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Comparison of Augmented State Track Fusion Methods for Non-full-rate Communication

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Abstract - For linear-Gaussian non-deterministic dynamics, that is, systems with non-zero process noise, it is well known that tracklet fusion based on equivalent measurement is optimal only for full communication rate, i.e., if the local posterior probabilities or estimates are communicated and fused after each observation and update time. Despite this constraint, tracklet fusion has become very popular because it performs well in many real world problems even when communication is not at full rate. By including local state estimates at multiple times, augmented state (AS) tracklet fusion computes the optimal global estimate despite this communication constraint. A similar method with this property is distributed accumulated state density (DASD) fusion, which computes decorrelated local pseudo estimates by means of a relaxed evolution model. This paper compares these two methods by examining their underlying principles. Numerical results compare their performance and also with that of a centralized Kalman filter. The results show that they have many properties such as the estimation accuracy in common despite their different derivations.

Keywords: Track fusion, tracklet fusion, augmented state, accumulated state density, centralized Kalman filter, distributed Kalman filter.

1 Introduction

Multiple sensors can provide better estimation performance because each additional sensor contributes more information. Centralized fusion of the measurements from all sensors at a single node is theoretically optimal because the information in the measurements is not degraded by any intermediate processing. However, centralized fusion is not always feasible when communication bandwidth is limited. Thus many systems use a distributed estimation or fusion architecture where the individual sensors process their measurements to generate local estimates and error covariances, which are then sent to a fusion node to be combined into global state estimates and estimation error covariances.

In most distributed estimation or fusion systems, the local estimates that are fused are optimal state estimates given the local sensor measurements and computed using only local sensor model information [1]. Since the local estimation errors are not independent, fusion algorithms have to address this cross sensor dependence or correlation. Some fusion algorithms address dependences that can be characterized by cross-sensor covariances of the local estimation errors. Examples include maximum likelihood estimate [2] [3], best linear unbiased estimate (BLUE) [4] [5], and minimum variance (MV) estimate [6] [7]. Since cross-covariances are used in fusion, the local sensor models have to be communicated to the fusion site along with the local estimates. Furthermore, the fused estimate may not be globally optimal because it is only the best estimate given the local estimates characterized by local and cross-sensor estimation error covariances.

A popular approach for distributed fusion is tracklet fusion or tracklet, equivalent-measurement, or channel-filter fusion, [8]–[10]. A tracklet uses the current and predicted local estimates to find the new information received since the last fusion time. However, tracklet fusion does not generate the optimal global estimate when process noise is present and the fusion rate is lower than the sensor observation rate. Optimality can be regained if the state is augmented to be the entire state trajectory for all observation times since the most recent fusion [11]–[14]. In particular, [14] shows that the centralized Kalman filter (CKF) estimate can be obtained even when the augmented state includes only the states of the most recent two or three time instants. This augmented state is equivalent to the accumulated state density (ASD) used for exact memoryless track fusion [15]–[17].

Even though augmented state (AS) tracklet fusion and distributed accumulated state density (DASD) fusion are both optimal in computing the CKF estimate, the local computations are different. In particular, the augmented state estimate is the best estimate given the local measurements. On the other hand, the local ASD estimate is non-optimal on a local perspective because the prediction step uses a relaxed evolution model.

The difference in the equations is due to the different forms of fusion equations. The convex combination fusion equation in DASD requires a relaxed evolution model in local processing. Thus, the local estimate is not the best estimates given the local data. On the other hand, augmented tracklet fusion is a natural generalization of the usual tracklet fusion by using augmented state to restore conditional independence of the tracklet measurements given the state. Thus, the local estimates are equivalent to the optimal estimates given the local data.

This paper compares the two fusion methods that use augmented state estimates. We discuss the difference in the derivations that result in different local processing and different global fusion algorithms. We also compare their performance by means of simulation experiments. Our goal is to understand the similarities and differences in the two methods. This may allow us to develop a method that has the best features of both approaches.

The rest of this paper is structured as follows. Section II presents the track fusion problem. Section III reviews the distributed accumulated state density (DASD) filter fusion. Section IV provides a similar review of augmented state fusion. Section V describes the simulation experiment, and Section VI presents the results. Section VII contains the conclusions.

2 Track Fusion Problem

This section presents the target and sensor models for the tracking problem and a track fusion architecture. Even though we call the problem track fusion, the results are applicable to general distributed estimation problems when the state is not that of a moving target.

2.1 State and Measurement Models

The state to be estimated is modeled by the linear system

$$x_{k+1} = F_k x_k + G_k w_k \tag{1}$$

where $x_k \in R^n$ is the state at time t_k with $k = 0,1,2,...,$ F_k and G_k are matrices representing the system dynamics, and w_k is a zero-mean Gaussian white random process with covariance Q_k.

We assume that the state is observed by S sensors with

$$z_k^s = H_k^s x_k + v_k^s \tag{2}$$

for $s = 1,...,S$, and $k = 1,2,....$, where z_k^s is the measurement of the s-th sensor at time t_k, H_k^s is the measurement matrix, and v_k^s is a zero-mean white noise process with covariance R_k^s. The measurement noises are assumed to be independent with each other and the process noise. The initial state x_0 is independent of the noises with mean $\bar{x}_0$ and covariance $\bar{P}_0$. For simplicity, we assume synchronous observations by all sensors but the results can be generalized to non-synchronous measurements with appropriate modifications of the algorithms.

2.2 Fusion Architecture

Define the cumulative measurements of sensor s to be $Z_l^s = (z_j^s)_{j=1}^l$. Let $x_{k|l}^s$ and $P_{k|l}^s$ be the optimal (local) estimate of x_k and its error covariance given Z_l^s. Local Kalman filtering at sensor s consists of the following prediction and update steps.

Prediction

$$x_{k|k-1}^s = F_{k-1} x_{k-1|k-1}^s \tag{3}$$

$$P_{k|k-1}^s = F_{k-1} P_{k|k-1}^s F_{k-1}{}^T + G_{k-1} Q_{k-1} G_{k-1}{}^T \tag{4}$$

Update

$$(P_{k|k}^s)^{-1} x_{k|k}^s = (P_{k|k-1}^s)^{-1} x_{k|k-1}^s + i_k^s \tag{5}$$

$$(P_{k|k}^s)^{-1} = (P_{k|k-1}^s)^{-1} + I_k^s \tag{6}$$

with initial conditions $x_{0|0}^s$ and $P_{0|0}^s$, $i_k^s \triangleq (H_k^s)^T (R_k^s)^{-1} z_k^s$, and $I_k^s \triangleq (H_k^s)^T (R_k^s)^{-1} H_k^s$. Each local processor only knows its own sensor model.

At the fusion time t_K, each local processor communicates some local estimate and its error covariance to the fusion site. When fusion takes place after each sensor observation, i.e., $K = \bar{K} + 1$, where $t_{\bar{K}}$ is the last fusion time, then communicating the local estimates $x_{K|K}^s$ and error covariances $P_{K|K}^s$ is sufficient for reconstruction of the optimal CKF estimate after fusion. This is the standard tracklet fusion method. When $K - \bar{K} > 1$, the local processors have to communicate augmented state estimates for the fusion site to reconstruct the CKF estimates. The following two sections discuss two different methods of computing the local augmented state estimates and the fusion algorithms.

In discussing augmented state estimation, it is convenient to define $Z_{k:l}^s \triangleq (z_j^s)_{j=l}^k$ as the sensor s measurements from t_l to t_k, $Z_{k:l} \triangleq (Z_{k:l}^s)_{s=1}^S$ as all measurements from t_l to t_k, and $Z_k \triangleq (Z_k^s)_{s=1}^S$ as all cumulative measurements.

3 Distributed Accumulated State Density (DASD)

The DASD filter is a distributed, memoryless filter [17], which means that the fusion center does not fuse the received data to update a central track but combines them without using the central track. The result is the global estimate, which is not required for future fusion steps. Similar to the Distributed Kalman Filter [18], the idea of the DASD is to obtain a product representation of the fused posterior density. In contrast to the DKF, the DASD requires larger communication bandwidths as the transmitted parameters are rather high dimensional. However, in contrast to the DKF, the local ASD computation does not require knowledge of all measurement models except for the number of sensors.

3.1 Underlying Principle of Approach

Let $X_{k:l} = [x_k^T, \ldots\ldots, x_l^T]^T$ be the augmented state from t_l to t_k with $k > l$. We present the approach for the general problem of computing $p(X_{k:l} \mid Z_k)$, the conditional probability density of $X_{k:l}$ given Z_k in terms of some pseudo conditional probability densities $\tilde{p}(X_{k:l} \mid Z_k^s)$. From Bayes rule, we have

$$p(X_{k:l} \mid Z_k) = C^{-1} p(Z_{k:l} \mid X_{k:l}) p(X_{k:l} \mid Z_{l-1}) \tag{7}$$

where C is a normalizing constant. Since the sensor measurements $Z_{k:l}^s$ for the S sensors are conditionally independent given the augmented state $X_{k:l}$, (7) becomes

$$p(X_{k:l} \mid Z_k) = C^{-1} \left(\prod_{s=1}^{S} p(Z_{k:l}^s \mid X_{k:l}) \right) p(X_{k:l} \mid Z_{l-1}) \tag{8}$$

Define $\tilde{p}(X_{k:l} \mid Z_{l-1})$ so that

$$p(X_{k:l} \mid Z_{l-1}) = \left(\tilde{p}(X_{k:l} \mid Z_{l-1}) \right)^S \tag{9}$$

Then (8) becomes

$$p(X_{k:l} \mid Z_k) = C^{-1} \prod_{s=1}^{S} \tilde{p}(X_{k:l} \mid Z_k^s) \tag{10}$$

with

$$\tilde{p}(X_{k:l} \mid Z_k^s) = C_s^{-1} p(Z_{k:l}^s \mid X_{k:l}) \tilde{p}(X_{k:l} \mid Z_{l-1}) \tag{11}$$

The probability $\tilde{p}(X_{k:l} \mid Z_k^s)$ is not necessarily the true conditional probability of $X_{k:l}$ given the local measurements Z_k^s because (11) uses $\tilde{p}(X_{k:l} \mid Z_{l-1})$ instead of $p(X_{k:l} \mid Z_{l-1})$.

When the probability densities are Gaussian, (10) becomes a convex combinations of the local estimates and information matrices, and (11) becomes the local processing equations for ASD. Equation (9) leads to the relaxed evolution model.

3.2 Local Processing and Fusion Equations

The derivations for the following equations can be found in [17].

3.2.1 Local processing

Let $X_{k:l|k}^s$ and $\mathbf{P}_{k:l|k}^s$ be the mean and covariance of $\tilde{p}(X_{k:l} \mid Z_k^s)$, and $X_{k:l|k-1}^s$ and $\mathbf{P}_{k:l|k-1}^s$ be the mean and covariance of $\tilde{p}(X_{k:l} \mid Z_{k-1}^s)$. These are not the local optimal estimates and their error covariances since from (11), the DASD filter only computes pseudo estimates at the sensor platforms. Then local processing consists of the following prediction and update steps.

Prediction

$$X_{k:l|k-1}^s = \begin{bmatrix} F_{k-1} x_{k-1|k-1}^s \\ X_{k-1:l|k-1}^s \end{bmatrix} \tag{12}$$

$$\mathbf{P}_{k:l|k-1}^s = \begin{bmatrix} F_{k-1} P_{k-1|k-1}^s F_{k-1}^T + S G_{k-1} Q_{k-1} G_{k-1}^T & \mathbf{F}_{k-1} \mathbf{P}_{k-1:l|k-1}^s \\ \mathbf{P}_{k-1:l|k-1}^s \mathbf{F}_{k-1}^T & \mathbf{P}_{k-1:l|k-1}^s \end{bmatrix} \tag{13}$$

where $\mathbf{F}_{k-1} = \begin{bmatrix} F_{k-1} & 0_{n \times n(k-2-l)} \end{bmatrix}$ and the initial conditions are $X_{l|l}^s = x_{l|l}^s$ and $\mathbf{P}_{l|l}^s = S P_{l|l}^s$.

Update

For local information parameters representing the measurement z_k^s, the update formulas are given by

$$(\mathbf{P}_{k:l|k}^s)^{-1} X_{k:l|k}^s = (\mathbf{P}_{k:l|k-1}^s)^{-1} X_{k:l|k-1}^s + J_k i_k^s \tag{14}$$

$$(\mathbf{P}_{k:l|k}^s)^{-1} = (\mathbf{P}_{k:l|k-1}^s)^{-1} + J_k I_k^s J_k^T \tag{15}$$

where $J_k = [I_n, 0_{n \times n(k-l-1)}]^T$ is a $n(k-l) \times n$ matrix that selects the x_k in X_k to generate the measurement z_k^s.

3.2.2 Communication

At the fusion time t_K, each local node sends the pseudo estimate $X_{K:l|K}^s$ and pseudo error covariance $\mathbf{P}_{K:l|K}^s$ to the fusion node.

3.2.3 Fusion processing

The global estimate $X_{K:l|K}$ and error covariance $\mathbf{P}_{K:l|K}$ are obtained by the following fusion equations

$$\mathbf{P}_{K:l|K}^{-1} X_{K:l|K} = \sum_{s=1}^{S} (\mathbf{P}_{K:l|K}^s)^{-1} X_{K:l|K}^s \tag{16}$$

$$\mathbf{P}_{K:l|K}^{-1} = \sum_{s=1}^{S} (\mathbf{P}_{K:l|K}^s)^{-1} \tag{17}$$

4 Augmented State Track Fusion

When the communication (and fusion) rate is lower than the observation rate, the new measurements $Z_{K:\bar{K}+1}^s$ collected by the sensors since the last fusion time are no longer conditionally independent given the state at a single time because of the common process noise. Thus equivalent measurement or tracklet fusion no longer produces the optimal global estimate. However, the measurements $Z_{K:\bar{K}+1}^s$

for the S sensors are conditionally independent given the augmented state $X_{K:\bar{K}+1}$. This conditional independence motivates the augmented state fusion algorithm, which is similar but not the same as the DASD fusion algorithm presented in Section III.

4.1 Underlying Principle of Approach

Let t_K and $t_{\bar{K}}$ be the current and last fusion times. Then

$$p(X_{K:\bar{K}+1} \mid Z_K) = C^{-1} p(Z_{K:\bar{K}+1} \mid X_{K:\bar{K}+1}) p(X_{K:\bar{K}+1} \mid Z_{\bar{K}}) \quad (18)$$

where C is a normalizing constant. Since the sensor measurements $Z^s_{K:\bar{K}+1}$ for the S sensors are conditionally independent give the augmented state $X_{K:\bar{K}+1}$, (18) becomes

$$p(X_{K:\bar{K}+1} \mid Z_K) = C^{-1} \left(\prod_{s=1}^{S} p(Z^s_{K:\bar{K}+1} \mid X_{K:\bar{K}+1}) \right) p(X_{K:\bar{K}+1} \mid Z_{\bar{K}}) \quad (19)$$

The likelihood $p(Z^s_{K:\bar{K}+1} \mid X_{K:\bar{K}+1})$ represents the new information on the augmented state received by sensor s since $t_{\bar{K}}$ and can be computed from the local conditional probabilities by

$$p(Z^s_{K:\bar{K}+1} \mid X_{K:\bar{K}+1}) = C_s\, p(X_{K:\bar{K}+1} \mid Z^s_K) / p(X_{K:\bar{K}+1} \mid Z^s_{\bar{K}}) \quad (20)$$

Equation (19) is the fusion equation that combines the conditional probability density at the last fusion time with the local likelihoods of the individual sensors. These likelihoods represent the new information in the tracklet of measurements received since the last fusion time and can be computed from true local conditional probabilities densities. When the probability densities are Gaussian, (19) and (20) become the augmented state (AS) tracklet fusion equations.

The difference between DADS and AS tracklet fusion is that DASD forces the fused conditional probability density into a product of local conditional probability densities (10) when conditional dependence does not support the factorization. On the other hand, AS fusion decomposes the fused conditional probability into products of local likelihoods and the conditional probability after the last fusion.

4.2 Local Processing and Fusion Equations

4.2.1 Local processing

The local processor performs the prediction and update functions to estimate the local state and its error covariance. In addition, it also computes $X^s_{k|k} = [(x^s_{k|k})^T, .., (x^s_{\bar{K}+1|k})^T]^T$, the estimate of the augmented state vector $X_{k:\bar{K}+1}$ given the measurements Z^s_k, and $\mathbf{P}^s_{k|k}$, its error covariance by the following equations.

Prediction

$$X^s_{k|k-1} = \begin{bmatrix} F_{k-1} x^s_{k-1|k-1} \\ X^s_{k-1|k-1} \end{bmatrix} \quad (21)$$

$$\mathbf{P}^s_{k|k-1} = \begin{bmatrix} P^s_{k|k-1} & \mathbf{F}_{k-1}\mathbf{P}^s_{k-1|k-1} \\ \mathbf{P}^s_{k-1|k-1}\mathbf{F}^T_{k-1} & \mathbf{P}^s_{k-1|k-1} \end{bmatrix} \quad (22)$$

where $\mathbf{F}_{k-1} = \left[F_{k-1} \; 0_{n\times n(k-2-\bar{K})} \right]$ and the initial conditions are $X^s_{\bar{K}|\bar{K}} = x^s_{\bar{K}|\bar{K}}$ and $\mathbf{P}^s_{\bar{K}|\bar{K}} = P^s_{\bar{K}|\bar{K}}$.

Update

$$(\mathbf{P}^s_{k|k})^{-1} X^s_{k|k} = (\mathbf{P}^s_{k|k-1})^{-1} X^s_{k|k-1} + J_k i^s_k \quad (23)$$

$$(\mathbf{P}^s_{k|k})^{-1} = (\mathbf{P}^s_{k|k-1})^{-1} + J_k I^s_k J^T_k \quad (24)$$

where $J_k = [I_n, 0_{n\times n(k-\bar{K}-1)}]^T$ is a $n(k-\bar{K})\times n$ matrix that selects the x_k in X_k to generate the measurement z^s_k.

Note that local processing only uses the local sensor model. This is different from DASD local processing that uses a relaxed evolution model with explicit dependence on the number of sensors. These local estimates and error covariances are optimal or exact given the local measurements.

4.2.2 Communication

At the fusion time t_K, each local node s sends its augmented state estimate $X^s_{K|K}$ ($n(K-\bar{K})$ vector) and error covariance $\mathbf{P}^s_{K|K}$ ($n(K-\bar{K})\times n(K-\bar{K})$ matrix) to the fusion node.

4.2.3 Fusion processing

The fusion node computes recursively the predictions $X^s_{K|\bar{K}}$ and $\mathbf{P}^s_{K|\bar{K}}$ for sensor s using the following equations

$$X^s_{k|\bar{K}} = \begin{bmatrix} F_{k-1} x^s_{k-1|\bar{K}} \\ X^s_{k-1|\bar{K}} \end{bmatrix} \quad (25)$$

$$\mathbf{P}^s_{k|\bar{K}} = \begin{bmatrix} F_{k-1} P^s_{k-1|\bar{K}} F^T_{k-1} + G_{k-1} Q_{k-1} G^T_{k-1} & \mathbf{F}_{k-1}\mathbf{P}^s_{k-1|\bar{K}} \\ \mathbf{P}^s_{k-1|\bar{K}}\mathbf{F}^T_{k-1} & \mathbf{P}^s_{k-1|\bar{K}} \end{bmatrix} \quad (26)$$

with initial conditions $X^s_{\bar{K}|\bar{K}}$ and $\mathbf{P}^s_{\bar{K}|\bar{K}}$ received at the last communication time. It also computes $X_{K|\bar{K}}$ and $\mathbf{P}_{K|\bar{K}}$ from the last fused estimate $X_{\bar{K}|\bar{K}}$ and error covariance $\mathbf{P}_{\bar{K}|\bar{K}}$ using similar equations.

The global estimate $X_{K|K} = [x_{K|K}^T, .., x_{\bar{K}+1|K}^T]^T$ of the augmented state and its error covariance $\mathbf{P}_{k|k}$ are given by

$$\mathbf{P}_{K|K}^{-1} X_{K|K} = \mathbf{P}_{K|\bar{K}}^{-1} X_{K|\bar{K}} + \sum_{s=1}^{S} \left((\mathbf{P}_{K|K}^{s})^{-1} X_{K|K}^{s} - (\mathbf{P}_{K|\bar{K}}^{s})^{-1} X_{K|\bar{K}}^{s} \right) \quad (27)$$

$$\mathbf{P}_{K|K}^{-1} = \mathbf{P}_{K|\bar{K}}^{-1} + \sum_{s=1}^{S} \left((\mathbf{P}_{K|K}^{s})^{-1} - (\mathbf{P}_{K|\bar{K}}^{s})^{-1} \right) \quad (28)$$

The global estimate $x_{K|K}$ and error covariance $P_{K|K}$ can be extracted from the augmented estimate and error covariance.

The augmented state fusion algorithm computes the optimal global estimate when the number of states equals the number of observations for all sensors between fusion times (note the difficulty for non-synchronous observations). Reducing the length or dimension of the augmented state produces a suboptimal global estimate but requires less communication bandwidth. It is shown in [14] that augmented state with a very short length such as 2 has performance similar to full augmented state under some conditions.

5 Simulation Experiment

We use the following simulation to compare the performance of the fusion algorithms.

5.1 Target Model

The target moves according to the 2 dimensional Ornstein-Uhlenbeck model used in [14] with the process noise intensity $q = 2\beta\sigma_{VEL}^2$ and

$$F_k \triangleq \exp\left(A\Delta t\right);\ A \triangleq \begin{bmatrix} 0 & I_2 \\ 0 & -\beta I_2 \end{bmatrix} \quad (29)$$

$$G_k Q_k G_k^T \triangleq \int_0^{\Delta t} e^{A\tau} \begin{bmatrix} 0 & 0 \\ 0 & qI_2 \end{bmatrix} e^{A^T\tau} d\tau \quad (30)$$

The initial condition at t_0 is $P_0 = \text{diag}[\sigma_{POS}^2 I_2, \sigma_{VEL}^2 I_2]$.

5.2 Measurement Model

Five sensors observe the position of the target, i.e., $H_k^s = [I_2\ , 0_{2\times 2}]$ for $s = 1, 2, ..., 5$ at time t_k with $t_{k+1} - t_k = \Delta t$. Communication and fusion take place at times $t_{K_1} < t_{K_2} < \text{L}$, with $K_{\ell+1} - K_\ell \triangleq \Delta K$, when each sensor sends its local estimate and error covariance for processing.

The nominal simulation parameters are: $\Delta t = 1$, $\sigma_{POS} = 10$, $\sigma_{VEL}^2 = 100$, $\beta = 0$, $q = 1.0$, and the covariance of the measurement noise of all sensors is given by

$$R_s = \begin{pmatrix} 100 & 10 \\ 10 & 100 \end{pmatrix}.$$

The total number of scans is 50.

6 Simulation Results

We evaluate the performance of the AS tracklet fusion, the distributed ASD, and a centralized Kalman filter (CKF). The scope of the evaluation is on communication issues. Thus, we consider six different scenarios in which the communication or other parameters differ. The figures plot the root mean square position error (RMSE) of 100 Monte Carlo simulations as a function of time.

6.1 Scenario 1. Perfect Communication.

The term "perfect communication" refers to a high bandwidth setup where all sensors are able to transmit their local data at each instant of time. Thus in Scenario 1, all sensors transmit their measurements at each time step to the fusion center. The fusion center then computes the global estimate by the three fusion methods.

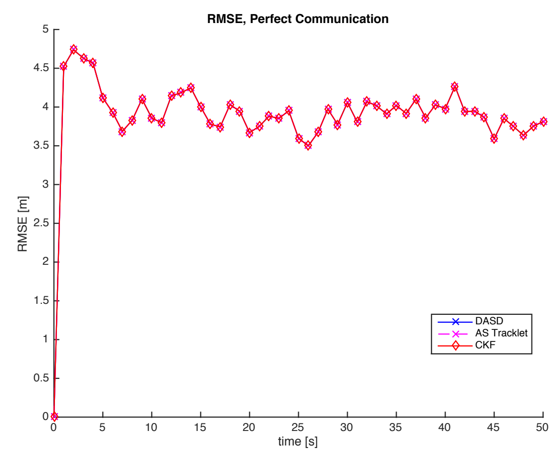

Fig. 1: Performance for perfect communication

As expected, Fig. 1 shows that CKF, DASD, and AS tracklet fusion produce the same results.

6.2 Scenario 2. Batch Communication.

In Scenario 2, all sensors keep their local data until the very last time step at 50s. Then, they transmit the batch containing measurements, ASD estimates, or augmented state estimates to the fusion center.

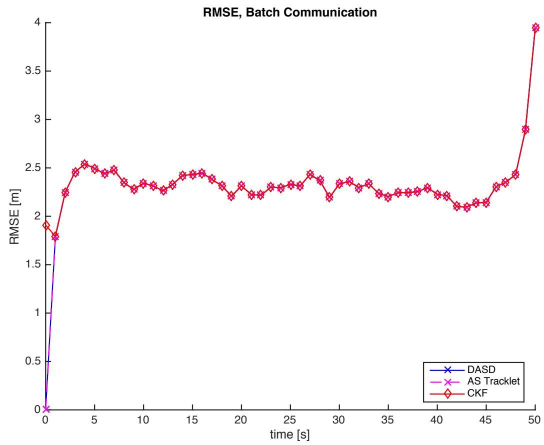

Fig. 2: Performance for batch communication

In Fig. 2, the RMSE results are computed from the smoothed estimates at time 50s. Thus they are smaller than those in Fig. 1. CKF, DASD, and AS tracklet fusion all have the same performance.

6.3 Scenario 3. Frequent Communication.

In Scenario 3, all sensors are able to transmit their local data at every n-th time step, where n = 10. The data transmitted refers to the complete lag.

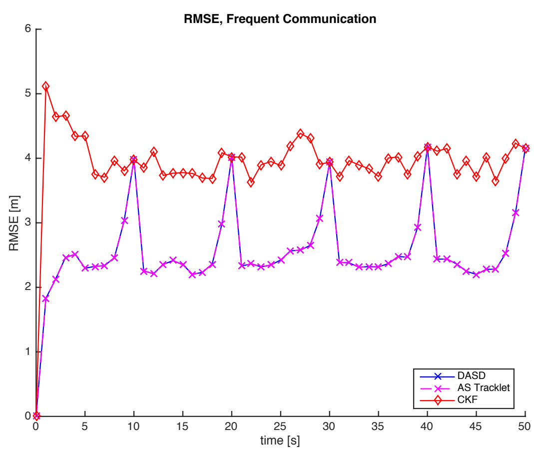

Fig. 3: Performance for frequent communication

The CKF results in Fig. 3 are the filter estimates and the DASD and AS fused estimates are smoothed after each fusion time. Thus CKF has larger RMSE than DASD and AS tracklet fusion estimates between fusion times.

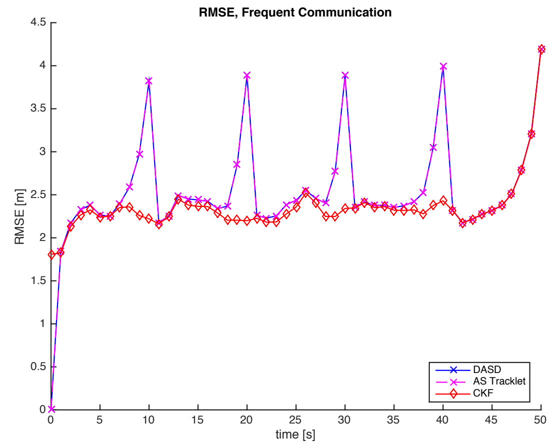

Fig. 4: Performance for frequent communication with smoothing by CKF.

As expected, Figure 4 shows that the smoothed estimates of CKF have smaller RMSE than the fused augmented state estimates.

6.4 Scenario 4. Random Losses.

In this scenario, the sensors try to send the data of a single time step after each update. However, three out of the five sensors fail to transmit successfully, so that only the data of two sensors will be received by the fusion center. The sensor indices of the transmission losses are permutated randomly.

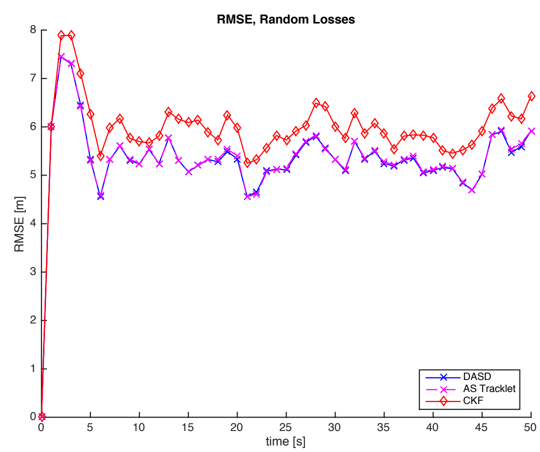

Fig. 5: Performance for communication with random loss

Since the sensor index of a communication failure is random, the augmented state estimate may contain more information than the measurement of a single time step. If some previous transmissions of sensor s have failed, the augmented state still contains information from measurements that are communicated. Thus DASD and AS tracklet fusion perform better than CKF in Fig. 5.

6.5 Scenario 5. Mismatched Number of Sensors.

Since local processing in the DASD algorithm depends on the number of sensors S, this scenario evaluates the effect of a model mismatch in the number of sensors. The real number of sensors is two but both local ASD processing and global fusion assume the number of sensors to be 500. Fig. 6 shows the results when the both local ASD processing and fusion have the same prior. More specifically, the priors are:

Sensor 1 and Sensor 2 ASD for $s = 1,2$:

$$x_0^s = (0,0,10,0)^T,$$

$$P_0^s = \begin{pmatrix} 10I_2 & 0_2 \\ 0_2 & 100I_2 \end{pmatrix} \cdot 500$$

Fusion Center ASD for l = 1,…,500:

$$x_0^l = (0,0,10,0)^T,$$

$$P_0^l = \begin{pmatrix} 10I_2 & 0_2 \\ 0_2 & 100I_2 \end{pmatrix} \cdot 500$$

Fusion is performed only at time 50s. It turns out that the DASD fusion center can compensate this model mismatch when it assumes the same number of sensors as local ASD processing and has the same prior. This can be explained as follows. When the fusion center of the DASD does not receive the ASD of a sensor, it predicts the ADS from the previous transmission. If there was never any transmission, as when the number of assumed sensors is larger than the true number, the prior is used to make the prediction. When both the fusion center and the local ASD processing have the same prior, the sensor number mismatch has no effect on the fusion equation. This can be seen from (8) to (11) of Section III A. The fusion center of the DASD computes the estimate under the assumption that the fictitious sensors never had any detection. Fig. 6 shows that the optimal estimate can be recovered despite the mismatch.

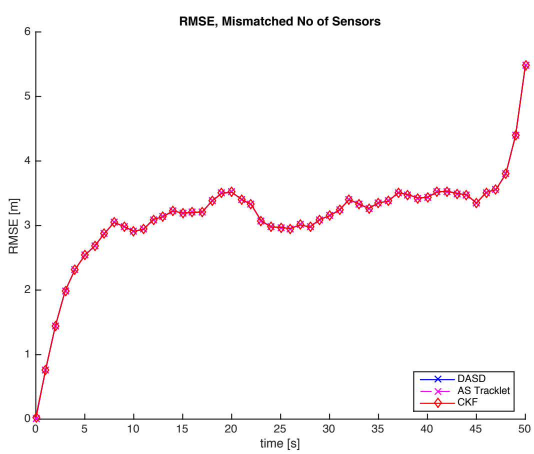

Fig. 6: RMSE for mismatched number of sensors. The DASD fusion center can compensate the mismatch by means of a common prior.

Since the assumption of a common prior is not always satisfied, we consider a different scenario when local ASD processing and fusion assume the same incorrect number of sensors but the fusion center is not aware of an initial prior. As a consequence the fusion center cannot predict the fictitious estimates and only relies on the transmissions.

This is the case of a model match between local ASD processing and fusion processing. Figure 7 shows that DASD fusion has degraded performance as compared to CKF or augmented state tracklet fusion.

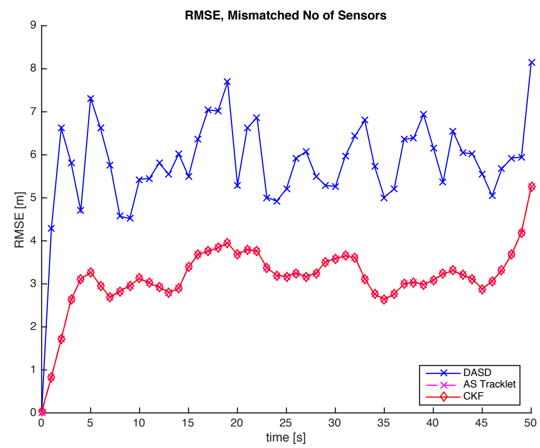

Fig. 7: RMSE for mismatched number of sensors. The DASD fusion center and local processing have different priors.

6.6 Scenario 6. Communication Outage

Communication links in real applications often cannot be assumed to be stable and reliable during the complete tracking process. In this scenario, the communication breaks down for ten time steps after 10s and after 30s again.

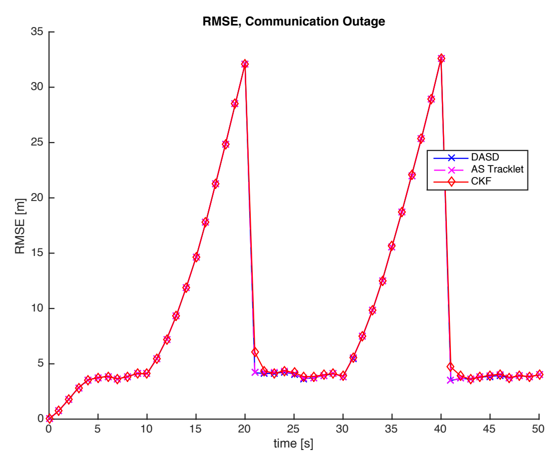

Fig 8: Performance for communication outages.

Fig. 8 shows that all fusion methods have the same performance.

7 Conclusions

We review two fusion methods that use augmented state estimates involving the states at multiple times. Although the local processing and fusion equations are different, both methods compute the optimal CKF estimate when the target dynamics involves non-zero process noise and the fusion rate is lower than the sensor observation rate. The difference in the equations is due to the different derivations required to obtain the fusion equations.

Simulation results show that both methods have good performance as compared to CKF. In particular augmented state tracklet fusion has the same performance as CKF at the fusion times. When there is a mismatch in the number of sensors, DASD fusion computes the optimal estimate when both fusion and local processing assume the same number of sensors and same prior. When they have different priors, performance degradation is observed.

Since DASD and augmented state tracklet fusion has similar performance, and [14] shows that augmented states with very short lengths are adequate for most fusion problems, either methods can be used for track fusion. However, track association performance can be improved significantly by using augmented state estimates of the tracks [19]. Since these are true estimates computed by the local augmented state estimation equations, augmented state tracklet fusion may be a better approach for track fusion than DASD fusion.

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