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Chernoff Fusion of Gaussian Mixtures for Distributed Maneuvering Target Tracking

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***Abstract*—A fusion methodology for tracks represented by Gaussian mixtures is proposed for distributed maneuvering target tracking with unknown correlation information between the local agents. For this purpose, Chernoff fusion is applied to the Gaussian mixtures provided by the local interacting multiple-model (IMM) filters. Chernoff fusion of Gaussian mixtures is achieved using a recently proposed method in the literature involving a sigma-point approximation. The results show that the fusion of Gaussian mixtures in a distributed maneuvering target tracking scenario brings a moderate improvement over fusing only moment matched Gaussian densities.**

***Index Terms*—Distributed estimation; maneuvering target tracking, IMM filter, Chernoff fusion, covariance intersection, sigma-points.**

I. Introduction

In a distributed sensor network, one of the main challenges is to handle unknown correlation between the local estimates. In a target tracking scenario where the local agents process their own information, even if the individual sensors collect measurements about the target conditionally independently, the common process noise of the target causes correlation between the local estimation errors [1]. A solution proposed for the correlation caused by the common process noise is to communicate Kalman filter gains [2] between the local agents which requires extra communication in the sensor network. In addition to the common process noise, correlation between the local estimates can also be caused by previous communication between the local agents. It is possible to account for the common information between the local agents (caused by previous communication) using the techniques of information decorrelation [3–5]). In many practical cases, keeping the log of the correlated information between the local estimates and compensating for them are either computationally or communication-wise too costly or infeasible. Therefore techniques which can work under unknown correlation information are of interest. The most well-known track fusion techniques which would give consistent estimates independent of the amount of correlation between the local estimates are the covariance intersection (CI) [6, 7] and the largest ellipsoid algorithm (LEA) [8, 9]. In this work we are going to be interested in CI and its generalization to density functions.

Target tracking before the last two decades was mostly dominated by Gaussian density based state estimators (e.g. Kalman filter (KF), extended KF (EKF), unscented KF (UKF) [10]). This was indeed a result of the computational restrictions of the era which made such filters actually the only possible choices. As a result, the early approaches to track fusion considered only the fusion of locally estimated means and covariances. With the advent of more sophisticated state estimators like Gaussian sum filters [11], interacting multiple model (IMM) filters [12], [13, Sec. 11.6] and particle filters [14], the need for fusing density functions became more apparent. The effects of the increase in the computational resources have also been observed in multiple target tracking where computationally costly multiple hypothesis trackers (MHTs) [15] started to be used. Since MHTs inherently hold mixtures for targets, the problem of fusion of local mixtures appears in multiple target tracking as well (even if Gaussian based state estimators are used in local trackers). The more recent developments in multiple target tracking leading to the probability hypothesis density (PHD) filters [16] made the need for density/intensity fusion methods even more significant.

The optimal fusion of density functions is investigated in detail in [17]. The generalization of CI to probability density functions was first proposed by Mahler in [18] and two years later, independently, by Hurley in [19]. This generalization is called by different names by different authors: Chernoff fusion [5]; geometric mean density [20]; exponential mixture densities [21]. In [18], Mahler also proposed the application of both the optimal approach [17] and Chernoff fusion to multitarget densities. The consistency and conservativeness properties of Chernoff fusion are investigated in [20]. Explicit formulae are derived for Chernoff fusion of Bernoulli, Poisson and independent cluster process multitarget densities in [22].

In this study, we consider a distributed maneuvering target tracking scenario where the estimates of the local agents are to be fused. We assume that the local agents run IMM filters for handling target maneuvers and the output of the local trackers are Gaussian mixtures. As a result the track fusion problem we consider involves the fusion of the Gaussian mixture densities. We assume that the correlation between local estimates is unknown and therefore apply Chernoff fusion using the methodology recently proposed in [23] which is based on a sigma-point approximation. We derive the required fusion expressions to be employed in the local IMM filters. As an alternative methodology, we consider making the track fusion with the single Gaussian densities obtained by reducing the output Gaussian mixtures of the IMM filter using moment matching. Comparisons show that the track fusion with the Gaussian mixtures provides a moderate improvement over the single Gaussian version. To the authors' best knowledge the Chernoff fusion methodology has not been applied in a distributed IMM filtering framework for making track fusion using Gaussian mixtures before.

The organization of the paper is given as follows. In Section II, we give a brief overview of maneuvering target tracking and IMM filter. Section III first presents the correlation independent fusion methodology CI [6, 7] and its generalization (Chernoff fusion), then, it describes the method taken from [23] utilized for performing Chernoff fusion of Gaussian mixtures. The fusion expressions to be applied in IMM filter which are the main contribution of the current work are given in Section IV. Section V presents the simulation results. The conclusions are drawn in Section VI

II. Maneuvering Target Tracking and IMM Filter

Maneuvering target tracking is a sub-area of target tracking which is interested in the model mismatch between the true target motion and assumed motion model in the tracking filter. When not detected and compensated, the maneuvers can degrade the performance of the tracker and might even lead to filter divergence. See [13, Chapter 11] for the history and a survey of the tracking methods proposed for maneuvering target tracking. The most commonly used solution for the maneuvering targets is the multiple model approach [13, Section 11.6] where multiple Kalman filters utilizing different motion models are executed in the tracker. The most well known instances of these methods are the so called Generalized Pseudo Bayesian (GPB) Methods [24, 25] and Interacting Multiple Model (IMM) filter [26].

IMM filter, which was invented by Blom and Bar-Shalom [12], is the most common maneuvering target tracking algorithm used in the literature. It is a state estimation algorithm developed for the following Jump Markov Linear System (JMLS).

$$x(k+1) = A_i x(k) + B_j w_i(k), \quad (1a)$$
$$z(k) = C_i x(k) + v_i(k) \quad (1b)$$

where

- $i \in \{1, \ldots, r\}$ denotes the model which is assumed to evolve according to a homogeneous Markov chain with the transition probability matrix $\Pi = [p_{ij}]$ where p_{ij} is the probability of a transition from the ith model to the jth model;
- $x(k) \in \mathbb{R}^n$ is the state vector;
- $z(k) \in \mathbb{R}^m$ is the measurement vector;
- $w_i(k) \sim \mathcal{N}(w_i(k), 0, Q_i)$ is the white process noise sequence for the ith model, $i = 1, \ldots, r$;
- $v_i(k) \sim \mathcal{N}(v_i(k), 0, R_i)$ is the white measurement noise sequence for the ith model, $i = 1, \ldots, r$, independent of the process noise sequence;
- A_i, B_i, C_i are the model parameter matrices for the ith model, $i = 1, \ldots, r$.

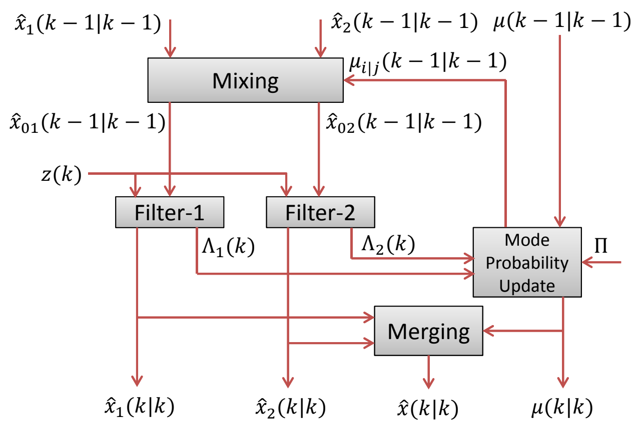

Fig. 1. Block diagram of the IMM filter with two models.

Here, the notation $\mathcal{N}(x, \bar{x}, P)$ denotes a Gaussian density over the variable x with mean $\bar{x}$ and covariance P. The calculation of the exact posterior state distribution for the model (1) is not computationally feasible. Hence the use of suboptimal solutions is necessary. IMM filter approximates the posterior state as a Gaussian mixture given as follows.

$$p(x_k|Z_0^k) \approx \sum_{i=1}^{r} \mu_i(k|k)\mathcal{N}(x_k, \hat{x}_i(k), P_i(k|k)) \quad (2)$$

where

- $Z_0^k \triangleq \{z(0), z(1), \ldots, z(k)\}$ is the cumulative set of measurements from time 0 to k;
- $\mu_i(k|k) \triangleq E[M_i(k)|Z_0^k]$ is the posterior mode probability where the $M_i(k)$ is the event that the target assumes the ith model at time k.
- $\hat{x}_i(k|k) \triangleq E[x(k)|Z_0^k, M_i(k)]$ is the mode-conditioned state estimate;
- $P_i(k|k)$ is the mode-conditioned state covariance defined as

$$P_i(k|k) \triangleq E[(x(k) - \hat{x}_i(k|k)) \times (x(k) - \hat{x}_i(k|k))^{\mathrm{T}}|Z_0^k, M_i(k)]. \quad (3)$$

Below we provide a brief overview of the single step of the IMM filter which is adopted from [13]. Suppose that we have the previous model conditioned estimates $\{\hat{x}_i(k-1|k-1)\}_{i=1}^r$, the associated covariances $\{P_i(k-1|k-1)\}_{i=1}^r$ and the previous mode probabilities $\{\mu_i(k-1|k-1)\}_{i=1}^r$. The updated state estimates $\{\hat{x}_i(k|k)\}_{i=1}^r$, covariances $\{P_i(k|k)\}_{i=1}^r$ and mode probabilities $\{\mu_i(k|k)\}_{i=1}^r$ are obtained using the following steps.

1) **Interaction (Mixing)**:

$$\mu_{i|j}(k-1|k-1) = \frac{p_{ij}\mu_i(k-1|k-1)}{\mu_j(k|k-1)}, \quad (4a)$$

$$\hat{x}_{0j}(k-1|k-1) = \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1)$$

$$\times \hat{x}_i(k-1|k-1) \tag{4b}$$

$$P_{0j}(k-1|k-1) = \sum_{i=1}^{r} \mu_{i|j}(k-1|k-1) \times \Big(\big[P_i(k-1|k-1) + [\hat{x}_i(k-1|k-1) - \hat{x}_{0j}(k-1|k-1)] \times [\hat{x}_i(k-1|k-1) - \hat{x}_{0j}(k-1|k-1)]^{\mathrm{T}} \big] \Big) \tag{4c}$$

where the predicted mode probability $\mu_j(k|k-1)$ is given by

$$\mu_j(k|k-1) = \sum_{i=1}^{r} p_{ij}\mu_i(k-1|k-1). \tag{4d}$$

2) **Mode-Conditioned Filtering:**
   - **Prediction Update:**

$$\hat{x}_j(k|k-1) = A_j\hat{x}_{0j}(k-1|k-1), \tag{5a}$$

$$P_j(k|k-1) = A_j P_{0j}(k-1|k-1)A_j^{\mathrm{T}} + B_j Q_j B_j^{\mathrm{T}}. \tag{5b}$$

   - **Measurement Update:**

$$\hat{x}_j(k|k) = \hat{x}_j(k|k-1) + K_j(k)\nu_j(k), \tag{6a}$$

$$P_j(k|k) = [I_{n_x} - K_j C_j]P_j(k|k-1), \tag{6b}$$

$$\nu_j(k) = z(k) - C_j\hat{x}_j(k|k-1), \tag{6c}$$

$$S_j = C_j P^j(k|k-1)C_j^{\mathrm{T}} + R_j, \tag{6d}$$

$$K_j = P_j(k|k-1)C_j^{\mathrm{T}} S_j^{-1} \tag{6e}$$

where I_n is the identity matrix of size $n \times n$.

3) **Mode-Probability Update:**

$$\mu_j(k|k) = \frac{\mu_j(k|k-1)\Lambda_j(k)}{\sum_{i=1}^{r}\mu_i(k|k-1)\Lambda_i(k)} \tag{7a}$$

where

$$\Lambda_j(k) = \mathcal{N}(z(k), \hat{x}_j(k|k-1), S_j). \tag{7b}$$

Being a Bayesian state estimator, the intrinsic output of the IMM filter is the Gaussian mixture (2). However, sometimes only a single mean and covariance is desired as the output for higher level processing. The single mean $\hat{x}(k|k)$ and covariance $P(k|k)$ are obtained using the following operations.

4) **Merging:**

$$\hat{x}(k|k) = \sum_{i=1}^{r} \mu_j(k|k)\hat{x}_j(k|k), \tag{8a}$$

$$P(k|k) = \sum_{i=1}^{r} \mu_j(k|k)\Big(P_j(k|k) + [\hat{x}_j(k|k) - \hat{x}(k|k)][\hat{x}_j(k|k) - \hat{x}(k|k)]^{\mathrm{T}}\Big). \tag{8b}$$

The calculation above amounts to reducing the Gaussian mixture (2) to a single Gaussian by moment matching and it is obviously a lossy operation. A block diagram for a single step of the IMM filter for two models (i.e., $r = 2$ in the JMLS (1)) is given in Figure 1.

III. CI and Chernoff Fusion

Covariance intersection (CI) [6, 7] is one of the main approaches to decentralized fusion [5]. Its main advantage is that it enables consistent fusion under unknown correlation information. The consistency in this context is defined as the fused covariance being always larger than or equal to the optimally fused covariance that would be obtained if the correlation information was available. See [27] for more details about the optimality and consistency properties of CI. The main procedure of CI is described as follows. Let us have two local estimates $x_1 \in \mathbb{R}^n$ and $x_2 \in \mathbb{R}^n$ and their positive definite covariances $P_1 \in \mathbb{R}^{n\times n}$ and $P_2 \in \mathbb{R}^{n\times n}$. CI calculates the fused estimate x_{CI} and covariance P_{CI} as

$$P_{\mathrm{CI}}^{-1}x_{\mathrm{CI}} = wP_1^{-1}x_1 + (1-w)P_2^{-1}x_2 \tag{9a}$$

$$P_{\mathrm{CI}}^{-1} = wP_1^{-1} + (1-w)P_2^{-1} \tag{9b}$$

where $w \in [0,1]$ is selected to be the solution w^* of the following optimization problem.

$$w^* \triangleq \arg\min_{w\in[0,1]} L\left(\left(wP_1^{-1} + (1-w)P_2^{-1}\right)^{-1}\right). \tag{10}$$

Here, the function $L : \mathbb{S}_{\geq 0}^{n\times n} \to \mathbb{R}_{\geq 0}$ represents an uncertainty measure from the space of symmetric positive semi-definite matrices ($\mathbb{S}_{\geq 0}^{n\times n}$) into non-negative real numbers ($\mathbb{R}_{\geq 0}$) and is usually selected either as the trace or the determinant of the matrix argument.

CI can be generalized to the fusion of density functions [18, 19]. The corresponding generalization is called as *Chernoff fusion* [5]. Given two density functions $p_{x,1}(\cdot)$ and $p_{x,2}(\cdot)$ representing the same random variable x, the fused density $p_{x,\mathrm{CF}}(\cdot)$ is obtained as

$$p_{x,\mathrm{CF}}(x) = \frac{p_{x,1}^{w}(x)p_{x,2}^{1-w}(x)}{\int p_{x,1}^{w}(x)p_{x,2}^{1-w}(x)\,\mathrm{d}x} \tag{11}$$

where the subscript CF stands for Chernoff fusion and w is selected to be the solution w^* of the optimization problem given below.

$$w^* = \arg\min_{w\in[0,1]} \mathcal{L}\left(\frac{p_{x,1}^{w}(x)p_{x,2}^{1-w}(x)}{\int p_{x,1}^{w}(x)p_{x,2}^{1-w}(x)\,\mathrm{d}x}\right). \tag{12}$$

Here, the function $\mathcal{L}(\cdot)$ represents an uncertainty measure from the set of density functions into real numbers. For example, the matrix uncertainty measure trace in CI corresponds to the uncertainty measure variance ($E_x[x^{\mathrm{T}}x] - E_x[x^{\mathrm{T}}]E_x[x]$) in Chernoff fusion and the matrix uncertainty measure determinant in CI corresponds to the uncertainty measure entropy ($E_x[-\log p(x)]$) in Chernoff fusion. See [20] for details about the consistency and conservativeness properties of Chernoff fusion formula (11).

A. Chernoff Fusion for Gaussian Mixtures

When the densities $p_{x,1}(\cdot)$ and $p_{x,2}(\cdot)$ in (11) are selected to be Gaussian mixtures, the application of Chernoff fusion formula (11) requires taking non-integer powers of the Gaussian mixtures. In general, a non-integer power of a Gaussian mixture is not a Gaussian mixture (See e.g. [28, Sec. VI]) and there is no exact analytical expression for it. In [23], a sigma-point approximation is proposed for approximating an arbitrary non-integer power of a Gaussian mixture, which was an improvement over the first-order approximation proposed in [29] for the same problem. Suppose we call the wth power of the Gaussian mixture $p(x) = \sum_{i=1}^{N} \alpha_i \mathcal{N}(x; x_i, P_i)$ as $q(x) \triangleq p^w(x)$. The function $q(x)$ is approximated in [23] as

$$q(x) \approx \sum_{i=1}^{N} \beta_i \mathcal{N}(x; x_i, w^{-1} P_i) \tag{13}$$

where the unknown weights $\{\beta_i\}_{i=1}^N$ are found by solving the following weighted non-negative least squares problem.

$$\underset{\beta}{\text{minimize}}\ (\mathbf{M}\beta - \mathbf{b})^{\mathrm{T}} \mathbf{W} (\mathbf{M}\beta - \mathbf{b}) \tag{14a}$$

$$\text{subject to } 0 \le \beta_i,\ i = 1, \ldots, N. \tag{14b}$$

In (14), the elements of the vector $\mathbf{b} \in \mathbb{R}^{N(2n+1)\times 1}$, the matrix $\mathbf{M} \in \mathbb{R}^{N(2n+1)\times N}$ and the diagonal matrix $\mathbf{W} \in \mathbb{R}^{N(2n+1)\times N(2n+1)}$ are defined as

$$[\mathbf{M}]_{(2n+1)(i-1)+j,m} \triangleq \mathcal{N}(s_i^j; x_m, w^{-1} P_m), \tag{15a}$$

$$[\mathbf{b}]_{(2n+1)(i-1)+j,1} \triangleq p^w(s_i^j), \tag{15b}$$

$$[\mathbf{W}]_{(2n+1)(i-1)+j,(2n+1)(i-1)+j} = \alpha_i \pi_i^j \tag{15c}$$

for $i, m = 1, \ldots, N$ and $j = 1, \ldots, 2n+1$ where the notation $[\cdot]_{i,j}$ denotes the i,jth element of the argument matrix. In (15), $\{s_i^j\}_{j=1}^{2n+1}$ denote the sigma-points for the ith component of $p(\cdot)$ generated by unscented transform [10] and $\{\pi_i^j\}_{j=1}^{2n+1}$ are their weights. The non-negative least squares (NNLS) problem (14) is solved using *Lawson-Hanson algorithm* [30, Chapter 23] a modified version of which is available in MATLAB as the function `lsqnonneg(·)`.

The study [23] proposes to fuse Gaussian mixtures based on the approximate power operation described above and calls the resulting Chernoff fusion method as sigma-point Chernoff fusion (SPCF). An overview of SPCF is given below. Suppose the mixtures to be fused are given as follows.

$$p_{x,1}(x) = \sum_{i=1}^{M} \mu_i \mathcal{N}(x; \phi_i, \Phi_i), \tag{16a}$$

$$p_{x,2}(x) = \sum_{j=1}^{N} \nu_j \mathcal{N}(x; \psi_j, \Psi_j). \tag{16b}$$

Suppose the wth and $(1-w)$th powers of $p_{x,1}(\cdot)$ and $p_{x,2}(\cdot)$, respectively, obtained by the procedure above are called $q_{x,1}(\cdot)$ and $q_{x,2}(\cdot)$ and given as

$$q_{x,1}(x) = \sum_{i=1}^{M} \widehat{\mu}_i(w) \mathcal{N}(x; \phi_i, w^{-1}\Phi_i), \tag{17}$$

$$q_{x,2}(x) = \sum_{j=1}^{N} \widehat{\nu}_j(w) \mathcal{N}(x; \psi_j, (1-w)^{-1}\Psi_j) \tag{18}$$

where the dependency of the weights on w is emphasized. Given $q_{x,1}(\cdot)$ and $q_{x,2}(\cdot)$, the rest of the fusion amounts to applying the so called "naive" fusion formula [5] (i.e., the fusion formula that would be valid if the local quantities were independent.[1]) to fuse the resultant mixtures (17) and (18).

Multiplication of the Gaussian mixtures $q_{x,1}(\cdot)$ and $q_{x,2}(\cdot)$ results in

$$\begin{aligned} &q_{x,1}(x) q_{x,2}(x) \\ &= \sum_{i=1}^{M} \sum_{j=1}^{N} \widehat{\mu}_i(w) \widehat{\nu}_j(w) \mathcal{N}\left(x; \phi_i, \frac{\Phi_i}{w}\right) \mathcal{N}\left(x; \psi_j, \frac{\Psi_j}{1-w}\right) \\ &= \sum_{i=1}^{M} \sum_{j=1}^{N} \widehat{\mu}_i(w) \widehat{\nu}_j(w) \pi_{ij}(w) \mathcal{N}\left(x; \tilde{x}_{ij}(w), \widetilde{P}_{ij}(w)\right) \end{aligned}$$

where

$$\pi_{ij}(w) \triangleq \mathcal{N}\left(\phi_i; \psi_j, \frac{\Phi_i}{w} + \frac{\Psi_j}{1-w}\right) \tag{21a}$$

$$\widetilde{P}_{ij}^{-1}(w) = w\Phi_i^{-1} + (1-w)\Psi_j^{-1} \tag{21b}$$

$$\widetilde{P}_{ij}^{-1}(w) \tilde{x}_{i,j}(w) = w\Phi_i^{-1}\phi_i + (1-w)\Psi_j^{-1}\psi_j. \tag{21c}$$

Therefore, we have

$$p_{x,\text{SPCF}}(x) = \frac{\begin{pmatrix} \sum_{i=1}^{M} \sum_{j=1}^{N} \widehat{\mu}_i(w) \widehat{\nu}_j(w) \pi_{ij}(w) \\ \times \mathcal{N}\left(x; \tilde{x}_{ij}(w), \widetilde{P}_{ij}(w)\right) \end{pmatrix}}{\sum_{i=1}^{M} \sum_{j=1}^{N} \widehat{\mu}_i(w) \widehat{\nu}_j(w) \pi_{ij}(w)} \tag{22}$$

where w is selected to be the solution w^* of the optimization problem given as

$$w^* = \arg \min_{w \in [0,1]} \mathcal{L}\left(p_{x,\text{SPCF}}(\cdot)\right). \tag{23}$$

In this work we are going to use the variance as the optimizing criterion since it is analytically computable for Gaussian mixtures, i.e., $\mathcal{L}(p_x(x)) = E_x[x^{\mathrm{T}}x] - E_x[x^{\mathrm{T}}]E_x[x]$, which gives

$$w^* = \arg \min_{w \in [0,1]} \frac{\begin{pmatrix} \sum_{i=1}^{M} \sum_{j=1}^{N} \widehat{\mu}_i(w) \widehat{\nu}_j(w) \pi_{ij}(w) \\ \times \left[\operatorname{tr}\left(\widetilde{P}_{ij}(w)\right) + \|\tilde{x}_{ij}(w) - \tilde{x}(w)\|_2^2\right] \end{pmatrix}}{\sum_{i=1}^{M} \sum_{j=1}^{N} \widehat{\mu}_i(w) \widehat{\nu}_j(w) \pi_{ij}(w)} \tag{24}$$

where

$$\tilde{x}(w) \triangleq \sum_{i=1}^{M} \sum_{j=1}^{N} \widehat{\mu}_i(w) \widehat{\nu}_j(w) \pi_{ij}(w) \tilde{x}_{ij}(w), \tag{25}$$

and the notation $\|\cdot\|_2$ denotes the Euclidean norm of the argument vector; the operator $\operatorname{tr}(\cdot)$ gives the trace of the argument matrix.

[1] The naive fusion formula is given as

$$p_{\text{naive}}(x) = \frac{p_{x,1}(x) p_{x,2}(x)}{\int p_{x,1}(x) p_{x,2}(x)\, \mathrm{d}x}. \tag{19}$$

IV. Distributed IMM Filtering

In this section, we are going to consider a distributed maneuvering target tracking framework where local agents use IMM filters to track targets. Without loss of generality, we are going to consider only two local agents. The aim is going to be to make distributed estimation where each agent receives processed information from the other agent and fuses its local estimate(s) with the remote information.

We consider two distinct fusion strategies depending on the nature of the communicated information from/to the IMM filters.

- **Strategy-1:** The communicated pieces of information are in the form of Gaussian mixtures. These are the mixtures (2) provided by the IMM filters. The Gaussian mixtures are communicated by sending the mode-probabilities along with mode conditioned means and covariances. We here consider a general case where the local IMM filters might use different (number of) models. It is also assumed that when a local agents receives remote information in the form of a Gaussian mixture, it does not know which models the components of the mixture correspond to.
- **Strategy-2:** The communicated pieces of information are in the form of single Gaussian densities moment-matched to IMM filter outputs. The mean and the covariance of the moment-matched Gaussian density are given as in (8). The Gaussian densities are communicated by sending the merged mean and covariance (8).

In both strategies, feedback is assumed. In other words, at each iteration, local IMM filters use the last fused mode-probabilities, estimates and their covariances as their initial condition. We are going to examine the required fusion operations in each strategy in separate subsections below. For each strategy, we are going to assume that we are in one of the local agents and we are trying to fuse the received remote information with the local quantities.

A. Fusion Strategy-1

For this strategy, the received remote information is in the form of a Gaussian mixture density denoted as

$$f^R(x) = \sum_{i=1}^{r_R} \mu_i^R(k|k)\mathcal{N}\left(x; \hat{x}_i^R(k|k), P_i^R(k|k)\right) \tag{26}$$

and it has to be fused with the estimates of each model in the local IMM filter. Note that each model in the IMM filter has a mode probability, mode-conditioned state estimate and covariance denoted as $\mu_j^L(k|k)$, $\hat{x}_j^L(k|k)$ and $P_j^L(k|k)$, respectively, and these quantities are to be replaced by the fused quantities denoted as $\mu_j^F(k|k)$, $\hat{x}_j^F(k|k)$ and $P_j^F(k|k)$. The information in the jth model of the local IMM filter is given as the weighted Gaussian density below.

$$f_j^L(x) = \mu_j^L(k|k)\mathcal{N}\left(x; \hat{x}_j^L(k|k), P_j^L(k|k)\right). \tag{27}$$

The fusion operation is illustrated in Figure 2. With Strategy-1, we consider the naive and SPCF fusion operations. For

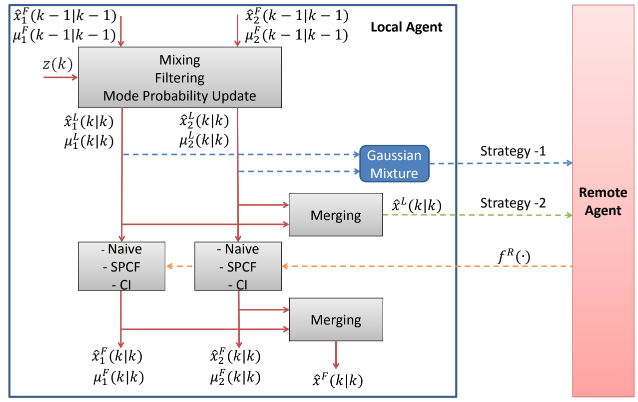

Fig. 2. Proposed fusion strategies with feedback.

the sake of brevity we drop the time stamps $(k|k)$ in the expressions.

1) Naive Fusion: The Naive Fusion approach neglects the correlation between the local and remote estimates and produces the fused information given as

$$f_j^F(x) = f_j^L(x)f^R(x) \tag{28a}$$

$$= \mu_j^L \sum_{i=1}^{r} \mu_i^R \mathcal{N}(x; \hat{x}_i^R, P_i^R)\mathcal{N}(x; \hat{x}_j^L, P_j^L) \tag{28b}$$

$$= \mu_j^L \sum_{i=1}^{r_R} \mu_i^R \mathcal{N}(\hat{x}_i^R; \hat{x}_j^L, P_i^R + P_j^L)\mathcal{N}(x; \hat{x}_{i|j}, P_{i|j}) \tag{28c}$$

$$= \mu_j^L c_j^F \sum_{i=1}^{r_R} \eta_{ij}^F \mathcal{N}(x; \hat{x}_{i|j}, P_{i|j}) \tag{28d}$$

where

$$\hat{x}_{i|j} = P_{i|j}((P_j^L)^{-1}\hat{x}_j^L + (P_i^R)^{-1}\hat{x}_i^R), \tag{29a}$$

$$P_{i|j} = \left((P_j^L)^{-1} + (P_i^R)^{-1}\right)^{-1}, \tag{29b}$$

$$c_j^F = \sum_{i=1}^{r_R} \mu_i^R \mathcal{N}(\hat{x}_i^R; \hat{x}_j^L, P_i^R + P_j^L), \tag{29c}$$

$$\eta_{ij}^F = \frac{\mu_i^R}{c_j^F} \mathcal{N}(\hat{x}_i^R; \hat{x}_j^L, P_i^R + P_j^L). \tag{29d}$$

Since the local information for the jth filter is in the form (27), the fused information (28d) should also be brought into the same form. For this purpose, the mixture in (28d) is reduced to a single moment-matched Gaussian density which gives

$$f_j^F(x) = \tilde{\mu}_j^F \mathcal{N}(x, \hat{x}_j^F, P_j^F) \tag{30}$$

where

$$\tilde{\mu}_j^F = \mu_j^L c_j^F, \tag{31a}$$

$$\hat{x}_j^F = \sum_{i=1}^{r_R} \eta_{ij}^F \hat{x}_{i|j}, \tag{31b}$$

$$P_j^F = \sum_{i=1}^{r_R} \eta_{ij}^F \left(P_{i|j} + [\hat{x}_{i|j} - \hat{x}_j^F][\hat{x}_{i|j} - \hat{x}_j^F]^{\mathrm{T}}\right). \tag{31c}$$

Finally, the fused mode probabilities $\tilde{\mu}_j^F$ have to be normalized across different modes in the local filter as

$$\mu_j^F = \frac{\tilde{\mu}_j^F}{\sum_{i=1}^{r_L} \tilde{\mu}_i^F} \tag{32}$$

to obtain

$$f_j^F(x) = \mu_j^F \mathcal{N}(x, \hat{x}_j^F, P_j^F). \tag{33}$$

2) SPCF: This is the proposed sigma-point Chernoff fusion technique. Chernoff fusion would yield the fused information given as

$$f_j^F(x) = \left(f_j^L(x)\right)^w \left(f^R(x)\right)^{1-w} \tag{34a}$$

$$= (\mu_j^L)^w \mathcal{N}^w\left(x; \hat{x}_j^L, P_j^L\right) \left[\sum_{i=1}^{r_R} \mu_i^R \mathcal{N}\left(x; \hat{x}_i^R, P_i^R\right)\right]^{1-w} \tag{34b}$$

The wth power of the Gaussian density $\mathcal{N}\left(x; \hat{x}_j^L, P_j^L\right)$ in (34b) can be taken exactly to give the weighted Gaussian given below.

$$\mathcal{N}^w\left(x; \hat{x}_j^L, P_j^L\right) = a_j^L(w) \mathcal{N}\left(x; \hat{x}_j^L, w^{-1} P_j^L\right) \tag{35}$$

where

$$a_j^L(w) \triangleq \frac{\left(\left|2\pi w^{-1} P_j^L\right|\right)^{0.5}}{\left(\left|2\pi P_j^L\right|\right)^{w/2}}. \tag{36}$$

For the calculation of $(1-w)^{\text{th}}$ power of the Gaussian mixture in (34b), the approximation described in Section III-A is used to obtain

$$\left[\sum_{i=1}^{r_R} \mu_i^R \mathcal{N}\left(x; \hat{x}_i^R, P_i^R\right)\right]^{1-w} \approx \sum_{i=1}^{r_R} \beta_i^R(w) \mathcal{N}\left(x; \hat{x}_i^R, (1-w)^{-1} P_i^R\right) \tag{37}$$

Hence, we have

$$f_j^F(x) \approx (\mu_j^L)^w a_j^L(w) \sum_{i=1}^{r_R} \beta_i^R(w) \mathcal{N}\left(x; \hat{x}_j^L, w^{-1} P_j^L\right) \times \mathcal{N}\left(x; \hat{x}_i^R, (1-w)^{-1} P_i^R\right) \tag{38a}$$

$$= (\mu_L^j)^w a_j^L(w) \sum_{i=1}^{r_R} \beta_i^R(w) \mathcal{N}\left(x; \hat{x}_{i|j}, P_{i|j}\right) \times \mathcal{N}\left(\hat{x}_i^R; \hat{x}_j^L, w^{-1} P_j^L + (1-w)^{-1} P_i^R\right) \tag{38b}$$

$$= (\mu_j^L)^w a_j^L(w) c_j^F(w) \sum_{i=1}^{r_R} \eta_{ij}^F(w) \mathcal{N}\left(x; \hat{x}_{i|j}(w), P_{i|j}(w)\right) \tag{38c}$$

where

$$\hat{x}_{i|j}(w) = P_{i|j}(w)\left(w(P_L^j)^{-1}\hat{x}_L^j + (1-w)(P_R^i)^{-1}\hat{x}_R^i\right) \tag{39a}$$

$$P_{i|j}(w) = \left(w(P_L^j)^{-1} + (1-w)(P_R^i)^{-1}\right)^{-1} \tag{39b}$$

$$c_j^F(w) = \sum_{i=1}^{r_R} \beta_i^R(w) \mathcal{N}\left(\hat{x}_i^R; \hat{x}_j^L, w^{-1} P_j^L + (1-w)^{-1} P_i^R\right), \tag{39c}$$

$$\eta_i^F(w) = \frac{\beta_i^R(w)}{c_j^F(w)} \mathcal{N}\left(\hat{x}_i^R; \hat{x}_j^L, w^{-1} P_j^L + (1-w)^{-1} P_i^R\right). \tag{39d}$$

Reducing the mixture in (38c) to a single moment-matched Gaussian density gives

$$f_j^F(x) = \tilde{\mu}_j^F \mathcal{N}(x, \hat{x}_j^F(w), P_j^F(w)) \tag{40}$$

where

$$\tilde{\mu}_j^F(w) = (\mu_j^L)^w a_j^L(w) c_j^F(w), \tag{41a}$$

$$\hat{x}_j^F(w) = \sum_{i=1}^{r_R} \eta_{ij}^F(w) \hat{x}_{i|j}(w), \tag{41b}$$

$$P_j^F(w) = \sum_{i=1}^{r_R} \eta_{ij}^F(w) \Big(P_{i|j}(w) + [\hat{x}_{i|j}(w) - \hat{x}_j^F(w)][\hat{x}_{i|j}(w) - \hat{x}_j^F(w)]^{\mathrm{T}}\Big). \tag{41c}$$

The fused mode probabilities $\tilde{\mu}_j^F$ have to be normalized across different modes in the local IMM filter as

$$\mu_j^F(w) = \frac{\tilde{\mu}_j^F(w)}{\sum_{i=1}^{r_L} \tilde{\mu}_i^F(w)} \tag{42}$$

to obtain

$$f_j^F(x) = \mu_j^F(w) \mathcal{N}(x, \hat{x}_j^F(w), P_j^F(w)). \tag{43}$$

The optimal value w^* of w is obtained by minimizing the trace of $P_j^F(w)$.

B. Fusion Strategy-2

In this fusion strategy, the remote information is the Gaussian density moment-matched to the output Gaussian mixture of the remote IMM filter as in (8). Hence the received remote information is given as

$$f^R(x) = \mathcal{N}\left(x; \hat{x}^R(k|k), P^R(k|k)\right). \tag{44}$$

The information in the jth model of the local IMM filter is the same as the weighted Gaussian density in (27). Since both pieces of information is in the form of a weighted Gaussian density, we can use CI for the fusion.

CI gives the fused information for the jth model of the local IMM filter as follows.

$$f_j^F(x) = \left(f_j^L(x)\right)^w \left(f^R(x)\right)^{1-w} \tag{45a}$$

$$= (\mu_j^L)^w \mathcal{N}^w\left(x; \hat{x}_j^L, P_j^L\right) \mathcal{N}^{1-w}\left(x; \hat{x}^R, P^R\right) \tag{45b}$$

Using the result in (36), we get

$$\begin{aligned} f_F^j(x) =&(\mu_L^j)^w a_j^L(w) a^R (1-w) \mathcal{N}\left(x;\hat{x}_j^L, w^{-1}P_j^L\right) \\ &\times \mathcal{N}\left(x;\hat{x}^R,(1-w)^{-1}P^R\right) \quad (46a)\\ =&(\mu_L^j)^w a_j^L(w) a^R (1-w) \mathcal{N}\left(x;\hat{x}_{j|R}(w), P_{j|R}(w)\right) \\ &\times \mathcal{N}\left(\hat{x}^R;\hat{x}_j^L, w^{-1}P_j^L+(1-w)^{-1}P^R\right) \quad (46b)\\ =&\tilde{\mu}_j^F(w)\mathcal{N}\left(x;\hat{x}_{j|R}(w),P_{j|R}(w)\right) \quad (46c)\end{aligned}$$

where

$$\hat{x}_{j|R}(w) = P_{j|R}^{-1}(w)\left(w(P_L^j)^{-1}\hat{x}_L^j + (1-w)(P_R)^{-1}\hat{x}_R\right) \quad (47a)$$

$$P_{j|R}(w) = \left(w(P_L^j)^{-1} + (1-w)(P_R)^{-1}\right)^{-1} \quad (47b)$$

$$\begin{aligned}\tilde{\mu}_j^F(w) =&(\mu_L^j)^w a_j^L(w) a^R (1-w) \\ &\times \mathcal{N}\left(\hat{x}^R;\hat{x}_j^L, w^{-1}P_j^L+(1-w)^{-1}P^R\right) \quad (47c)\end{aligned}$$

The fused mode probabilities $\tilde{\mu}_j^F$ have to be normalized across different modes in the local IMM filter as

$$\mu_j^F(w) = \frac{\tilde{\mu}_j^F(w)}{\sum_{i=1}^{r_L}\tilde{\mu}_i^F(w)} \quad (48)$$

to obtain

$$f_j^F(x) = \mu_j^F(w)\mathcal{N}(x,\hat{x}_j^F(w),P_j^F(w)). \quad (49)$$

where $\hat{x}_j^F(w) = \hat{x}_{j|R}(w)$ and $P_j^F(w) = P_{j|R}(w)$. The optimal value w^* of w is obtained by minimizing the trace of $P_j^F(w)$.

V. Simulation Results

In order to test the performances of the strategies and the fusion methods, a 2D tracking scenario in which the outputs of two local agents using IMM filters have to be fused. For this purpose, random trajectories for a target with sampling time $T = 1$s are generated using a JMLS which has two 2D nearly constant velocity models with different process noise standard deviation (std) values. The process noise std for Model-1 and Model-2 are selected as $\sigma_{p1} = 1\text{m/s}^2$, $\sigma_{p2} = 35\text{m/s}^2$ respectively. The target trajectories are started from the origin with x-y components of the velocity vector selected independently and uniformly in the interval $[100, 200]$m/s. The diagonal elements of the transition probability matrix for the JMLS are given as $p_{11} = p_{22} = 0.9$. The Cartesian x-y position measurements are collected with measurement noise std $\sigma_r = 200$ generated independently for each local agent. The local IMM filters use the true JMLS parameters. For SPCF and CI approaches, the optimal value w^* of the parameter w is obtained by making a search over a uniform grid of 20 elements in the interval $[0, 1]$.

A total of 250 Monte-Carlo runs are made where in each run a different realization of the target trajectory (of length 100 seconds) and target measurements are used. The RMS position errors for the Naive (Strategy-1), SPCF (Strategy-1) and CI (Strategy-2) are obtained. In order to serve as a baseline, a centralized IMM filter which uses the measurements of both local agents is also implemented. Similarly the RMS errors of one of the local agents are also calculated.

TABLE I
Average (both ensemble and time) RMS position error values.

Method	Local	Cent.	Naive	SPCF	CI
RMS (m)	178.9	134.8	2502.2	163.3	170.3

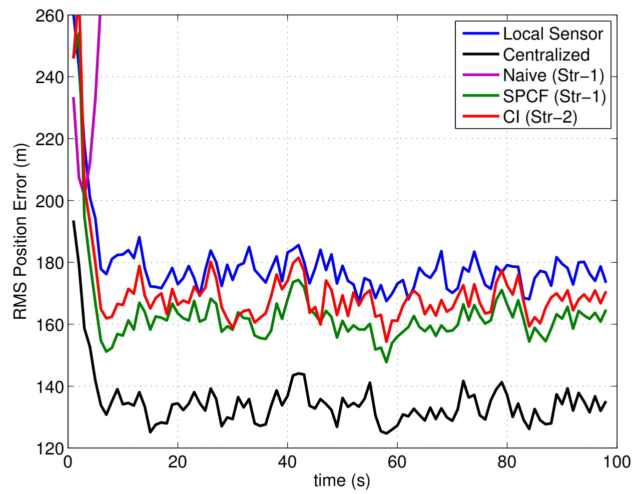

Fig. 3. RMS position errors for the local agent along with centralized fusion, SPCF, Naive and CI approaches.

Figure 3 shows the RMS position errors of the local agent along with those of the centralized fusion, Naive, SPCF and CI methodologies. Table I lists the time average of the RMS position errors illustrated in Figure 3. It is seen that Naive approach diverges at the beginning of the scenario. This is expected due the fact that the Naive approach does not compensate for correlated information in the fusion operation. Due to the feedback in the fusion operation, there is a constant accumulation of correlated information in the local filter estimates which makes the local IMM filters using the Naive approach inconsistent immediately leading to divergence. The SPCF approach yields the closest results to the centralized approach which is optimal. The CI fusion performance can occasionally get very close to the results obtained by the local agent. The use of Gaussian mixtures in SPCF instead of the moment-matched single Gaussian densities in CI brings an improvement of around 7 meters into the fusion operation.

VI. Conclusions

In this study, a distributed track fusion methodology is proposed for tracks represented by Gaussian mixtures. The unknown correlation between the tracks is accounted for by adopting the correlation independent methodology of Chernoff fusion to avoid double counting a.k.a. rumor propagation. For Chernoff fusion a method recently proposed in the literature is adapted to the IMM framework. The results show that making track fusion with Gaussian mixtures instead of moment-matched single Gaussian densities can yield a moderate advantage in a 2D tracking scenario.

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