The Sequential Monte Carlo Multi-Bernoulli Filter for Extended Targets

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Abstract—The multi-Bernoulli (MB) filter for extended targets has been derived recently. However, the implementation of the extended target (ET) MB filter for nonlinear non-Gaussian models has not been presented. In this paper, we propose the sequential Monte Carlo (SMC) implementation of the ET-MB filter for estimating multiple extended targets by using the SMC technique and measurement partitioning algorithm. Simulation results demonstrate that the estimation performance of the SMC-ET-MB filter is superior to that of the standard SMC-MB filter for extended target measurement models.

Index Terms-Random finite set, multi-Bernoulli filter, extended targets, sequential Monte Carlo

I. INTRODUCTION

In multi-target tracking, the objective is to simultaneously estimate the number of targets and their states from a sequence of noisy measurements. Generally, each target is assumed to be a point which produces at most one measurement per scan. This assumption is valid when the target is far away from the sensor or the resolution of sensors is low. For the high resolution sensor, or the distance between the target and sensor is small, the sensor may be able to resolve individual features on the target. Each target may generate more than one measurement per scan, and the assumption of point targets is not appropriate. Hence extended target tracking arises. An extended target is defined as a target that potentially generates more than one measurement per scan [1].

Extended target tracking has attracted more attention in recent years. Among various extended target tracking approaches, we are interested in the random finite set (RFS) approach. The RFS approach provides another kind of methods for target tracking [2],[3],[4]. In the RFS approach, targets' states and measurements are treated as RFSs. With RFS models, Mahler has proposed the multi-target Bayes filter that propagates the multi-target posterior density recursively [2],[5]. Since the optimal multi-target Bayes filter is generally intractable, some approximated multi-target Bayes filters have been proposed, such as the probability hypothesis density (PHD) filter [5] which propagates the first order moment of the multi-target density, cardinality PHD (CPHD) filter [6] which propagates the first order moment and cardinality distribution of the multi-target density, and the multi-Bernoulli (MB) filter [2],[7] which propagates the parameters of an MB distribution to approximate the multi-target density. These filters have been implemented by using Gaussian mixture (GM) and sequential Monte Carlo (SMC) techniques [7],[8],[9],[10]. The PHD, CPHD, and MB filters are usually considered for estimating multiple point targets. Using a Poisson model of extended target measurements [11], Mahler has derived the PHD filter for extended targets [12]. The GM implementation of the ET-PHD filter has been presented in [1],[13]. A Gaussian inverse Wishart implementation of the ET-PHD filter has been proposed to jointly estimate targets' states and extensions. The CPHD filter for extended targets has been derived in [14], and the GM implementation of the ET-CPHD filter was presented in [15]. Subsequently, a Gamma Gaussian inverse Wishart implementation of the ET-CPHD filter was proposed to jointly estimate targets' states and extensions.

Recently, the MB filter for extended targets has been proposed in [17]. A GM implementation of the ET-MB filter for linear Gaussian models was proposed in [18]. However, the GM-ET-MB filter cannot be directly applied to nonlinear non-Gaussian models. With the assumption of point targets, the SMC-MB filter has been proposed for nonlinear models, which obtains higher estimation accuracy than SMC-PHD and SMC-CPHD filters, as the SMC-MB filter does not need the extra clustering method to extract target states. Hence, in this paper we propose the SMC implementation of ET-MB filter for extended targets. Using SMC techniques and the existing measurement partitioning algorithm, the SMC-ET-MB filter for estimating multiple extended targets is presented in this paper.

The rest of this paper is organized as follows. Section II reviews the ET-MB filter for extended targets. The SMC implementation of the ET-MB filter is presented in Section III. Numerical results for a simulation scenario are offered in Section IV. Finally, the conclusion is drawn in Section V.

II. THE ET-MB FILTER

The MB filter propagates parameters of an MB distribution to approximate the multi-target Bayes filter. It propagates a time varying number of target tracks in time. Initially, the MB filter was proposed for handing point targets. Recently, based on a Poisson model measurement likelihood proposed by Gilholm [11], Zhang [17] has derived the MB filter for extended targets. In this section, the ET-MB filter is reviewed, for more details see [17]. The ET-MB filter consists of prediction and update. Prediction: If at time k-1, the posterior multi-target density

$$\pi_{k-1} = \{ (r_{k-1}^{(i)}, p_{k-1}^{(i)}(x_{k-1})) \}_{i=1}^{M_{k-1}}$$
(1)

is given, where $r_{k-1}^{(i)}$ denotes the existing probability of the *i*th hypothesized track, $p_{k-1}^{(i)}(\cdot)$ denotes the probability density of the *i*th hypothesized track, and M_{k-1} is the total number of hypothesized tracks at time k-1, then the predicted multitarget density is described by

$$\pi_{k|k-1} = \{ (r_{P,k|k-1}^{(i)}, p_{P,k|k-1}^{(i)}(x_k)) \}_{i=1}^{M_{k-1}} \cup \\ \{ (r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)}(x_k)) \}_{i=1}^{M_{\Gamma,k}}$$
(2)

where

$$r_{P,k|k-1}^{(i)} = r_{k-1}^{(i)} \langle p_{k-1}^{(i)}(x_{k-1}), p_{S,k}(x_{k-1}) \rangle$$
(3)

$$p_{P,k|k-1}^{(i)}(x_k) = \frac{\left\langle f_{k|k-1}(x_k|x_{k-1}), p_{k-1}^{(i)}(x_{k-1})p_{S,k}(x_{k-1})\right\rangle}{\left\langle p_{k-1}^{(i)}(x_{k-1}), p_{S,k}(x_{k-1})\right\rangle}$$
(4)

 $\langle \cdot, \cdot \rangle$ denotes a inner product defined between two realvalued functions α and β by $\langle \alpha, \beta \rangle = \int \alpha(x)\beta(x)dx$ (or $\sum_{i=0}^{\infty} \alpha(i)\beta(i)$, when α and β are sequences), $f_{k|k-1}(\cdot|x_{k-1})$ denotes a single target transition density given state x_{k-1} at time k, $p_{S,k}(\cdot)$ is the survival probability of a target at time k, and $\{(r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)}(x_k))\}_{i=1}^{M_{\Gamma,k}}$ are the parameters of birth targets at time \vec{k} .

Update: If at time k, the predicted multi-target density is

$$\pi_{k|k-1} = \{ (r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)}(x_k)) \}_{i=1}^{M_{k|k-1}}$$
(5)

then the posterior multi-target density

$$\pi_{k} = \{ (r_{L,k}^{(i)}, p_{L,k}^{(i)}(x_{k})) \}_{i=1}^{M_{k|k-1}} \cup \\ \{ (r_{U,k}(W_{k}), p_{U,k}(x_{k}; W_{k})) \}_{W_{k} \in \wp \angle Z_{k}}$$
(6)

can be computed as follows.

The legacy components:

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \times \frac{1 - \left\langle p_{k|k-1}^{(i)}(x_k), (1 - e^{-\gamma_k(x_k)}) p_{D,k}(x_k) \right\rangle}{1 - r_{k|k-1}^{(i)} \left\langle p_{k|k-1}^{(i)}(x_k), (1 - e^{-\gamma_k(x_k)}) p_{D,k}(x_k) \right\rangle}$$
(7)

$$p_{L,k}^{(i)}(x_k) = p_{k|k-1}^{(i)}(x_k) \times \frac{1 - (1 - e^{-\gamma_k(x_k)})p_{D,k}(x_k)}{1 - \langle p_{k|k-1}^{(i)}(x_k), (1 - e^{-\gamma_k(x_k)})p_{D,k}(x_k) \rangle}$$
(8)

The measurement-updated components:

$$r_{U,k}(W_k) = \frac{\omega_{\wp}}{d_{W_k}} \times \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1 - r_{k|k-1}^{(i)}) \langle p_{k|k-1}^{(i)}(x_k), \psi_{W_k}(x_k) \rangle}{(1 - r_{k|k-1}^{(i)} \langle p_{k|k-1}^{(i)}(x_k), (1 - e^{-\gamma_k(x_k)}) p_{D,k}(x_k) \rangle)^2}$$
(9)

$$p_{U,k}(x_k; W_k) = \frac{\sum_{i=1}^{M_k|k-1} \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} p_{k|k-1}^{(i)}(x_k) \psi_{W_k}(x_k)}{\sum_{i=1}^{M_k|k-1} \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} \langle p_{k|k-1}^{(i)}(x_k), \psi_{W_k}(x_k) \rangle}$$
(10)

$$\omega_{\wp} = \frac{\prod_{W_k \in \wp} d_{W_k}}{\sum_{\wp \neq Z_k} \prod_{W_k \in \wp} d_{W_k}} \tag{11}$$

$$d_{W_{k}} = \delta_{|W_{k}|,1} +$$

$$\sum_{k|k-1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} \langle p_{k|k-1}^{(i)}(x_{k}), \psi_{W_{k}}(x_{k}) \rangle}{\frac{1}{1 - \frac{(i)}{1 - \frac{(i$$

$$1 - r_{k|k-1}^{(i)} \langle p_{k|k-1}^{(i)}(x_k), (1 - e^{-\gamma_k(x_k)}) p_{D,k}(x_k) \rangle$$
(13)

$$\psi_{W_k}(x_k) = p_{D,k}(x_k) e^{(-\gamma_k(x_k))} \gamma_k(x_k)^{|W_k|} \prod_{z_k \in W_k} \frac{g_k(z_k|x_k)}{\kappa_k(z_k)}$$
(14)

where the notation $\wp \angle Z_k$ denotes that \wp partitions the measurement set Z_k into nonempty cells W_k , the notation $W_k \in \wp$ denotes that the set W_k is a cell in the partition, $|W_k|$ is the number of elements in W_k , $\delta_{i,j}$ is the Kronecker delta function, Z_k is the measurement set at time k, $\gamma_k(\cdot)$ is the expected number of measurements generated from each target, $g_k(\cdot|x_k)$ is the single measurement likelihood given state x_k at time k, $p_{D,k}(x_k)$ is the detection probability given state x_k at time k, and $\kappa_k(\cdot)$ is the clutter intensity at time k.

III. THE SMC-ET-MB FILTER

The ET-MB filter has been derived in [17], but its implementations were not presented. Recently, a GM implementation of the ET-MB filter for linear Gaussian models was proposed in [18]. However, the GM-ET-MB filter does not directly apply to nonlinear system models. For nonlinear models, Vo [7] has proposed SMC-MB filter for multiple point targets estimation. The SMC-MB filter has higher estimation accuracy than the SMC-PHD and SMC-CPHD filters. Hence, in this section we propose the SMC implementation of the ET-MB filter for multiple extended targets estimation. Since the prediction of the SMC-ET-MB filter is exactly the same as that of the standard SMC-MB filter [7], the prediction is omitted. The update of the SMC-ET-MB filter is very easily derived. For clarity, the derivation of the update of the SMC-ET-MB filter is given in Appendix. The update of the SMC-ET-MB filter is described as follows.

A. Update

 d_V

i=1

If at time k-1, the predicted multi-target density is specified by $\pi_{k|k-1} = \{(r_{k|k-1}^{(i)}, p_{k|k-1}^{(i)}(x_k))\}_{i=1}^{M_{k|k-1}}$, and each $p_{k|k-1}^{(i)}(x_k)$ is composed of a set of weighted particles,

$$p_{k|k-1}^{(i)}(x_k) = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} \delta(x_k - x_{k|k-1}^{(i,j)})$$
(15)

then the updated multi-target density

$$\pi_{k} = \{ (r_{L,k}^{(i)}, p_{L,k}^{(i)}(x_{k})) \}_{i=1}^{M_{k|k-1}} \cup \\ \{ (r_{U,k}(W_{k}), p_{U,k}(x_{k}; W_{k})) \}_{W_{k} \in \wp \angle Z_{k}}$$
(16)

can be computed as follows.

The legacy components are

$$r_{L,k}^{(i)} = r_{k|k-1}^{(i)} \frac{1 - \varrho_{L,k}^{(i)}}{1 - r_{k|k-1}^{(i)} \varrho_{L,k}^{(i)}}$$
(17)

$$p_{L,k}^{(i)} = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{L,k}^{(i,j)} \delta(x_k - x_{k|k-1}^{(i,j)})$$
(18)

where

$$\varrho_{L,k}^{(i)} = \sum_{j=1}^{L_{k|k-1}^{(i)}} (1 - e^{-\gamma_k (x_{k|k-1}^{(i,j)})}) p_{D,k} (x_{k|k-1}^{(i,j)}) w_{k|k-1}^{(i,j)}$$
(19)

$$\tilde{w}_{L,k}^{(i,j)} = \frac{w_{k|k-1}^{(i,j)} (1 - (1 - e^{(-\gamma_k (x_{k|k-1}^{(i,j)}))}) p_{D,k} (x_{k|k-1}^{(i,j)}))}{1 - \varrho_{L,k}^{(i)}}$$
(20)

$$w_{L,k}^{(i,j)} = \frac{\tilde{w}_{L,k}^{(i,j)}}{\sum_{j=1}^{L_{k|k-1}^{(i)}} \tilde{w}_{L,k}^{(i,j)}}$$
(21)

The measurement-updated components are

$$r_{U,k}(W_k) = \frac{\omega_{\wp}}{d_{W_k}} \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}(1-r_{k|k-1}^{(i)})\varrho_{U,k}^{(i)}(W_k)}{(1-r_{k|k-1}^{(i)}\varrho_{L,k}^{(i)})^2}$$
(22)

$$p_{U,k}(x_k; W_k) = \sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{L_{k|k-1}^{(i,j)}} w_{U,k}^{(i,j)}(W_k) \delta(x_k - x_{k|k-1}^{(i,j)})$$
(23)

where

$$\omega_{\wp} = \frac{\prod_{W_k \in \wp} d_{W_k}}{\sum_{\wp \angle Z_k} \prod_{W_k \in \wp} d_{W_k}}$$
(24)

$$d_{W_k} = \delta_{|W_k|,1} + \sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)} \varrho_{U,k}^{(i)}(W_k)}{1 - r_{k|k-1}^{(i)} \varrho_{L,k}^{(i)}}$$
(25)

$$\varrho_{U,k}^{(i)}(W_k) = \sum_{j=1}^{L_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} p_{D,k}(x_{k|k-1}^{(i,j)}) e^{(-\gamma_k(x_{k|k-1}^{(i,j)}))} \times \gamma_k(x_{k|k-1}^{(i,j)})^{|W_k|} \prod_{z_k \in W_k} \frac{g_k(z_k | x_{k|k-1}^{(i,j)})}{\kappa_k(z_k)}$$
(26)

$$\tilde{w}_{U,k}^{(i,j)}(W_k) = \frac{\frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} w_{k|k-1}^{(i,j)} p_{D,k}(x_{k|k-1}^{(i,j)}) e^{(-\gamma_k(x_{k|k-1}^{(i,j)}))} \times}{\sum_{k=1}^{\gamma_k(x_{k|k-1}^{(i,j)})^{|W_k|}} \prod_{\substack{z_k \in W_k}} \frac{g_k(z_k|x_{k|k-1}^{(i,j)})}{\kappa_k(z_k)}}{\sum_{k=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} \varrho_{U,k}^{(i)}(W_k)}}$$
(27)

$$w_{U,k}^{(i,j)}(W_k) = \frac{\tilde{w}_{U,k}^{(i,j)}(W_k)}{\sum_{i=1}^{M_{k|k-1}} \sum_{j=1}^{L_{k|k-1}^{(i)}} \tilde{w}_{U,k}^{(i,j)}(W_k)}$$
(28)

B. Measurement Set Partitioning Algorithm

to the ET-PHD and ET-CPHD filters Similar [1],[13],[16],[19], the ET-MB filter also requires partitioning algorithms that partitions the measurement set into non-empty cells. As the number of possible partitions grows very large with the increase of the number of measurements. It is computationally infeasible to consider all the partitions, hence some approximated partitioning algorithms were developed [1]. The distance partitioning, K-means++ clustering, and distance partitioning with subpartitioning methods have been successfully applied to the GM-ET-PHD filter for estimating multiple extended targets [1], [13]. These partitioning algorithms have been shown to reduce the number of partitions with little impact on tracking performance [1],[13]. The distance partitioning method shows a better estimation performance than the K-means++ clustering method [1], hence the distance partitioning method is considered in this paper.

C. Resampling and Multi-Target State Extraction

To reduce the effect of degeneracy of the particles, the resampling is performed for each hypothesized track. The resampling and multi-target state extraction approaches are the same as that of the standard SMC-MB filter, for details see [7].

IV. SIMULATION RESULTS

A. Scenario

Consider a two dimensional scenario with a time-varying number of extended targets observed in clutter and missed detection. The surveillance region is $[0, \pi]$ rad $\times [0, 2000]$ m. A maximum of 6 extended targets appears in the scenario, and targets appear and terminate at a random time. The true target tracks are shown in Fig. 1.

The kinematic state $x_k = [p_{x,k}, \dot{p}_{x,k}, p_{y,k}, \dot{p}_{y,k}, \omega_k]^T$ consists of the position component $(p_{x,k}, p_{y,k})$, velocity component $(\dot{p}_{x,k}, \dot{p}_{y,k})$, and the turn rate ω_k . Assume that each target follows a coordinated turn (CT) dynamical model [20], i.e.

$$x_k = F_{k-1}(\omega_{k-1})x_{k-1} + w_{k-1} \tag{29}$$

the process noise w_{k-1} is a zero mean Gaussian noise with the known covariance Q_{k-1} , where

$$F(\omega_{k-1}) = \begin{bmatrix} 1 & \frac{\sin\omega_{k-1}T}{\omega_{k-1}} & 0 & -\frac{1-\cos\omega_{k-1}T}{\omega_{k-1}} & 0\\ 0 & \cos\omega_{k-1}T & 0 & -\sin\omega_{k-1}T & 0\\ 0 & \frac{1-\cos\omega_{k-1}T}{\omega_{k-1}} & 1 & \frac{\sin\omega_{k-1}T}{\omega_{k-1}} & 0\\ 0 & \sin\omega_{k-1}T & 0 & \cos\omega_{k-1}T & 0\\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$
(30)

$$Q_{k-1} = \begin{bmatrix} \frac{T^3}{3}\tilde{q}_w & \frac{T^2}{2}\tilde{q}_w & 0 & 0 & 0\\ \frac{T^2}{2}\tilde{q}_w & T\tilde{q}_w & 0 & 0 & 0\\ 0 & 0 & \frac{T^3}{3}\tilde{q}_w & \frac{T^2}{2}\tilde{q}_w & 0\\ 0 & 0 & \frac{T^2}{2}\tilde{q}_w & T\tilde{q}_w & 0\\ 0 & 0 & 0 & 0 & T\tilde{q}_\omega \end{bmatrix}$$
(31)

The sampling period is T = 1 s. $\tilde{q}_w = 25 \text{ m}^2/\text{s}^3$ and $\tilde{q}_\omega = 3.05 \times 10^{-4} \text{ rad}^2/\text{s}^3$ are related to process noise intensities.

The measurement model is a noisy bearing and range measurement, which is described by

$$z_k = \begin{bmatrix} \arctan\left(\frac{p_{x,k} - p_{\mathsf{Se},x}}{p_{y,k} - p_{\mathsf{Se},y}}\right) \\ \sqrt{(p_{x,k} - p_{\mathsf{Se},x})^2 + (p_{y,k} - p_{\mathsf{Se},y})^2} \end{bmatrix} + v_k, \quad (32)$$

where $(p_{\text{Se},x}, p_{\text{Se},y}) = (0,0)$ m is the position of the sensor, The measurement noise v_k follows a zero mean Gaussian distribution with the known covariance $R_k = \text{diag}\{\sigma_{\theta}, \sigma_r\}^2$, where $\sigma_{\theta} = 2\pi/180$ rad and $\sigma_r = 20$ m are the standard deviation of measurements for bearing and range portions, respectively.

Clutter is modeled as a Poisson RFS with intensity [8]

$$\kappa_k = \lambda_c V u(z_k),\tag{33}$$

 $_{i=1}^{4}$

where λ_c is the average clutter intensity, V is the volume of the surveillance area, and $u(\cdot)$ is the uniform density over the surveillance area. The clutter density is set to $\lambda_c = 1.6 \times 10^{-3} (\text{rad m})^{-1}$ (an average of 10 clutter measurements per scan).

The birth process is modeled as an MB-RFS, which is set to

$$\pi_{\Gamma,k} = \{ (r_{\Gamma,k}^{(i)}, p_{\Gamma,k}^{(i)}(x_k)) \}$$

where

$$\begin{split} r_{\Gamma,k}^{(1)} &= r_{\Gamma,k}^{(2)} = 0.02 \\ r_{\Gamma,k}^{(3)} &= r_{\Gamma,k}^{(4)} = 0.03 \\ p_{\Gamma,k}^{(i)}(x_k) &= \mathcal{N}(x_k; m_{\Gamma,k}^{(i)}, P_{\Gamma,k}^{(i)}) \end{split}$$

$$\begin{split} m_{\Gamma,k}^{(1)} &= [-1500, 0, 250, 0, 0]^T, \ m_{\Gamma,k}^{(2)} &= [-250, 0, 1000, 0, 0]^T, \\ m_{\Gamma,k}^{(3)} &= [250, 0, 750, 0, 0]^T, \ m_{\Gamma,k}^{(4)} &= [1000, 0, 1500, 0, 0]^T, \\ \text{and} \ P_{\Gamma,k}^{(i)} &= \text{diag}\{30, 30, 30, 30, 3\pi/180\}^2. \ \text{The survival probability is} \ p_{S,k} &= 0.99, \ \text{and the detection probability is} \ p_{D,k} &= 0.99. \ \text{The expected number of generated measurements is set} \\ \text{to} \ \gamma_k &= 10 \ \text{for each target.} \ \text{At each time step, a maximum} \\ \text{of} \ L_{\text{max}} &= 1000 \ \text{and minimum of} \ L_{\text{max}} &= 300 \ \text{particles are} \\ \text{imposed for each hypothesized track. Each hypothesized track} \end{split}$$



Fig. 1. True target tracks in $r\theta$ plane (the start/end positions for each track are denoted by \circ/\triangle , and the sensor is denoted by \Box)



Fig. 2. True tracks and measurements in xy positions versus time.

is pruned with a threshold of $T_r = 10^{-4}$, and the maximum of hypothesized tracks is set to 100.

B. The Metric

The optimal sub-pattern assignment (OSPA) metric [21] is considered for evaluating the filtering performance, since it can jointly capture difference in cardinality and individual elements between two finite sets. For two arbitrary finite sets $X = \{x_1, \dots, x_m\}$ and $Y = \{y_1, \dots, y_n\}$, the OSPA distance is defined as follows.

$$d_{p}^{(c)}(X,Y) := \begin{cases} \left(\frac{1}{n} \left(\min_{\pi \in \Pi_{n}} \sum_{i=1}^{m} d^{(c)}(x_{i}, y_{\pi_{(i)}})^{p} + c^{p}(n-m) \right) \right)^{\frac{1}{p}}, \\ m \leq n \\ \overline{d}_{p}^{(c)}(Y,X), \quad m > n \\ 0, \qquad m = n = 0 \end{cases}$$
(34)



Fig. 3. True tracks and estimates in xy positions versus time for the SMC-ET-MB filter.



Fig. 4. True tracks and estimates in xy positions versus time for the SMC-MB filter.

where $d^{(c)}(x, y) := \min(c, ||x - y||), || \cdot ||$ is the Euclidean norm, and Π_n denotes the set of permutations on $\{1, 2, \dots, n\}$. The order parameter p determines the sensitivity to outliers, and the cut-off parameter c determines the relative weighting of the penalties assigned to cardinality and localization errors, see [21] for more details. In this paper the parameters are set to p = 2 and c = 200.

C. Monte Carlo Runs

To verify the effectiveness of the SMC-ET-MB filter for extended targets, 200 Monte Carlo (MC) runs are performed. In each MC simulation, the target trajectories are the same, but measurements (stem from targets and clutter) are independently generated. Fig. 2 shows the true target tracks and measurements versus time. Figs. 3 and 4 plot the true tracks and estimates for the SMC-ET-MB and SMC-MB filters,



Fig. 5. The cardinality statistics versus time for (a) SMC-ET-MB filter, (b) SMC-MB filter.



Fig. 6. The OSPA distance.

respectively. In Figs. 3 and 4 it can be seen that the SMC-ET-MB filter can estimate the number and states of multiple extended targets correctly, and the SMC-ET-MB filter shows better estimation performance than the standard SMC-MB filter. Fig. 5shows the cardinality statistics for SMC-ET-MB and SMC-MB filters. From Fig. 5 we can see that the average cardinality of the SMC-ET-MB filter converges to the true value, while the standard SMC-MB filter shows a significant bias in the number of targets. Fig. 6 plots the OSPA distance for the SMC-ET-MB and SMC-MB filters. It is shown that SMC-ET-MB filter greatly outperforms the standard SMC-MB filter for extended target models.

V. CONCLUSION

This paper proposes the SMC implementation of the ET-MB filter for estimating extended targets. It is shown that the SMC-ET-MB filter is able to estimate multiple extended targets

correctly. In this paper, we adopt the existing partitioning algorithm and consider a simple scenario. The measurement partitioning algorithm still needs further studies for multiple extended targets estimation, specifically for complex scenarios, such as multiple crossed tracks, multiple parallel tracks and so on [1],[19]. The SMC-ET-MB filter can be extended to multiple model for maneuvering extended targets. The targets' extensions are not considered in the ET-MB filter, in future work the joint estimation of the targets' kinematic states and extensions will be considered. These have been implemented for ET-PHD and ET-CPHD filters [19],[16].

Appendix

Substituting (15) into $\langle p_{k|k-1}^{(i)}(x_k), (1 - e^{-\gamma_k(x_k)})p_{D,k}(x_k)\rangle$, then

$$\langle p_{k|k-1}^{(i)}(x_k), (1 - e^{-\gamma_k(x_k)}) p_{D,k}(x_k) \rangle$$

$$= \langle \sum_{j=1}^{L_k^{(i)}} w_{k|k-1}^{(i,j)} \delta(x_k - x_{k|k-1}^{(i,j)}), (1 - e^{-\gamma_k(x_k)}) p_{D,k}(x_k) \rangle$$

$$= \sum_{j=1}^{L_k^{(i)}} w_{k|k-1}^{(i,j)} \int \delta(x_k - x_{k|k-1}^{(i,j)}) (1 - e^{-\gamma_k(x_k)}) p_{D,k}(x_k) dx_k$$

$$= \sum_{j=1}^{L_k^{(i)}} w_{k|k-1}^{(i,j)} (1 - e^{-\gamma_k(x_{k|k-1}^{(i,j)})}) p_{D,k}(x_{k|k-1}^{(i,j)})$$
(35)

and (19) is obtained. Substituting (35) into (7), then (17) is obtained.

Substituting (15),(35) into (8), then

$$p_{L,k}^{(i)}(x_{k}) = \sum_{j=1}^{L_{k}^{(i)}} w_{k|k-1}^{(i,j)} \delta(x_{k} - x_{k|k-1}^{(i,j)}) \frac{1 - (1 - e^{-\gamma_{k}(x_{k})})p_{D,k}(x_{k})}{1 - \varrho_{L,k}^{(i)}} = \sum_{j=1}^{L_{k}^{(i)}} \frac{1 - (1 - e^{-\gamma_{k}(x_{k|k-1}^{(i,j)})})p_{D,k}(x_{k|k-1}^{(i,j)})}{1 - \varrho_{L,k}^{(i)}} w_{k|k-1}^{(i,j)} \times \delta(x_{k} - x_{k|k-1}^{(i,j)})$$
(36)

and (20),(21) are obtained.

(:)

Substituting (15) and (14) into $p_{k|k-1}^{(i)}(x_k)\psi_{W_k}(x_k)$, then

$$P_{k|k-1}^{(i)}(x_{k})\psi_{W_{k}}(x_{k})$$

$$= \sum_{j=1}^{L_{k}^{(i)}} w_{k|k-1}^{(i,j)} \delta(x_{k} - x_{k|k-1}^{(i,j)}) p_{D,k}(x_{k}) e^{(-\gamma_{k}(x_{k}))} \gamma_{k}(x_{k})^{|W_{k}|} \times \prod_{\substack{z_{k} \in W_{k}}} \frac{g_{k}(z_{k}|x_{k})}{\kappa_{k}(z_{k})}$$

$$= \sum_{j=1}^{L_{k}^{(i)}} w_{k|k-1}^{(i,j)} p_{D,k}(x_{k|k-1}^{(i,j)}) e^{(-\gamma_{k}(x_{k|k-1}^{(i,j)}))} \gamma_{k}(x_{k|k-1}^{(i,j)})^{|W_{k}|} \times \prod_{\substack{z_{k} \in W_{k}}} \frac{g_{k}(z_{k}|x_{k|k-1}^{(i,j)})}{\kappa_{k}(z_{k})} \delta(x_{k} - x_{k|k-1}^{(i,j)})$$
(37)

Integrate (37), then

$$\begin{aligned} \langle p_{k|k-1}^{(i)}(x_k), \psi_{W_k}(x_k) \rangle \\ &= \sum_{j=1}^{L_k^{(i)}} w_{k|k-1}^{(i,j)} p_{D,k}(x_{k|k-1}^{(i,j)}) e^{(-\gamma_k (x_{k|k-1}^{(i,j)}))} \gamma_k (x_{k|k-1}^{(i,j)})^{|W_k|} \times \\ &\prod_{z_k \in W_k} \frac{g_k (z_k | x_{k|k-1}^{(i,j)})}{\kappa_k (z_k)} \end{aligned}$$

$$(38)$$

and (26) is obtained. Substituting (35), (38) into (9),(12), and (22), (25) are obtained.

Substituting (37),(38) into (10), then

$$\frac{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} \sum_{j=1}^{L_{k}^{(i)}} w_{k|k-1}^{(i,j)} p_{D,k}(x_{k|k-1}^{(i,j)}) e^{(-\gamma_{k}(x_{k|k-1}^{(i,j)}))} \times \gamma_{k}(x_{k|k-1}^{(i,j)}) \frac{1}{\sum_{k \in W_{k}} \frac{g_{k}(z_{k}|x_{k|k-1}^{(i,j)})}{\kappa_{k}(z_{k})} \delta(x_{k} - x_{k|k-1}^{(i,j)})}{\sum_{i=1}^{M_{k|k-1}} \frac{r_{k|k-1}^{(i)}}{1 - r_{k|k-1}^{(i)}} \varrho_{U,k}^{(i)}(W_{k})}$$
(39)

and (27),(28) are obtained.

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