGS (LBFGS) quasi-Newton method or the conjugate gradient (CG) method [10]. The obtained  $\hat{\theta}$  is treated then as the underlying parameters.

In order to give a training data dependent observed RSS model that takes into account all error sources, we compute

according to [6] the Gaussian posterior probability of an observed RSS value at any position  $\mathbf{p}_*$  by

$$p(r(\mathbf{p}_*)|\mathcal{D};\boldsymbol{\theta}) \sim \mathcal{N}\left(\overline{\mu}(\mathbf{p}_*), \overline{k}(\mathbf{p}_*)\right)$$
 (6)

where

$$\overline{\mu}(\mathbf{p}_*) = \mathbf{k}^T(\mathbf{p}_*, \mathbf{P})\mathbf{C}^{-1}(\mathbf{P}, \mathbf{P})(\mathbf{y}(\mathbf{P}) - \mathbf{m}(\mathbf{P})) + m(\mathbf{p}_*)$$
(7)

and

$$\overline{k}(\mathbf{p}_*) = \sigma_n^2 + \sigma_s^2 - \mathbf{k}^T(\mathbf{p}_*, \mathbf{P})\mathbf{C}^{-1}(\mathbf{P}, \mathbf{P})\mathbf{k}(\mathbf{p}_*, \mathbf{P}).$$
 (8)

Note that in (7) and (8),

$$\mathbf{k}(\mathbf{p}_*,\mathbf{P}) \triangleq [k(\mathbf{p}_*,\mathbf{p}_1),k(\mathbf{p}_*,\mathbf{p}_2),\ldots,k(\mathbf{p}_*,\mathbf{p}_M)]^T.$$

Although the GPR model is more advanced to use, it still ignores the spatial correlation in the measurements collected for different reference network nodes. More advanced RSS model that is able to take into account this aspect will be our future work.

#### C. Evaluation Set of Sample Positions

Apart from the deployment information and the RSS model, we need also an evaluation set of sample positions for which localization accuracy will be evaluated. An evaluation set can be selected for instance to contain uniform grids or a plurality of trajectories that cover the area where positioning is of interest.

# III. RSS THRESHOLD OPTIMIZATION

This section aims to give a general procedure for RSS threshold optimization, which may lead to enhanced performance of some low-cost proximity based positioning algorithms.

Before performing the threshold optimization, the prerequisites, cf. Step 1 in Fig. 2, are first obtained. Concretely,

- Obtain the sensor deployment information, including for instance the floor plan, reference network node positions, transmit power, etc.
- Obtain an RSS model, possibly calibrated, for each reference network node *i*, *i* = 1, 2, ..., *N*, characterized by one of the model parameter vectors:
  - 1)  $\hat{\theta}_i = [\hat{A}_i, \hat{B}_i, \hat{\sigma}_i^2]^T$  in the linear log-distance model, cf. **Example I** given in Section II-B.
  - 2)  $\hat{\theta}_i = [\hat{A}_{1,i}, \hat{B}_{1,i}, \bar{B}_{2,i}, \hat{\sigma}_{1,i}^2, \hat{\sigma}_{2,i}^2]^T$  in the dualslope piece-wise linear model, cf. **Example II** given in Section II-B.
  - 3)  $\hat{\theta}_i = [\hat{A}_i, \hat{B}_i, \hat{\sigma}_{s,i}^2, \hat{\sigma}_{n,i}^2, \hat{l}_{c,i}]$  in the nonlinear GPR model, cf. **Example III** given in Section II-B.

Herein, we use the subscript i in the model parameters to indicate their attribution to node i.

• Obtain an evaluation set,  $\mathcal{X}^*$ , which contains known sample positions  $\mathbf{p}_i^* \triangleq [x_i^*, y_i^*, z_i^*]^T$ ,  $i = 1, 2, \ldots, |\mathcal{X}^*|$  that might be considered be of varied importance quantified by the weighting factors  $w_i^*$ ,  $i = 1, 2, \ldots, |\mathcal{X}^*|$ .

The key parts of step 2 in Fig. 2 are as follows:

- 1) For every candidate RSS threshold  $P_{th}$  (unit in dBm) in a finite interval  $[P_{th}^{\min}, P_{th}^{\max}]$ , sequentially do:
  - a) For every sample position p<sub>i</sub><sup>\*</sup> ∈ X<sup>\*</sup>, evaluate some localization accuracy metric f(P<sub>th</sub>) based on the proximity information and the RSS model parameters calibrated in Step 3.
    b) Taking the average by

$$\overline{f}(P_{th}) = \sum_{i=1}^{|\mathcal{X}^*|} w_i^* \cdot f(\mathbf{p}_i^*, P_{th}) \qquad (9)$$

where  $\overline{f}(P_{th})$  is a short-hand notation of  $\overline{f}(\text{RSS model}, \text{deployment}, P_{th})$ , and  $f(\mathbf{p}_i^*, P_{th})$  is a short-hand notation of  $f(\text{evaluation set}, \text{RSS model}, \text{deployment}, P_{th})$ .

2) Solve 
$$P_{th}^{opt} = \underset{P_{th}}{\text{optimize}} f(P_{th}).$$

In step 3 of Fig. 2, the device(s) are configured with the optimized reporting threshold  $P_{th}^{opt}$  to generate proximity reports. These proximity reports can be used then for positioning purposes. The RSS threshold optimization is supported by an architecture, and we characterize the architecture by describing the logical entities performing different steps in the procedure. Furthermore, different node candidates are discussed for the logical entities. Figure 3 provides a signaling chart, where most of the key steps are performed by a *computation entity*. It can also be so that the offline processing is performed in a configuration entity, and the online processing in a separate *fusion entity*. Another possibility is that the calibration efforts are performed in a dedicated *calibration entity*. These logical entities can be implemented separately or jointly in a mobile device, a reference network node, or some other network node. The necessary communication in the two phases between a device and logical entity may be via a link with a reference network node, via some other communication link, or internally in the device.

Some remarks are given below to conclude this section:

- Finite *L* levels are assumed in the search interval  $[P_{th}^{\min}, P_{th}^{\max}]$ .  $P_{th}^{\min}$  and  $P_{th}^{\max}$  can be predefined, for instance, according to the sensor reading limits or expert knowledge, if any. Alternatively, we could set the lowest/highest RSS threshold candidate as the minimum/maximum RSS value of the whole batch of data (used in the offline phase for RSS model calibration).
- In the sequel, we consider a representative localization accuracy metric, namely the localization root-mean-square-error (RMSE). This metric is often used to evaluate the performance of a localization algorithm. However, in this paper we are not focusing on any specific algorithm and compute the best achievable localization RMSE with the aid of the Cramér-Rao bound (CRB) analysis. More precisely, we have

$$f(\mathbf{p}_i^*, P_{th}) \triangleq \sqrt{\operatorname{tr}(\operatorname{FIM}^{-1}(\mathbf{p}_i^*, P_{th}))}$$
(10)

where the fisher information matrix (FIM) of  $\mathbf{p}_{i}^{*}$ , FIM $(\mathbf{p}_{i}^{*}, P_{th})$ , is computed in a snapshot manner and under the assumption that the sample positions are



Fig. 3. Signaling chart illustrating the proposed RSS threshold optimization procedure. The steps marked with dashed lines and in red color are optional.

deterministic. Extension to Barankin bound [11] or Bayesian type CRB [12] will be considered in our future work. But we believe that the results shouldn't vary much.<sup>1</sup> We also stress that some other localization metrics, such as percentile and localization outage probability, could also be used.

- An optimal RSS threshold can also be determined to optimize some localization metric of a particular localization or tracking algorithm. A concrete example has been given in [4], where  $P_{th}^{opt} = -82$  dBm has been found to minimize the Monte-Carlo RMSE over different trajectories.
- Only a single RSS threshold has been trained so far. Extension to multiple thresholds for different reference network nodes is straightforward but comes at an exponentially increased computational complexity. For instance, when we need to train an RSS threshold  $P_{th,i}$ for each reference network node individually, all  $L^N$ combinations of  $[P_{th,1}, P_{th,2}, \ldots, P_{th,N}]$  need to be considered.

# IV. EXPERIMENTAL VALIDATION

In Section III, we have shown how to optimize a single RSS threshold for enhanced localization performance. In what follows, we aim to validate the idea experimentally using a batch of real RSS measurements collected in an indoor bluetooth low-energy (BLE) network. The most attractive features of the BLE network as compared to other wireless networks

TABLE I.	3-D POSITION OF THE REFERENCE NETWORK NODES (BLE
BEACONS)	AND THE AMOUNT OF RSS MEASUREMENTS COLLECTED BY
EACH	NODE DURING THE OFFLINE MEASUREMENT CAMPAIGN.

	D III ( )	
Node ID	Position (x-,y-,z-) m	# Measurements
# 1	(2.27, 24.26, 2.60)	638 samples
# 2	(16.47, 20.34, 2.35)	1226 samples
# 3	(14.83, 11.48, 0.71)	1217 samples
# 4	(30.89, 20.36, 2.35)	1928 samples
# 5	(29.13, 14.90, 2.54)	2197 samples
# 6	(49.00, 20.34, 2.35)	203 samples
# 7	(37.74, 4.10, 2.60)	1214 samples
# 8	(18.82, 4.00, 2.60)	642 samples
# 9	(45.51, 13.30, 2.25)	1442 samples
# 10	(29.66, 4.10, 2.60)	1437 samples

lie in the low power consumption and efficient monitoring procedures in devices. As trade-off, more BLE beacons need to be deployed due to the shorter communication range compared to techniques based on higher transmission powers.

## A. Sensor Deployment and Measurement Campaign

We consider a typical office environment at Ericsson, Linköping, Sweden. In total N = 10 BLE beacons are placed rather uniformly in the area. The floor plan as well as the known beacon positions are shown in two-dimensional (2-D) space in Fig. 4, wherein a local coordinate system is used. The BLE beacons serve as transmitters and broadcast beacon information regularly. The transmit power is  $P_T = -58$  dBm. A moderate scale measurement campaign was conducted during normal work hours. Throughout the measurement campaign, the mobile device (equipped with BLE chipset) receives data packages from the BLE beacons and measures the RSS. A total number of M = 12144 RSS measurements were collected along 52 predefined tracks. During the measurement campaign, the mobile device was held approximately 1.3 meter above the ground. For clarity, we depict the 52 tracks all together in Fig. 4 and use different colors to indicate the quality of the observed RSS. Besides, Table I gives the 3-D positions of the BLE beacons as well as the total number of RSS measurements collected per beacon. The obtained RSS measurements were then uploaded to the computation entity (in this case a laptop) via Wi-Fi for RSS model fitting and threshold optimization. In the above training phase, we assumed full knowledge about the position of all BLE beacons and tracks.

## B. Fitted RSS Models

In Section II-B, we enumerated three different RSS models. In the first two linear models, we take into account the truncation effect by setting  $P_{dec} = -99$  dBm. Next, we perform RSS model calibration repeatedly for each BLE beacon using the real RSS measurements. Due to space constraint, we only show some representative results. Specifically, we show the calibrated linear log-distance model, dual-mode piece-wise linear model, and nonlinear GPR model all for beacon #4in Fig. 5 and Fig. 6, respectively. It is straightforward to see that the piece-wise log-distance mode can better represent the data as compared to the simplest log-distance model, but they are only able to represent the predicted mean RSS value as a simple function of the Euclidean distance between the mobile device and the reference network node. In contrast, the GPR model is able to take into account some additional information hidden in the training data about the deployment area. As was

<sup>&</sup>lt;sup>1</sup>When computing the Barankin bound, the sample positions in the evaluation set are treated as deterministic and unknown. While for computing Bayesian type CRB, such as the posterior-CRB, the evaluation set should contain several trajectories that follow certain motion model.



Fig. 4. Illustration of the deployment area and the calibration set of sample positions and RSS measurements (with the strength indicated by different colors). The BLE beacons are indexed and marked by red \*.



Fig. 5. Scatter plot of the collected RSS measurements (marked by red circles) versus the calibrated log-distance model (blue line) and piece-wise linear log-distance model (black lines) for the 4th BLE beacon. The calibrated parameters for the log-distance model are  $\hat{A}_4 = -60.0145$  dB,  $\hat{B}_4 = -2.1156$  dB, and  $\hat{\sigma}_4 = 7.45$  dB; while the calibrated parameters for the piece-wise log-distance model are  $\hat{A}_{1,4} = -66.81$  dB,  $\hat{B}_{1,4} = -0.87$  dB,  $\hat{B}_{2,4} = -3.50$  dB,  $\hat{\sigma}_{1,4} = 8.30$  dB, and  $\hat{\sigma}_{2,4} = 7.27$  dB, and the corresponding critical distance is set to 0.8 meter (in log-scale) for this beacon.

demonstrated in Fig. 6(a), concrete walls should have more adverse impact on the mean RSS value than glass walls.

#### C. RSS Threshold Optimization

In order to perform the RSS threshold optimization, we first generate a set  $\mathcal{X}^*$  with 3038 sample positions  $\mathbf{p}_i^*$  spread uniformly over the area, as shown in Fig. 7. The weighting factors are set equally as  $w_i^* = 1/|\mathcal{X}^*|$  for all sample positions in  $\mathcal{X}^*$ . We note that this evaluation set  $\mathcal{X}^*$  shouldn't be confused with the calibration data set used for RSS model calibration. As the localization accuracy metric, we adopt the best achievable RMSE as defined in (10). Herein, we assume that the z-component of all sample positions is fixed to 1.3 meter and known a priori. In other words, we only concern about position estimation in x- and y-directions. As a consequence,  $f(\mathbf{p}_i^*, P_{th})$  boils down to  $f([x_i^*, y_i^*], P_{th})$ .

We repeat the steps for RSS optimization as given in Section III for the three different RSS models. In Fig. 8, we depict the overall best achievable localization RMSE as a function of the RSS threshold  $P_{th}$ , which ranges from  $P_{th}^{\min} = -99$ 



Fig. 6. Illustration of the training data set (marked by black +) and the calibrated GPR model with parameters  $\hat{A}_4 = -69.85$  dB,  $\hat{B}_4 = -1.41$  dB,  $\hat{\sigma}_{s,4} = 3.75$  dB,  $\hat{\sigma}_{n,4} = 2.78$  dB,  $\hat{l}_{c,4} = 5.55$  meter, for the 4th BLE beacon: (a) depicts the posterior mean and (b) depicts the posterior variance of (6).

dBm to  $P_{th}^{\text{max}} = -70$  dBm with an increment 1 dBm. It is not surprising to have convex profiles of  $\overline{\text{RMSE}}_{\text{pos}}(P_{th})$ with respect to  $P_{th}$  in all cases. The reason is that too large or too small threshold gives very little information about an unknown location. In order to better explain this, let us reconsider the example shown in Fig. 2. Therein, when  $P_{th}^{opt}$ is set to  $-\infty$  or equivalently the coverage area is infinitely large, the receiver will receive [1, 1, 1] everywhere; Similarly, when  $P_{th}^{\text{opt}}$  is set to  $+\infty$  or equivalently the coverage area is null, the receiver will always receive [0,0,0]. Despite the use of different RSS models, the final RSS thresholds remain similar. In addition, we illustrate in Fig. 9 the best achievable localization RMSE at each sample position of the evaluation set,  $\mathcal{X}^*$ , but only for the conventional linear log-distance RSS model and  $P_{th} = P_{th}^{\text{opt}} = -82$  dBm. Therein, we can clearly see that the localization performance is quite good in the center of this floor, where many beacons can be received, but can be extremely bad in perimeter areas, where few beacons are received. This can be seen from Fig. 10, where the average number of received BLE beacons reaches the maximum in the center of this floor and decreases when moving to the boundary. Appendix B gives detailed derivations for computing this quantity based on the linear log-distance RSS model. Similar results can be observed for the other two RSS models and are omitted here due to space constraint. Lastly, we note that in Fig. 9 it is obvious to see a few narrow stripe-like areas where the best achievable RMSE is relatively big. The



Fig. 7. Illustration of an evaluation set of uniformly distributed sample positions (marked by blue dots),  $\mathcal{X}^*$ .



Fig. 8. Overall best achievable localization RMSE versus threshold candidates for the linear log-distance model in subfigure-I, piece-wise linear log-distance model in subfigure-II, and Gaussian process regression model in subfigure III, respectively. The optimal RSS threshold is marked by red circle.

reason is that in these areas the mobile device and the most influencing BLE beacons are nearly co-linearly located, hence the geometric dilution of precision (GDOP) or simply the geometry for positioning is extremely poor.

#### D. Verification

Given the optimized RSS threshold, we next verify the performance improvement achieved by  $P_{th}^{\text{opt}}$  (in dBm) as compared to a too-low threshold, say  $P_{th} = -99$  dBm. Assuming that we are now in the online positioning phase and a mobile device moves along a line in the x-direction as shown in Fig. 11. Instead of plotting the position estimation uncertainty of any specific algorithm, we use the  $1\sigma$  uncertainty of both xdirection and y-direction extracted from the diagonal elements of the CRB. The resulting  $1\sigma$  uncertainty circle is centered at the true position, because the CRB is derived for the unbiased estimator. In Fig. 11, we plot the  $1\sigma$  circle at some specific positions on the trajectory along with the average number of connected BLE beacons evaluated at these positions. From the results shown in Fig. 11, we can clearly see the performance improvement. When we use  $P_{th} = -99$  dBm as the threshold for proximity based indoor localization, almost all the BLE beacons are within the RSS threshold but very little information is hidden in the proximity measurements simply because



Fig. 9. Illustration of the best achievable localization RMSE evaluated at each sample position of the evaluation set,  $\mathcal{X}^*$ .



Fig. 10. Illustration of the average number of communicating BLE beacons at each sample position of the evaluation set,  $X^*$ .

any point in the deployment area is a good candidate. When we decrease  $P_{th}$  from -99 dBm to  $P_{th}^{opt} = -82$  dBm, the uncertainty circle shrinks despite that the average number of BLE beacons within the RSS threshold decreases as well.

## E. Concluding Remarks

We have shown in the previous subsections that the proposed RSS threshold optimization procedure is capable of providing enhanced localization accuracy. But still the localization uncertainty can be quite huge in the areas where the mobile device receives very few ( $\leq 2$ ) BLE beacons within the RSS threshold. We believe that this problem can be well solved by installing more BLE beacons in the deployment area. We also believe that even more reasonable RSS model can be achieved by fully exploiting the deployment information. such as the height and orientation of the transmit antenna, mixed LOS/NLOS propagation condition, wall effect, etc. In our experiments, we merely took advantage of the geometry of the reference network nodes. Apart from these, optimizing the network nodes' positions to give the best geometric dilution of precision (GDOP) would help further improve the localization accuracy, however, at additional cost. As a summary, we strongly believe that proximity based indoor localization is able to provide not-too-coarse position estimate for a wide range of event-based applications.

#### V. CONCLUSION

In this paper, we have proposed a general RSS threshold optimization procedure for indoor positioning using wireless



Fig. 11. Online localization accuracy obtained for different RSS thresholds. In the figures, blue dots constitute the trajectory of the moving receiver. At some particular positions (painted in red),  $1\sigma$  uncertainty circle is also plotted in red. Besides, the average number of connected BLE beacons is given right above the red dots. (a) depicts the results obtained for  $P_{th}^{\rm opt} = -82$  dBm corresponding to the conventional linear log-distance model; and (b) depicts the results obtained for  $P_{th} = -99$  dBm.

networks. The importance of this work is to provide a fundamental baseline for converting a continuous RSS measurement to a binary proximity measurement for analyzing time series of binary proximity reports. Given the prior knowledge about the deployment information and the RSS model, a reasonable RSS threshold can be found via optimizing an adequate performance metric. As a concrete example, we have exemplified how to optimize the RSS threshold for three salient RSS models so that the best achievable localization RMSE of any unbiased position estimator is minimized. Moreover, we have conducted experimental validation of the proposed procedure in a live BLE network deployed at an office area at Ericsson. The results have shown largely enhanced localization performance when using the optimized RSS threshold, which underpins our statement that inappropriately selected RSS threshold will result in little information in the proximity measurements.

# APPENDIX A

In the following, the Fisher information matrix (FIM) for estimating an unknown deterministic position  $\mathbf{p} = [x, y, z]^T$ will be computed based on the proximity information converted from the RSS measurements, more precisely,

$$c_i = \begin{cases} 0, & r_i \le P_{th} \\ 1, & r_i > P_{th} \end{cases}$$

where  $c_i$  is introduced here to denote the proximity information obtained through comparing a threshold  $P_{th}$  with the instantaneous RSS value,  $r_i$ , measured at a receiver in communication with the *i*th reference network node.  $c_i$  being equal to '0 'indicates that the receiver is not connected with reference network node *i* or '1 'indicates a successful connection of them. Given the whole batch of RSS measurements collected after the offline phase, the FIM computation may be performed either in the receiver (e.g. a mobile device) or in the transmiter (e.g., one of the reference network nodes), but the computation remains the same as shown below. No matter where the computation will be performed, it is assumed that the total number of reference network nodes as well as their positions are known.

The FIM for estimating  $\mathbf{p}$  is defined as FIM( $\mathbf{p}$ ), which can be easily proven to be equal to

$$\operatorname{FIM}(\mathbf{p}) = \sum_{i=1}^{N} \operatorname{FIM}_{i}(\mathbf{p}), \tag{11}$$

owing to the independence assumption on the measurements collected from different reference network nodes. Hence, it is much simpler to work with

$$\operatorname{FIM}_{i}(\mathbf{p}) \triangleq \mathbb{E} \left[ \nabla_{\mathbf{p}} \ln \left( \operatorname{Pr}\{c_{i}; \mathbf{p}, P_{th}\} \right) \nabla_{\mathbf{p}}^{T} \ln \left( \operatorname{Pr}\{c_{i}; \mathbf{p}, P_{th}\} \right) \right]$$

where the expectation is taken with respect to a discrete-valued probability  $Pr\{c_i; \mathbf{p}, P_{th}\}$ . Next we show how to compute  $Pr\{c_i; \mathbf{p}, P_{th}\}$  for the three different RSS models given in Section II-B. The nice statistical property in common is the Gaussian distribution of a RSS measurement, which facilitates the subsequent derivations.

Linear log-distance model: Owing to the Gaussian nature of the RSS, namely,

$$r_i \sim \mathcal{N}(\hat{\mu}_i(\mathbf{p}), \hat{\sigma}_i^2),$$

where

$$\hat{\mu}_i(\mathbf{p}) \triangleq \hat{A}_i + 10\hat{B}_i \log_{10} \left(\frac{||\mathbf{p} - \mathbf{p}_{\mathbf{r}}||}{d_0}\right)$$

we have,

$$\Pr\{c_i; \mathbf{p}, P_{th}\} = \begin{cases} G\left(\frac{P_{th} - \hat{\mu}_i(\mathbf{p})}{\hat{\sigma}_i}\right), & c_i = 0\\ 1 - G\left(\frac{P_{th} - \hat{\mu}_i(\mathbf{p})}{\hat{\sigma}_i}\right), & c_i = 1 \end{cases},$$
(12)

where

$$G\left(\frac{t-\mu}{\sigma}\right) \triangleq \frac{1}{2} \left[1 + \operatorname{erf}\left(\frac{t-\mu}{\sqrt{2}\sigma}\right)\right].$$

It is easy to derive that

$$\operatorname{FIM}_{i}(\mathbf{p}) \triangleq \begin{bmatrix} f_{i,xx} & f_{i,xy} & f_{i,xz} \\ f_{i,yx} & f_{i,yy} & f_{i,yz} \\ f_{i,zx} & f_{i,zy} & f_{i,zz} \end{bmatrix}$$
(13)

where for any combination of  $m \in \{x, y, z\}$  and  $n \in \{x, y, z\}$ ,

$$f_{i,mn} = \mathbb{E}\left[\frac{\frac{\partial}{\partial m} \Pr\{c_i; \mathbf{p}, P_{th}\}}{\Pr\{c_i; \mathbf{p}, P_{th}\}} \cdot \frac{\frac{\partial}{\partial n} \Pr\{c_i; \mathbf{p}, P_{th}\}}{\Pr\{c_i; \mathbf{p}, P_{th}\}}\right]$$
$$= \sum_{c_i \in \{0,1\}} \frac{\frac{\partial}{\partial m} \Pr\{c_i; \mathbf{p}, P_{th}\} \cdot \frac{\partial}{\partial n} \Pr\{c_i; \mathbf{p}, P_{th}\}}{\Pr\{c_i; \mathbf{p}, P_{th}\}}$$
$$= \left(\frac{\partial G}{\partial m} \cdot \frac{\partial G}{\partial n}\right) \cdot \left(\frac{1}{G} + \frac{1}{1 - G}\right).$$
(14)

Note that G is a short-hand notation of  $G\left(\frac{P_{th}-\hat{\mu}_i(\mathbf{p})}{\hat{\sigma}_i}\right)$  in the above expressions. It is easy to verify that

$$\begin{split} \frac{\partial G}{\partial m} &\equiv \frac{\partial}{\partial m} \left\{ \frac{1}{2} \mathrm{erf}\left(\frac{P_{th} - \hat{\mu}_i(\mathbf{p})}{\sqrt{2}\hat{\sigma}_i}\right) \right\} \\ &= \frac{-10\hat{B}_i}{\ln(10)\sqrt{2\pi}\hat{\sigma}_i} \exp\left[-\frac{(P_{th} - \hat{\mu}_i(\mathbf{p}))^2}{2\hat{\sigma}_i^2}\right] \frac{m - m_{\mathrm{r},i}}{||\mathbf{p} - \mathbf{p}_{\mathrm{r},i}||^2}. \end{split}$$

Inserting the above result into (14) and performing some algebraic manipulations yields

$$f_{i,mn} = C_1 \cdot \frac{(m - m_{\mathbf{r},i})(n - n_{\mathbf{r},i})}{||\mathbf{p} - \mathbf{p}_{\mathbf{r},i}||^4},$$
(15)

where  $C_1$  is a variable in terms of the calibrated RSS model parameters and the RSS threshold, more precisely,

$$C_{1} = \frac{200\hat{B}_{i}^{2}}{\pi\hat{\sigma}_{i}^{2}\ln^{2}(10)} \cdot \frac{\exp\left[\frac{-(P_{th}-\hat{\mu}_{i}(\mathbf{p}))^{2}}{\hat{\sigma}_{i}^{2}}\right]}{1 - \operatorname{erf}^{2}\left(\frac{P_{th}-\hat{\mu}_{i}(\mathbf{p})}{\sqrt{2}\hat{\sigma}_{i}}\right)}$$

Combining the results in (15), (13), and (11), we can then easily obtain a final expression of  $FIM(\mathbf{p})$ .

Piece-wise linear log-distance model: In this case, we need to determine in the first place the underlying mode by comparing the distance  $||\mathbf{p} - \mathbf{p}_{\mathrm{r},i}||$  with the critical distance,  $d_{c,i}$ , defined in (3). When the mode number is one, we use  $\hat{A}_{i,1}, \hat{B}_{i,1}, \hat{\sigma}^2_{i,1}$  for computing FIM<sub>i</sub>(**p**) in light of the steps given in the previous case; Otherwise  $\hat{A}_{i,1}, \hat{B}_{i,2}, \hat{\sigma}^2_{i,2}$  will be used for the FIM computation.

Gaussian process regression model: Adopting the Gaussian posterior, cf. (6), as the observed RSS model yields

.

$$\Pr\{c_i; \mathbf{p}, P_{th}\} = \begin{cases} G\left(\frac{P_{th} - \overline{\mu}_i(\mathbf{p})}{\overline{k}_i(\mathbf{p})}\right), & c_i = 0\\ 1 - G\left(\frac{P_{th} - \overline{\mu}_i(\mathbf{p})}{\overline{k}_i(\mathbf{p})}\right), & c_i = 1 \end{cases}$$

where  $\overline{\mu}_i(\mathbf{p})$  and  $\overline{k}_i(\mathbf{p})$  are the predicted mean and variance as given in (7) and (8), respectively. Similarly, we have

$$\frac{\partial G}{\partial m} \equiv \frac{\partial}{\partial m} \left\{ \frac{1}{2} \operatorname{erf} \left( \frac{P_{th} - \overline{\mu}_i(\mathbf{p})}{\sqrt{2 \cdot \overline{k}_i(\mathbf{p})}} \right) \right\}$$
$$= C_2 \cdot \left[ \frac{\frac{\partial u_i(\mathbf{p})}{\partial m} v_i(\mathbf{p}) - \frac{\partial v_i(\mathbf{p})}{\partial m} u_i(\mathbf{p})}{v_i^2(\mathbf{p})} \right]$$

where

$$C_2 = \frac{1}{\sqrt{\pi}} \exp\left[-\frac{(P_{th} - \overline{\mu}_i(\mathbf{p}))^2}{2\overline{k}_i(\mathbf{p})}\right],$$
$$u_i(\mathbf{p}) = P_{th} - \overline{\mu}_i(\mathbf{p}), \quad v_i(\mathbf{p}) = \sqrt{2\overline{k}_i(\mathbf{p})}.$$

Due to space limitations, the explicit expressions of the first order derivatives as well as the final  $FIM(\mathbf{p}_*)$  are not shown.

## APPENDIX B

In the following, we show how to compute the average number of communicating BLE beacons for the linear logdistance RSS model, but the methodology also applies for the other two RSS models. The computation relies on (12). Before proceeding further, let us first define  $S_k$  as a set of all possible

combinations  $(c_1, c_2, \ldots, c_N)$  such that any of them fulfills the constraint  $\sum_{i=1}^{N} c_i = k$ . The cardinality of  $S_k$  can be easily computed by

$$\mathcal{S}_k| = \frac{N!}{k!(N-k)!}$$

The probability of having k connected beacons at any unknown location **p** is computed by

$$\Pr\{N_c = k\} = \sum_{\forall (c_1, \dots, c_N) \in \mathcal{S}_k} \left[ \prod_{i=1}^N \Pr\{c_i; \mathbf{p}, P_{th}\} \right].$$

The average number of connected beacons is finally computed to be

$$\bar{N}_c = \sum_{k=0}^N k \cdot \Pr\{N_c = k\}.$$

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