# Scheduling Multifunction Radar for Search and Tracking

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Abstract—Modern sensors that are increasingly flexible may be used to schedule various functions. Such sensors typically have competing demands on limited sensor resources such as the sensor timeline. Sensor management systems attempt to meet the overall goal of the sensor user by considering the user's priorities and the capabilities of the sensor.

This paper describes an algorithm for scheduling an electronically scanned array radar in order to perform the two functions of (i) searching for new targets and (ii) updating tracks on known targets. A previous formulation considered search theory approaches to determine radar dwell times. This formulation is extended from the search problem to address the tracking or update function for known targets. A key feature of the scheduling formulation permits the sensor user to specify the costs of not detecting targets within a surveillance volume and of not updating tracks on known targets. A single cost function is used to determine a myopic radar schedule that is optimised using an interior point method.

A simple example of a dynamically evolving target environment is used to illustrate the allocation of radar timeline for search and tracking functions.

Keywords: Sensor Management, Radar, Surveillance, Search, Tracking.

# I. INTRODUCTION

Recent technology advancements enable modern sensors to perform many tasks which support different functions. However, the operation of multifunction sensors requires the dynamic allocation of limited sensor resources to achieve overall mission objectives. The different functions may have competing demands on the sensor resources so automated techniques are needed to schedule resources based on the sensor capabilities and on the objectives [1], [2].

In this paper the problem of scheduling an electronically scanned phased array radar to jointly perform the two functions of (i) searching for previously undetected targets and (ii) updating tracks on previously detected targets is addressed. Previous work has typically considered the search and tracking functions as separate problems.

Mechanically scanned radars, where the radar is physically rotated with a fixed period, are quite constrained in the allocation of sensor resources. Track-While-Scan functions are commonly used to update tracks with radar measurements that arise when the mechanically scanned radar beam illuminates a given volume. In contrast, the agility of an electronically scanned phased array radar permits various radar parameters such as the pointing direction and dwell-time to be selected dynamically so that radar resources are optimised for achieving mission objectives. A multifunction radar scheduling system must account for a dynamic object environment, the characteristics of the radar and any constraints on the operation of the radar.

Various approaches have been applied to the problem of radar resource allocation for either the search function, the track function, or both functions [3], [4]. Many radar scheduling approaches focus on allocating time to tasks at the radar dwell level, including allowing for interleaving [5], pulse train length [6] and waveform considerations [7], [8]. Wintenby distinguished between this low level tasking and a higher level of abstraction, in which radar resources are allocated to search and track tasks, rather than specific waveforms [9]. Scheduling at the higher level of abstraction permits a more direct consideration of the relative importance of search and track tasks.

Methods of dividing radar resources between search and track functions include fixed, heuristic rules [10] or a fixed ratio of search to track effort [11], [12]. Methods to vary the allocation between functions tend to optimise the schedule according to some utility function, for example based on information theoretic measures [11], [13]–[15]. Sensor scheduling may be myopic (short term) so that the sensor allocations are decided for a single time step ahead. In contrast, non-myopic (long term) sensor scheduling plans multiple sensor allocations in the future [15], [18], [19].

The foundation for the sensor scheduling approach presented in this paper lies in search theory techniques for allocating sensor resources to detect targets [16]. Grid-based search scheduling techniques have been applied to the problem of non-myopic sensor management for allocating the resources of electronically scanned radar to search for multiple targets [19]. An approximately convex cost function is solved using an interior point method to optimise the time allocation of search actions within the surveillance region.

In this paper the scheduling framework of [19] is extended from considering the radar search function alone to include the target track function. A single cost function is developed that has two terms: (i) a term that addresses the cost of not detecting targets for the search problem, and (ii) a statedependent term that addresses the cost of error in the track state estimates. A system user may therefore explicitly define the relative importance of searching for targets that have not



Figure 1. Closed loop tracking and sensor scheduling

been detected previously compared to that of updating the set of tracks on previously detected targets.

Figure 1 illustrates the concept of closed loop tracking and sensor scheduling [1]. A sensor system yields a set of measurements  $z_t$  corresponding to time t. Then, for each of the detected targets, a tracker is used to determine a track state estimate that typically comprise a mean  $\hat{x}_{t|t}$  and a measure of uncertainty given by the covariance  $\hat{\mathbf{P}}_{t|t}$ , where the terminology t|t is used to denote that the estimate is obtained for time t given all of the measurement data obtained up to and including time t. An operator may then make decisions on the basis of the track estimates. In our framework there exists a feedback loop, whereby the track estimates are made available to a sensor scheduler, which also accepts user-specified costs associated with the functions of search and track update. Sensor resources such as radar dwell times for particular beams or actions  $au_{t,a}$  are allocated to yield sensor measurements for subsequent updates for the tracker. The overall goal is to allocate the sensor resources in order to provide an operator with a track picture that satisfies the operator's mission objectives for search and track update.

In section II the surveillance region is described together with background information on the sensor model that is used to develop the scheduling algorithm. Section III reiterates the myopic search cost function developed in [19] and then section IV presents a compatible track update cost function. The scheduling algorithm for combining the search and track update functions is described in section V. A simple example illustrating the implementation of a myopic scheduling algorithm is presented in section VI. Conclusions and future work are discussed in section VII.

## II. SURVEILLANCE REGION AND SENSOR

We consider an agile radar that performs sensing actions to either detect previously undetected targets or to update track estimates for known targets. From these actions, the radar generates measurements in 2D, range-azimuth space including estimates of the associated measurement error. The measurements are processed using a tracker to perform track state prediction and update [1]. For this paper we assume a single, stationary sensor however the method can be extended to multiple sensors whose motion is known.

# A. Grid

The radar surveillance region is quantised into a set of N non-overlapping cells  $S_1, \ldots, S_N$ . We assume these cells are fixed for the duration of the scenario. Note that the search optimisation strategy in [19] uses a subspace quantised into Gaussian cells, the exact quantisation strategy is relatively unimportant.

The use of a grid-based approach for the search problem can be justified by the diffuse undetected target density [19], but the distribution of track estimates is expected to have strong peaks over the same grid. In this case a grid-based method is likely to be highly inefficient - we discuss how the efficiency can be improved in section IV.

# B. Sensor timeline

The radar is assumed to be able to actively detect targets using a number of beams (or actions),  $a \in \mathcal{A} = \{1, \ldots, A\}$ , between which it must divide its time. Each beam is able to observe a sparse subset of the surveillance space with nonzero probability of detection. We denote the set of beams able to observe cell  $S_i$  by  $\mathcal{V}(S_i)$ .

We assume the radar timeline can be segmented into smaller, time indexed sections (or 'scans') of duration  $\Delta T = R_t$  so that time  $T_{t+1} = T_t + \Delta T$ . We aim to generate a schedule for the radar beams for each scan by allocating time to each beam  $\tau_{t,a}$ such that  $\sum_{a=1}^{A} \tau_{t,a} \leq R_t$ . The schedule of future scans can be recalculated after the detection actions for each scan have been performed. In this paper we only consider scheduling the next single scan (a myopic plan) however future work is to extend this to a finite planning horizon.

The scheduling method described in this paper can be extended to include multiple sensors by appending new beams to the set A, separating the total time constraints for each sensor.

# C. Probability of detection model

A model relating the time allocated to a beam during the scan at time t,  $\tau_{t,a}$ , to the probability of detecting a target is required. Any convex relation can be used, but for our scenario we use Albersheim's model [17] and assume the value is constant over each cell  $S_i$ . As in [19] we use a modified version of Albersheim's formula to allow for the eclipsing of targets during radar transmit.

$$P_d(\mathcal{S}_i, a, \tau_{t,a}) = \frac{(1-d) + (1-\gamma)e^{b(\mathcal{S}_i, a, \tau_{t,a})}}{1 + e^{b(\mathcal{S}_i, a, \tau_{t,a})}}.$$
 (1)

Where  $\gamma < 1$  is the probability of eclipsing and constant  $d = 1 + (1 - \gamma)e^{-c/(0.12c+1.7)}$  is used to fix the probability of detection to zero when no resources are allocated. Constant  $c = \ln(0.62/P_{fa})$  for a given false alarm rate. Factor  $b(S_i, a, \tau) = (\omega_{S_i,a}\tau - c)/(0.12c + 1.7)$  depends not only on the dwell time,  $\tau$ , but also on the range of cell  $S_i$  and the detection specifications of beam a. The sensor detection

performance may vary from beam to beam, and with time but is assumed to be constant for the duration of each scan. For beam a it is assumed that there is a specified probability of detection  $\bar{P}_d^a$  at a specified range  $\bar{r}^a$  given a specified dwell time  $\bar{\tau}^a$ , so that

$$\omega_{\mathcal{S}_{i},a} = \frac{1}{\bar{\tau}^{a}} \left(\frac{\bar{r}^{a}}{r(\mathcal{S}_{i})}\right)^{4} \times \left[\left(\frac{1}{0.12c+1.7}\right) \ln\left(\frac{1-\bar{P}_{d}^{a}-d}{\bar{P}_{d}^{a}-1+\gamma}\right) + c\right].$$
 (2)

From equations 1 and 2 we find the probability of miss to be  $P_m(S_i, a, \tau_{t,a}) = 1 - P_d(S_i, a, \tau_{t,a}) = \exp [\alpha(S_i, a, \tau_{t,a})]$ where

$$\alpha(\mathcal{S}_i, a, \tau_{t,a}) = \ln\left(\frac{d + \gamma e^{b(\mathcal{S}_i, a, \tau_{t,a})}}{1 + e^{b(\mathcal{S}_i, a, \tau_{t,a})}}\right).$$
 (3)

In the scheduling problem we will need a log-convex probability of miss but, as discussed in [19], the function  $\alpha(S_i, a, \tau_{t,a})$  is neither convex nor concave. We follow the example of [19] and 'convexify' this function by setting the second derivative to be zero where it would otherwise be negative - allowing the use of a convex optimisation solver later. Both  $\alpha$  and its first derivative  $\alpha'$  are unaltered but we set the second derivative to  $\tilde{\alpha}'' \equiv \max\{\alpha''(S_i, a, \tau_{t,a}), 0\}$ . This is similar to sequential convex programming [21], as we are forcing the Hessian to be positive semi-definite at each approximation to the solution but not explicitly defining a trusted region. This meets the convexity requirements of Newton's Method in section V while using a realistic radar detection model.

## III. TARGET SURVEILLANCE

In [19] a convex optimisation method to schedule agile radar beams for the search problem is presented. There is an unknown number of undetected targets within the surveillance region and we wish to divide array time for some defined scan between search beams to minimise the expected number of undetected targets after completion of the scan.

Following [11], the expected number of undetected targets at a given time t is assumed to follow a Poisson distribution with parameter  $\lambda$  that may vary in time and space. We denote the expected number of undetected targets in cell  $S_i$  by  $\lambda(S_i)$ , which is assumed to be constant for any cell but may vary between cells to approximate a Poisson distribution that is nonhomogeneous over the surveillance region. At time t we denote the predicted value of  $\lambda(S_i)$  as  $\lambda_{t|t-1}(S_i)$  and the filtered value (once measurements have been incorporated) as  $\lambda_{t|t}(S_i)$ .

In each time interval, say between time t - 1 and time t, the predicted undetected target density in a cell  $S_i$  may be computed as the sum of

- the number of new targets that arrive in cell  $S_i$ , and
- the number of targets that move from other cells or that simply remain in cell  $S_i$ ,

so that we have:

$$\lambda_{t|t-1}(\mathcal{S}_i) = \lambda^A(\mathcal{S}_i)\Delta T + \sum_{j=1}^N \Phi_{\Delta T}(\mathcal{S}_j, \mathcal{S}_i)\lambda_{t-1|t-1}(\mathcal{S}_j).$$
(4)

In the above  $\lambda^A(S_i)$  is the arrival rate for new targets in  $S_i$ and  $\Phi_{\Delta T}(S_j, S_i)$  is the transition kernel for targets that move from cell  $S_j$  to cell  $S_i$  in the time interval between time t-1and t, which is of duration  $\Delta T$ .

The arrival function  $\lambda^A(S_i)$  may incorporate three different types of arrivals:

- 1) new targets appearing anywhere within the surveillance region,
- new targets appearing at the boundary of the surveillance region,
- 3) new targets appearing at likely entry points, e.g. airports.

We define  $\lambda_S$  to be the average arrival rate of targets within the surveillance region,  $\lambda_B$  to be the average arrival rate of targets at the boundary of the surveillance region, where  $\mathcal{B} = \bigcup_{i \in \mathcal{I}_B} S_i$  is the set of cells at the surveillance boundary. Finally,  $\lambda_E^l$  to be the average arrival rate of targets within a region  $\mathcal{E}_l = \bigcup_{i \in \mathcal{I}_l} S_i$  associated with the *l*-th (of *L*) entry point and where  $\mathcal{I}_l$  is the set of corresponding cell indices that define the region in which targets may enter.

We sum the contributions from the various arrival mechanisms to obtain the arrival density

$$\lambda^{A}(\mathcal{S}_{i}) = \frac{\lambda_{S}\mathcal{A}(S_{i})}{\mathcal{A}_{S}} + \frac{\delta_{B}(S_{i})\lambda_{B}\mathcal{A}(S_{i})}{\sum_{\cup_{i}\in\mathcal{I}_{B}}\mathcal{A}(S_{i})} + \sum_{l=1}^{L}\frac{\delta_{E}^{l}(S_{i})\lambda_{E}^{l}\mathcal{A}(S_{i})}{\sum_{\cup_{i}\in\mathcal{I}_{l}}\mathcal{A}(S_{i})},$$
(5)

where  $\mathcal{A}(S_i)$  is the area of cell  $S_i$ ,  $\mathcal{A}_S$  is the total surveillance area and where  $\delta_B(S_i)$  and  $\delta_E(S_i)$  are indicator functions such that

$$\delta_B(S_i) = \begin{cases} 1, & \text{if } S_i \in \mathcal{B}, \\ 0, & \text{else.} \end{cases}$$
(6)

Similarly,

$$S_E^l(S_i) = \begin{cases} 1, & \text{if } S_i \in \mathcal{E}_l, \\ 0, & \text{else.} \end{cases}$$
(7)

After detection actions have been performed at time t, the distribution of undetected targets is updated to be Poisson with intensity

$$\lambda_{t|t}(\mathcal{S}_i) = P_m(\mathcal{S}_i|\tau_{t,a\in\mathcal{V}(\mathcal{S}_i)})\lambda_{t|t-1}(\mathcal{S}_i),\tag{8}$$

where  $P_m(S_i | \tau_{t,a \in \mathcal{V}(S_i)})$  is the probability of miss for cell  $S_i$  given the scheduled sensing functions that observe it. When the log-probability of miss is given by Albersheim's formula, the contributions from the search beams that observe cell  $S_i$  during the scan at time t are summed to obtain

$$P_m(\mathcal{S}_i | \tau_{t,a \in \mathcal{V}(\mathcal{S}_i)}) = \exp\left[\sum_{a \in \mathcal{V}(\mathcal{S}_i)} \alpha(\mathcal{S}_i, a, \tau_{t,a})\right].$$
(9)

A search cost  $c_s(S_i)$  for not detecting a target in a given cell is introduced. This cost is assumed to be constant for the whole cell and to not change over the single time step planning horizon. We wish to assign time to each beam to minimise the expected total cost over the region for the scan interval at time t. The optimisation problem for myopic search scheduling is presented in equation (8) of [19] and repeated here for completeness:

minimise 
$$\sum_{i=1}^{N} c_s(\mathcal{S}_i) \exp\left[\sum_{a \in \mathcal{V}(\mathcal{S}_i)} \alpha(\mathcal{S}_i, a, \tau_{t,a})\right] \lambda_{t|t-1}(\mathcal{S}_i)$$
  
subject to 
$$\sum_{a=1}^{A} \tau_{t,a} \le R_t, \tau_{t,a} \ge 0.$$
 (10)

As in [19] this is a convex function when using the convexified version of Albersheim's formula.

#### IV. TRACK UPDATES

We consider the track update scheduling problem. In a manner analogous to the search cost  $c_s(S_i)$ , we seek to develop a cost function for track updates that is of a similar form and can be applied over the same surveillance space.

Assume there is a set of targets  $j \in \{1, ..., M\}$  already being tracked in the surveillance region. These tracks may be cued by a different sensor or be historical tracks from the sensor under consideration. The targets are tracked so that the estimate,  $\hat{\mathbf{x}}_{t|t}^{j}$ , of the state of target j is available at time t.

We assume that the scan time  $R_t$  is short enough and that the surveillance region cells are large enough that if a track's predicted state estimate is within the cell then we can apply the update cost for that target entirely to that cell at that time. The predicted state of target j is denoted  $\hat{x}_{t|t-1}^{j}$  and so an indicator function for whether a track lies within a cell is defined by

$$\delta(\hat{\mathbf{x}}_{t|t-1}^{j} \in \mathcal{S}_{i}) = \begin{cases} 1, & \hat{\mathbf{x}}_{t|t-1}^{j} \in \mathcal{S}_{i} \\ 0, & \text{else.} \end{cases}$$
(11)

An update cost  $c_u(j)$  for not updating the track on target j is introduced. This, combined with the indicator function (11), gives the overall track update cost for cell  $S_i$  as the sum of all the track update costs for the tracks that lie within the cell:

$$c_u(\mathcal{S}_i, t) = \sum_{j=1}^M c_u(j)\delta(\hat{\mathbf{x}}_{t|t-1}^j \in \mathcal{S}_i).$$
(12)

The track update  $\cot c_u(j)$  can be anything, but to be of use for developing a scan schedule should be related to the information loss cost of not updating the track within the onestep planning horizon.

Radars generally use different waveforms depending on whether they are searching for a new target in a given direction or are updating a track on a target in that direction [8]. For now we consider a single set of beams that are able to perform both search and track update actions. Future work will divide the radar timeline further into separate search and track update functions.

Assuming that the probability of miss is the same regardless of whether a beam is performing search or track, the probability of miss for a track update is given by (9). Combining these, the objective for the track update problem becomes:

minimise 
$$\sum_{i=1}^{N} c_u(\mathcal{S}_i, t) \left[ \sum_{a \in \mathcal{V}(\mathcal{S}_i)} \alpha(\mathcal{S}_i, a, \tau_{t,a}) \right]$$
  
subject to 
$$\sum_{a=1}^{A} \tau_{t,a} \le R_t, \tau_{t,a} \ge 0.$$
 (13)

An efficient implementation of this method would only consider the subset of the A update beams that actually contain a track.

# V. COMBINED SEARCH AND TRACK UPDATE PROBLEM

If the search and update problems are defined on the same grid, then the two can be directly combined and sensing actions that observe the grid optimised. We assume for now that the radar beams can search for new targets and update tracks in the same action subject to the restriction on total radar time, so

minimise 
$$\sum_{i=1}^{N} c_s(\mathcal{S}_i) \exp\left[\sum_{a \in \mathcal{V}(\mathcal{S}_i)} \alpha(\mathcal{S}_i, a, \tau_{t,a})\right] \lambda_{t|t-1}(\mathcal{S}_i) + \sum_{i=1}^{N} c_u(\mathcal{S}_i, t) \exp\left[\sum_{a \in \mathcal{V}(\mathcal{S}_i)} \alpha(\mathcal{S}_i, a, \tau_{t,a})\right]$$
subject to 
$$\sum_{a=1}^{A} \tau_{t,a} \le R_t, \tau_{t,a} \ge 0.$$
 (14)

The probability of miss is non-increasing with increased resources so the restriction on total time allocated for the scan at time t can be replaced with the equality constraint:

$$\mathbf{e}^T \boldsymbol{\tau}_t = R_t, \tag{15}$$

where  $\boldsymbol{\tau}_t = [\tau_{t,1}, \dots, \tau_{t,A}]^T$  and **e** is an  $A \times 1$  vector of ones. As in [19] we solve this problem using Newton's Method for a related objective function, one with a log barrier for the non-negative time constraints:

$$B_{\theta}(\boldsymbol{\tau}_{t}) = \sum_{i=1}^{N} \left( c_{s}(\mathcal{S}_{i})\lambda_{t|t-1}(\mathcal{S}_{i}) + c_{u}(\mathcal{S}_{i},t) \right) \times \\ \exp\left[ \sum_{a \in \mathcal{V}(\mathcal{S}_{i})} \alpha(\mathcal{S}_{i},a,\tau_{t,a}) \right] - \theta \sum_{a=1}^{A} \ln \tau_{t,a}.$$
(16)

This is convex in  $\tau_t$  if  $\alpha(S_i, a, \tau_{t,a})$  is convex  $\forall a$ , as  $\exp(\cdot)$  is convex and  $-\ln(\cdot)$  is convex [20]. Its derivatives are:

$$\frac{\delta B_{\theta}(\boldsymbol{\tau}_{t})}{\delta \tau_{t,a^{j}}} = \sum_{i|a^{j} \in \mathcal{V}(\mathcal{S}_{i})} \left( \left( c_{s}(\mathcal{S}_{i})\lambda_{t|t-1}(\mathcal{S}_{i}) + c_{u}(\mathcal{S}_{i},t) \right) \cdot \alpha'(\mathcal{S}_{i},a^{j},\tau_{t,a^{j}}) \exp \left[ \sum_{a \in \mathcal{V}(\mathcal{S}_{i})} \alpha(\mathcal{S}_{i},a,\tau_{t,a}) \right] \right) - \frac{\theta}{\tau_{t,a^{j}}}$$
(17)

and

$$\frac{\delta^2 B_{\theta}(\boldsymbol{\tau}_t)}{\delta \tau_{t,a^j}^2} = \sum_{i|a^j \in \mathcal{V}(\mathcal{S}_i)} \left( \left( c_s(\mathcal{S}_i) \lambda_{t|t-1}(\mathcal{S}_i) + c_u(\mathcal{S}_i, t) \right) \cdot \left[ \tilde{\alpha}''(\mathcal{S}_i, a^j, \tau_{t,a^j}) + \alpha'(\mathcal{S}_i, a^j, \tau_{t,a^j})^2 \right] \cdot \exp\left[ \sum_{a \in \mathcal{V}(\mathcal{S}_i)} \alpha(\mathcal{S}_i, a, \tau_{t,a}) \right] \right) + \frac{\theta}{\tau_{t,a}^2}.$$
(18)

$$\frac{\delta^2 B_{\theta}(\boldsymbol{\tau}_t)}{\delta \tau_{t,a^j} \delta \tau_{t,a^k}} = \sum_{i|a^j \in \mathcal{V}(\mathcal{S}_i), a^k \in \mathcal{V}(\mathcal{S}_i)} \left( \left( c_s(\mathcal{S}_i) \lambda_{t|t-1}(\mathcal{S}_i) + c_u(\mathcal{S}_i, t) \right) \cdot \left[ \alpha'(\mathcal{S}_i, a^j, \tau_{t,a^j}) \alpha'(\mathcal{S}_i, a^k, \tau_{t,a^k}) \right] \cdot \exp\left[ \sum_{a \in \mathcal{V}(\mathcal{S}_i)} \alpha(\mathcal{S}_i, a, \tau_{t,a}) \right] \right), a^j \neq a^k$$
(19)

As  $\alpha(S_i, a_u, \tau_{t,a})$  is not inherently convex, we solve the equality-constrained auxiliary problem using Newton's Method [20] on the convexified Hessian forced locally by keeping  $\tilde{\alpha}'' \geq 0$ . The step direction is calculated by:

$$\begin{bmatrix} \Delta \tau_t \\ w \end{bmatrix} = \begin{bmatrix} \nabla^2 B_{\theta}(\boldsymbol{\tau}_t) & \mathbf{e} \\ \mathbf{e}^T & 0 \end{bmatrix} \setminus \begin{bmatrix} -\nabla B_{\theta}(\boldsymbol{\tau}_t) \\ 0 \end{bmatrix}.$$
(20)

The optimal dual variable, w, here serves to ensure that the equality constraint (15) is met. To start the minimisation from a strictly feasible point, we choose an initial  $\tau_t = \frac{R_t}{A} \mathbf{e}$ . We update this estimate  $\tau_t := \tau_t + \gamma \Delta \tau_t$  until a termination condition is met, or for a fixed number of steps.

Following [20], the step length is chosen to ensure the estimate does not reach the unstable log boundary  $\gamma = (1-\epsilon) \min\{1, \tau_{t,a}/-\Delta \tau_{t,a} \forall a : \Delta \tau_{t,a} < 0\}$  with a small scale factor  $\epsilon$  that keeps  $\tau_{t,a}$  certainly positive. Nominally choose  $\epsilon = 0.01$ .

Further, to ensure that  $B_{\theta}(\tau_t)$  decreases every step without performing an expensive line search, we use the common technique of backtracking [20]. In the case that a given step would increase the cost function, the step is repeated from the previous position in the same direction (i.e.  $\Delta \tau_{t,a}$  need not be re-calculated) but for a smaller step size  $\gamma := 0.5\gamma$ .

Every Newton step the log-barrier keeping the allocated times positive is tightened. In this experiment, as in [19], we commence with an initial  $\theta = 0.01$  and reduce this geometrically every step  $\theta := \theta/3$ .

# VI. EXAMPLE

A simple scenario is developed to illustrate how a multifunction sensor schedule can be generated for a dynamically evolving target scenario. A myopic scan schedule is found for a single, stationary radar that is able to perform search and track update functions in a 2D surveillance region where there are extant target tracks to be updated and assumptions are made about the underlying undetected target density.

# A. Scenario

We define the surveillance region as a polar coordinate grid centred around the sensor with grid cells formed by uniform range and azimuth divisions. The cells are chosen so that it is reasonable to assume that the probability of detection and the costs are spatially uniform across a cell. The range component is discretised from 20 km to 200 km with  $N_r = 100$  uniform range divisions. The azimuth component is discretised from 0 to  $2\pi$  radians with  $N_{\theta} = 30$  uniform azimuth bins so there are  $N_r \times N_{\theta} = 3000$  cells in all.

The sensor is an agile radar that can perform search and track update actions. For simplicity, we define A = 30 beams, each centred on an azimuth bin and including the adjacent bins. All of the range cells are observed in the three azimuth bins for each beam. For the centre azimuth bin of each beam, the specified probability of detection is  $\bar{P}_d^a = 0.5$  at range  $\bar{r}^a = 100$  for dwell time  $\bar{\tau}^a = 0.1$ s. For the two outer bins of each beam a, a 24dB reduction is assumed so  $\bar{r}^a = 100/4$ . The probability of eclipsing is set to  $\gamma = 0.1$  and  $P_{fa} = 10^{-6}$ .

The time the sensor spends on each of the beams ( $\tau_{a,t}$ , a = 1, ..., 30 at scan time t) is allocated myopically from a scan time of  $R_t = 1$  second, after which the schedule is executed and the target tracks and undetected target density updated according to the simulated observations. This is repeated for 30 scans (equivalent to 30 seconds).

The optimisation is the Newton's Method with step length and backtracking as described in section V and with a maximum of 15 steps.

We assume undetected targets follow a Poisson distribution as described in section III.

The total background arrival rate is assumed to be a constant  $\lambda_S = 0.05$  targets per second throughout the whole surveillance volume. In addition, there is a total boundary arrival rate of  $\lambda_B = 0.01$  targets per second to cells at the outer range in angle bins 16-21. Finally, an entry arrival region is defined in the third angle bin at ranges [100, 150] to have an extra  $\lambda_E = 0.01$  targets arriving per second.

The sensor scheduling method outlined in this paper does not require that the surveillance region is empty of targets when the sensor starts detecting. The scheduler can account for an initially high undetected target density (reflecting the targets that may inhabit the surveillance space when the sensor is turned on). We initiate the scenario with an undetected target density equivalent to the background arrival rate maintained for 1 second.

If the cells are chosen to be large and frequently observed, then the undetected targets may be assumed to be stationary, so the transition kernel in equation (4) is the identity matrix. The predicted undetected target density in cell  $S_i$  can then be simplified to

$$\lambda_{t|t-1}(\mathcal{S}_i) = \lambda^A(\mathcal{S}_i)\Delta T + \lambda_{t-1|t-1}(\mathcal{S}_i).$$
(21)

The Poisson arrival distribution is sampled across the whole surveillance region over a scan time to determine if a target is born at that time. If so, an "undetected track" is started at a random position drawn from the surveillance region weighted by undetected target density and with velocity (here in km per second) drawn from the continuous uniform distribution  $v_i \sim \mathcal{U}(-0.1, 0.1), i = x, y$ . This target's kinematics are updated at each time according to a constant velocity model (note that this is different to the assumption of stationary undetected targets for the density transition kernel). If a beam illuminates the undetected target position it is detected with the probability  $P_d = 1 - P_m$  and if detected it is added to the targets being tracked with an initial covariance as below (24). In this paper we assume that no false alarm measurements are produced.

The kinematic state of the targets evolve according to a constant velocity process with piecewise-constant white acceleration (see [1] p.204). The sensor scheduling method proposed does not specify a particular tracking method. In this example, the track state is estimated using a standard non-linear Extended Kalman Filter (EKF) with process noise intensity set to q = 0.1.

We introduce two targets, which we label A and B, that are tracked at the start of the scenario. A starts close to the sensor and moves out of the detection region, and B starts far from the sensor and moves slowly towards it. If the state space is  $\mathbf{x} = [x, v_x, y, v_y]^T$ , then

$$\mathbf{x}^A = [30, -10, 40, 0]^T \tag{22}$$

$$\mathbf{x}^B = [140, -4, -140, 4]^T.$$
(23)

At the beginning of the scenario each track has a large covariance, indicating large uncertainty that will incur a large cost:

$$\mathbf{P}_{0} = \begin{bmatrix} 1^{2} & 0 & 0 & 0\\ 0 & 0.1^{2} & 0 & 0\\ 0 & 0 & 1^{2} & 0\\ 0 & 0 & 0 & 0.1^{2} \end{bmatrix}.$$
 (24)

After each scan the radar beam time allocations are computed using Albersheim's formula based on the probability of detection for each target. Radar measurements  $\mathbf{z}_t = [r_t, \theta_t]^T$ , each consisting of range and azimuth components, are assumed to have a mean at the predicted target state and covariance R, the radar measurement error covariance which is given by

$$R = \begin{bmatrix} (0.1\text{km})^2 & 0\\ 0 & (0.1\text{rad})^2 \end{bmatrix}.$$
 (25)

The radar measurement error covariance R is used in an EKF update on the track covariance so that the new cost of not updating the track can be generated and used to determine the schedule for the next scan period.

We choose a track update cost that is a function of the ratio between the expected standard deviation (error) of the track estimate in each dimension at the end of the scan and some desired error

$$c_u(j) = \omega_r \cdot f\left(\frac{\sigma_r^j}{\sigma_r^d}\right) + \omega_\theta \cdot f\left(\frac{\sigma_\theta^j}{\sigma_\theta^d}\right), \text{ where }$$
(26)  
$$\omega_r = \text{range accuracy weight}$$

 $\omega_{\theta} = azimuth accuracy weight$ 

$$\sigma_r^d = \text{desired range error}$$
  

$$\sigma_{\theta}^d = \text{desired azimuth error}$$
  

$$\sigma_r^j = \text{estimated track range error}$$
  

$$\sigma_{\theta}^j = \text{estimated track azimuth error}$$
  

$$f(x) = \max\{x - 1, 0\}.$$

Ĵ

The contributions to the track update cost from the range and azimuth components are truncated at the desired errors, assuming that there is no operational benefit to further refining the track estimate.

The accuracy weights in (26) may be set according to the relative operational importance of these components, but in this example we let  $\omega_r = \omega_{\theta} = 1$ . Furthermore, the desired error is defined as a state-dependent function to reflect the relative operational importance of targets with different states:

$$\sigma_r^d(r) = \begin{cases} 0.1 & : r < 50\\ 0.2 & : 50 \le r < 100\\ 0.5 & : 100 \le r \end{cases}$$
(27)

 $\begin{bmatrix} 0.5 & : r < 50 \end{bmatrix}$ 

$$\sigma_{\theta}^{d}(r) = \begin{cases} 1.0 & :50 \le r < 100\\ 2.5 & :100 \le r. \end{cases}$$
(28)

Equations (27) and (28) indicate that a track at short range has low desired track errors in the range and azimuth components compared to a track at longer range.

To balance the search cost with the track costs, the cost of an undetected target is set to be uniformly 300 throughout the surveillance region. Note that the operator can be provided with  $c_u(j)$  for the initial state of a track and choose a directly comparable undetected target cost depending on her own priorities.

Target tracks are terminated if the targets depart from the surveillance region. A new track is initiated from measurements of previously undetected targets in which case the initial track state estimate covariance is given by  $\mathbf{P}_0$  in equation (24).

Beams are allocated time based on the undetected target cost of the cells observed by that beam and on the update cost of all tracks that fall along that azimuth, as given in the auxiliary objective function (16).

# B. Results

A myopic sensor schedule was derived using the optimisation method outlined in this paper for the example radar and target scenario described in the preceding section.

A representation of the resulting schedule is shown in figure 2, where the time allocations  $\tau_{t,a}$  for each beam *a* (vertical axis) are shown as a function of scan number *t* (horizontal axis). As required by the constraint (15) the sum of the time allocation for all beams must sum to the scan time. Throughout the duration of the scenario it can be seen that beam 3 receives a relatively high time allocation which is consistent with the presence of the entry arrival region in the centre of that beam. Beams 16-21 initially receive a large time allocation corresponding to the boundary arrival region covered by those beams. Target track information is overlaid



Figure 2. Myopic schedule time allocation (seconds). Tracks are overlaid (-), with  $\star$  indicating the track birth and  $\circ$  a measurement.



Figure 3. An example of the decrease in the cost with Newton steps for a single scan.

onto figure 2 so that the azimuth of a measurement is indicated by the corresponding beam position.

The target tracks receive sensor time for their update beams as required to maintain track state covariances that are low cost. A measurement is obtained on a target when sufficient time has been allocated to the beams that observe the target. Even when relatively little time is allocated to a given beam, the combination of the time allocated to the set of beams that observe a target can be enough to allow a detection on that target. For example the measurement obtained on the target in beam 27 at time 21 arises despite the low time allocation for that beam and for the two adjacent beams.

An example of the decrease in the objective function  $B_{\theta}$ for a single scan is shown in figure 3. Typically the time allocations for the set of beams A are stable within 15 Newton steps of the optimisation solver. During the course of the scenario, small perturbations in the undetected target density are amplified by the closed loop nature of the scheduler. The resultant sparse schedule, where some beams are allocated substantially more time than others appears as a 'chequerboard'



Figure 4. Surveillance region (grey) around sensor ( $\diamondsuit$ ) showing initial track position ( $\star$ ), radar measurements ( $\circ$ ) and track estimates (-). Entry arrival region (green) and boundary arrival region (blue) are indicated. Axis units are kilometres.



Figure 5. Undetected target distribution at t=30s for uniform and myopic schedules.

pattern in figure 2.

Figure 4 shows the surveillance region and the locations in Cartesian coordinates of targets A and B which each exist at the start of the scenario as well as other targets that appear later.

Figure 5 shows the undetected target density at the end of the scenario for two different sensor schedules. A uniform schedule in which the total scan time is allocated to each beam equally can be seen to retain a high undetected target density in the entry arrival region. In contrast, applying the myopic sensor schedule as described in this paper reduces the undetected target density in the entry arrival region.

Preliminary results using the joint track and search scheduling scheme outlined in this paper are promising. In particular the sensor scheduling scheme provides a means to specify the costs of different functions for implementation in a single optimisation framework. The initial results appear consistent with expectation and indicate benefits in fulfilling overall user objectives.

# VII. CONCLUSIONS AND FUTURE WORK

An approach for determining a sensor schedule for multiple functions such as target search and track update has been developed. The approach is illustrated with a simple example of an electronically scanned radar that is tasked to perform search and track update functions. The cost function provides a mechanism for the user to specify directly compatible costs for undetected targets and target track maintenance requirements, determining the relative priorities of different functions.

In the multifunction radar systems of interest the waveforms of track update beams are typically tailored to the state estimate of the corresponding target. Future work will consider the sensor scheduling problem where search beams and track update beams have distinct characteristics associated with their respective functions. An extension of the single scan (myopic) scheduling to accommodate multiple scan (nonmyopic) scheduling is planned to be conducted following the framework for the single function of search as outlined in [19]. Other appropriate cost functions for track update may be explored, as well as applying constraints for operational radars employing search and track functions. Functions other than search and track update for radar systems are also being considered for inclusion into a similar sensor scheduling approach.

Finally, a methodology for assessing the resultant sensor schedule is required. Ultimately, the scheduling algorithms for multifunction sensors must allow an operator's priorities to be considered in meeting their objectives.

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