18th International Conference on Information Fusion
Washington, DC - July 6-9, 2015

Fault Tolerant Fusion Approach Based on Information Theory Applied on GNSS Localization

Joelle Al Hage, Nourdine Aït Tmazirte, Maan E. El Najjar and Denis Pomorski
CRIStAL UMR 9189, University of Lille, Villeneuve d'Ascq, France
joelle.al-hage@ed.univ-lille1.fr

Abstract - In this work, we present an informational framework able to guarantee high integrity of data fusion method. The proposed framework is designed using a bank of Information Filters (IF) and information theory metrics in order to develop a monitoring method which able to detect and exclude faulty data. The decision of faults detection and exclusion are examined through a coherence test elaborated using the Conditional Informational Entropy (CIE) metric. The proposed test is directly connected to the convergence of the information filters. Since the Chi-square test is one of the classical used tests in the Faults Detection and Exclusion (FDE) procedure using Kalman Filter (KF), a relationship between the proposed test and the Chi-square one is established. Through real GPS measurements, the performance of the proposed framework is shown.

Keywords: Information theory, Information filter, GPS system, FDE, filter convergence.

1 Introduction

An autonomous navigation system depends on the high integrity of the state estimation using data fusion method. The characterization of the latter can be reached with an integrity monitoring algorithm which permits to detect and exclude faulty measurements. The addition of this functionality allows making the data fusion method a fault tolerant one.

The various existing integrity monitoring methods differs mainly on the statistic test; example of test applied includes Chi-square distribution, T-distribution, F-distribution [1]. But these approaches suffer from various weakness points. In fact, the choice of threshold is linked to a false alarm probability which is chosen in a heuristic manner inducing a lower integrity level. Moreover, these residual test schemes succeed easily on picking out the sporadic errors, but hardly detect the gradually increasing errors. Another limitation of these kinds of methods appears in case of multiple simultaneous faulty observations.

An informational framework for integrity monitoring is designed in order to integrate a diagnosis level which able to detect and exclude faulty measures before state estimation.

The KF is one of the traditional algorithms used in stochastic state estimation. Information filter, which is the informational form of the KF, has proved to be more efficient for multi-sources state estimation. Instead of using a moment representation with a state vector and a covariance matrix, an IF works with a canonical representation using information form of the co-variance matrix and of the state vector, called respectively the information matrix (fisher matrix) and the information vector. The difference in representation between the KF and the IF make the last more efficient in term of multi sources data fusion and especially for diagnosis [2][3]. Indeed, the IF correction step is a simple addition of predicted Fisher information (respectively information vector) and information contribution of each new observation. This property offers the possibility to develop powerful architectures based on filters synthesis for data fusion and diagnosis.

The proposed diagnosis method is a multi-levels hierarchical method consisting of a bank of IF. The decision of faults detection and exclusion are examined through a test elaborated using the CIE. Integrity monitoring is performed by applying a Non-linear IF (NIF) and coherence tests based on CIE. The IF makes possible to quantify the information innovation of each measurement. Then, a first coherence test is used to detect the appearance of faults and a second one is used to exclude the erroneous measurements. Particularly, the proposed FDE performance is notable in the case of multi-faults situation.

The elaborated tests are directly linked to the global convergence of the IF, hence their importance. These tests will converge exponentially [4][5][6] to a constant value and they will be largely sensitive to the appearance of faults. A theoretical study is elaborated to demonstrate the exponential convergence of these tests, and convergence boundaries are constructed. A comparison of the proposed tests and the Chi-square one is presented and advantages are dressed.

The proposed framework is applied to Global Navigation Satellite System (GNSS) localization in order to guarantee high integrity information. This system uses redundant GNSS measurements to detect faulty satellites. The measurements based on code C/A of L1 signal, enable to calculate the range between the receiver and satellites. The GNSS waves could be affected by various errors having origin from ionospheric delay, tropospheric delay and from multi-path trajectories [7].

This paper is organized as follow: section 2 presents the IF and a study about its convergence. Then, the exponential convergence needed in the detection and the exclusion test is presented in the same section, in addition to the relation between Chi-square and these tests. In section 3, we present the developed informational framework. Section 4 presents an application of this one to GPS system. Finally, conclusion is proposed in section 5.

2 Theoretical study of the information filter

2.1 Introduction to the information filter

Consider a system evolving according to the linear equation:

$$x_{k+1} = F.x_k + w(k) \tag{1}$$

x_k: The state vector at time k
F: The state transition matrix
$w(k)$: The process noise modeled as uncorrelated white noise with $E\{w(i)w^T(j)\}=\delta_{ij}Q(i)$.

The observation is supposed to be non-linear (we deal with the Extended form of KF (EKF) and its informational form the Extended IF (EIF)):

$$z_k = h(x_k) + v(k) \tag{2}$$

z_k:The observation vector,
$h(.)$: The observation model function and H its Jacobien,
$v(k)$: The observation noise modeled as uncorrelated white noise with $E\{v(i)v^T(j)\}=\delta_{ij}R(i)$

The observation is linearized around the predicted state:

$$z_k = h(x_k^*) + \frac{\partial h}{\partial x}|_{x^*}\Delta x + v(k) \tag{3}$$

Where x^* is the nominal reference trajectory and Δx is the error between the real and the nominal trajectory.

Instead of working with the state vector $x_{i/j}$ and covariance matrix $P_{i/j}$ as in KF, the IF deals with the information vector $y_{i/j}$ and the information matrix (Fisher matrix) $Y_{i/j}$ where:

$$y_{i/j} = P_{i/j}^{-1}\, x_{i/j} \tag{4}$$

$$Y_{i/j} = P_{i/j}^{-1} \tag{5}$$

The IF is described in two steps (Prediction/Correction):

Prediction:

$$\hat{Y}_{k/k-1} = [F\, Y_{k-1/k-1}^{-1}\, F^T + Q(k)]^{-1} \tag{6}$$

$$\hat{y}_{k/k-1} = \hat{Y}_{k/k-1}\, x_{k/k-1} \tag{7}$$

Correction:

$$Y_{k/k} = \hat{Y}_{k/k-1} + \sum_{i=1}^{n} I_i(k) \tag{8}$$

$$y_{k/k} = \hat{y}_{k/k-1} + \sum_{i=1}^{n} i_i(k) \tag{9}$$

Where $(I_i(k), i_i(k))$ are the information contributions of observation i:

$$I(k) = \sum_{i=1}^{n} I_i(k) = H^T(k)R^{-1}(k)H(k) \tag{10}$$

$$i(k) = \sum_{i=1}^{n} i_i(k) = H^T(k)R^{-1}(k)\, z_k \tag{11}$$

And n is the number of measurements.

As one can remark in equation (8) and (9), the correction step is a simple summation between all observations; hence the importance of the IF compared to the KF [2] which requires the inversion of a high dimensional matrix. This advantage will appear obviously in the FDE step mainly in the case of multiple faulty measurements.

2.2 The exponential convergence of the information filter

At first some Lemmas should be remembered [5]:

Lemma1: Let A be a positive definite matrix and $\lambda_m(A)$, $\lambda_M(A)$ denote respectively its minimum and its maximum eigen value then:

$$\lambda_m(A)\, I \leq A \leq \lambda_M(A)\, I \tag{12}$$

Lemma 2: let A and B be $n \times n$ real symmetric matrix; if $A \geq B$ then for any $n \times m$ real matrix C, we can write [8]:

$$C^T A C \geq C^T B C \tag{13}$$

Lemma 3: let A and B be positive definite matrix with: $a_1 I \leq A \leq a_2 I$ and $b_1 I \leq B \leq b_2 I$, then:

$$\frac{1}{1+b_2/a_1} A^{-1} \leq (A+B)^{-1} \leq \frac{1}{1+b_1/a_2} A^{-1} \tag{14}$$

In our application and without the loose of generality we may take the following assumptions in order to prove the exponential convergence of the IF:

- The Information matrix is bounded:

$$\alpha I \leq Y_{k/k} \leq \beta I \tag{15}$$

- The process covariance matrix Q is bounded and the process error is additive:

$$q_1\mathrm{I} \leq Q(k) \leq q_2\mathrm{I} \tag{16}$$

$$P_{k/k} = \bar{P}_{k/k} + Q(k) \tag{17}$$

- The model is ideal and the system is observable:

$$z_k = H(k)x_0 \tag{18}$$

In the state domain we define:

$$\tilde{x}_{k/k} = \hat{x}_{k/k} - x_0 \tag{19}$$

$\tilde{x}_{k/k}$ represents the state estimation error and x_0 is the reference value. Then, we obtain [5]:

$$\tilde{x}_{k/k} = \bar{P}_{k/k}\ P_{k-1/k-1}^{-1}\ \tilde{x}_{k-1/k-1} \tag{20}$$

Making the transformation to the information space and using equations (4) and (5), we obtain the informational form of the state estimation error ($\tilde{y}(k/k)$):

$$\tilde{y}_{k/k} = Y_{k/k}\ \bar{Y}_{k/k}^{-1}\ \tilde{y}_{k-1/k-1} \tag{21}$$

Where $\bar{Y}_{k/k}^{-1} = \bar{P}_{k/k}$

Now, we define a Lyapunov function in its informational form:

$$V_k = \tilde{y}_{k/k}^{T} Y_{k/k}^{-1}\ \tilde{y}_{k/k} \tag{22}$$

Replacing (21) in (22):

$$\begin{aligned} V_k &= \tilde{y}_{k-1/k-1}^{T}\ [\bar{Y}_{k/k} + \bar{Y}_{k/k}\ Q(k)\ \bar{Y}_{k/k}]^{-1}\ \tilde{y}_{k-1/k-1} \\ V_k &= \tilde{y}_{k-1/k-1}^{T}\ [Y_{k-1/k-1} + O_k]^{-1}\ \tilde{y}_{k-1/k-1} \end{aligned} \tag{23}$$

With O_k :

$$O_k = H^T(k)R^{-1}(k)H(k) + \bar{Y}_{k/k}\ Q(k)\ \bar{Y}_{k/k} \tag{24}$$

Given R and Q are symmetric and positive matrices using lemma 2 and equation (24), one can see that $O_k > 0$, so we obtain this inequality:

$$Y_{k-1/k-1} + O_k > Y_{k-1/k-1} \tag{25}$$

Using equations (23) and (25) V_k is expressed in function of V_{k-1} as:

$$V_k < \tilde{y}_{k-1/k-1}^{T}\ [Y_{k-1/k-1}]^{-1} \tilde{y}_{k-1/k-1} = V_{k-1} \tag{26}$$

This relation proves that V_k decreases; in other term it proves that the IF is stable (the filter will converge).

In order to prove the *exponential* convergence of the IF, we use Lemma 1 and 3:

Given O_k and $Y_{k-1/k-1}$ are symmetric and positive matrices and using lemma 1, we obtain:

$$\lambda_m(O_k)\ I \ \leq\ O_k \ \leq\ \lambda_M(O_k)\ I \tag{27}$$

$$\lambda_m(Y_{k-1/k-1})\ I \leq Y_{k-1/k-1} \leq \lambda_M(Y_{k-1/k-1})\ I \tag{28}$$

Based on lemma 3, we get:

$$[Y_{k-1/k-1} + O_k]^{-1} \leq \frac{1}{1+\lambda_m(O_k)/\lambda_M(Y_{k-1/k-1})}\ Y_{k-1/k-1}^{-1} \tag{29}$$

This means:

$$V_k \leq \frac{1}{1+\lambda_m(O_k)/\lambda_M(Y_{k-1/k-1})} V_{k-1} \tag{30}$$

By repeating this iterative procedure and by applying it to the lower bound we can find the upper and lower bound of V_k that appear having an exponential form:

$$\frac{1}{(1+\mu_2)^k} V_0 \leq V_k \leq \frac{1}{(1+\mu_1)^k} V_0 \tag{31}$$

With: $\mu_1 = \lambda_m(O_k)/\lambda_M(Y_{k-1/k-1})$

$\mu_2 = \lambda_M(O_k)/\lambda_m(Y_{k-1/k-1})$

$V_0 = \tilde{y}_0^T\ Y_0^{-1}\ \tilde{y}_0$ (from the initialisation step)

Using $Y_{k/k} \leq \beta\mathrm{I}$ we can write $V_k > \beta^{-1}|\tilde{y}_{k/k}|^2$,

Finally equation (32) is reached:

$$\frac{\alpha}{(1+\mu_2)^k} V_0 \leq |\tilde{y}_{k/k}|^2 \leq \frac{\beta}{(1+\mu_1)^k} V_0 \tag{32}$$

This equation shows that the error is bounded and it has an exponential form.

2.3 The relation between the Chi-square and the Information theory

In FDE algorithm, traditionally, we use KF with Chi-square test requiring fixing a false alarm probability [1]. Since we aim to propose a new test based on information theory, a relation and a comparison between the Chi-square and informational quantities are established.

The Generalized form of the Chi-Square (GCS) test could be written given two probability mass functions p and q [9]:

$$D_{\chi^2}(p,q) = \sum \frac{(p(\mathrm{x}) - q(\mathrm{x}))^2}{q(\mathrm{x})} \tag{33}$$

The Mutual Information (MI) $I(x,z)$ is known to be written in the form [10]:

$$I(x,z) = \sum_x \sum_z p(x,z) log \frac{p(x,z)}{p(x)p(z)} \tag{34}$$

In the literature, the relationship between GCS test and information quantities like Shannon entropy, MI, Kullback-Leibler… are addressed in finding an inequality between two statistical quantities like in [9] or by constructing an asymptotical limit as in [11]. In our proposed framework, we aimed to elaborate a statistical test based on information quantities. Note that we use the MI which is a variant form of the CIE. For this reason, an approximated equality between the GCS and the MI test is proposed. This comparison tries to show the equivalence between these two tests.

To see in more details the relation between GCS and MI, we can write [12]:

$$p(x,z) = p(x)p(z) + \delta \text{ with } \sum \delta = 0 \tag{35}$$

Using (35) the MI is expressed as in (36):

$$I(x,z) = \sum_x \sum_z (p(x)p(z) + \delta) \ \log(1 + \frac{\delta}{p(x)p(z)}) \tag{36}$$

Using Taylor decomposition around $\frac{\delta}{p(x)p(z)} = 0$ (meaning independence between x and z), and using $\log(1+x) \approx x - \frac{x^2}{2} + O(x^3)$, we obtain:

$$I(x,z) = \sum_x \sum_z (p(x)p(z) + \delta) \ [\frac{\delta}{p(x)p(z)} - \frac{1}{2}\frac{\delta^2}{p^2(x)p^2(z)} + O(\delta^3)] \tag{37}$$

Performing the calculation and replacing δ by its value $p(x,z) - p(x)p(z)$, the result become as in (38):

$$I(x,z) \approx \sum_x \sum_z \frac{1}{2}\frac{(p(x,z) - p(x)p(z))^2}{p(x)p(z)} \tag{38}$$

In other term:

$$I(x,z) \cong \frac{1}{2} D_{\chi^2}(p(x,z), p(x)p(z)) \tag{39}$$

One can conclude that the *GCS is an approximation of the MI* (under a restricted hypothesis: low correlation between variables x (the state vector) and z (the observation vector)). This proves the most generalized aspect of informational quantities.

3 Fault detection and exclusion using information theory

The MI of a multivariate Gaussian distribution is defined as [13][14];

$$I(x,z) = \frac{1}{2}\ln\frac{|P(x)|}{|P(x/z)|} \tag{40}$$

Where $P(x)$ is the covariance matrix.

Given the demonstration of the exponential convergence of IF in section 2, we define a first coherence test namely the Global Observation Mutual Information (GOMI) in its information form [14] by considering $P(x)$ as the predicted covariance matrix and $P(x/z)$ as the corrected covariance matrix:

$$I(x,z) = \frac{1}{2}\ln\frac{\left|\hat{Y}_{k/k-1} + \sum_{\mathrm{i}=1}^{n} I_{\mathrm{i}}(k)\right|}{\left|\hat{Y}_{k/k-1}\right|} \tag{41}$$

The GOMI is used to detect faulty measurements. It could be also expressed in the form of (42):

$$I(x,z) = \frac{1}{2}\ln\frac{\left|\hat{Y}_{k/k-1} + \sum_{\mathrm{i}=1}^{n} I_{\mathrm{i}}(k)\right|}{\left|Y_{k-1/k-1}\right|} \tag{42}$$

A second test is elaborated in order to exclude the faulty measurements from the fusion procedure using a bank of EIF. The Partial Observation Mutual Information (POMI) is defined as:

$$I_{\mathrm{j}}(x,z) = \frac{1}{2}\ln\frac{\left|\hat{Y}_{k/k-1} + \sum_{\substack{\mathrm{i}=1 \\ \mathrm{i}\neq\mathrm{j}}}^{n} I_{\mathrm{i}}(k)\right|}{\left|Y_{k-1/k-1}\right|} \tag{43}$$

So the proposed FDE algorithm detects the faulty measurement using the GOMI test and then excludes the erroneous measurements using the POMI test. This procedure can be repeated for the detection of multiple simultaneous faulty measurements.

At instant k, $Y_{k-1/k-1}$ is supposed to be out of fault or all detected faulty measurements are excluded.

In the proposed framework, the GOMI and the POMI tests are elaborated using the MI quantity. In section 2, we have shown the equivalence between the MI and GCS. The quantity $|\tilde{y}_{k/k}|^2$ (equation (32)), which is the test of the convergence of the IF, has a form of the Chi-square test (simple form of GCS). So the relation between filters convergence and the elaborated tests (GOMI, POMI) is highlighted.

However, we can remark that some differences exist between the GOMI test and the GCS one. The main one is that the GOMI test deals with covariance matrices (i.e. uncertainties of measurements). In other terms, the GCS test use the mean value of the probability distribution of the state estimation, when the GOMI and POMI tests can benefit from all the information contained in the probability distribution. This difference improves the integrity of the FDE method.

Figure 1 resume the proposed FDE approach explained in this section.

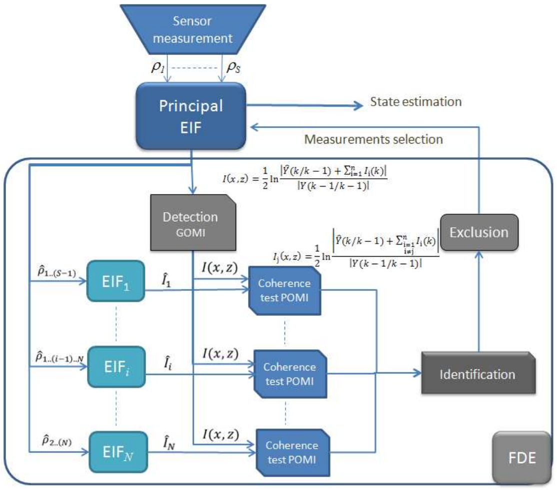

Figure 1. Fault detection and exclusion structure based on the IF approach

4 Application to GPS positioning system

In order to test the performance of the proposed approach, a real experiment is conducted on a GPS constellation. Data acquisition has been carried out with CyCab vehicle. In this work, we use corrected measurements of GPS RTK Thales Sagitta 02 and an open GPS Septentrio polaRx2e@.

In order to test developed algorithms with real data, the "goGPS" software package (http://www.gogps-project.org) is used.

The data acquisition has been carried around CRIStAL Lab of the university Lille 1. The trajectory is about 250m and 530 epochs.

The process model is supposed evolving according to the equation (1) [15].

The state vector is composed of the eight following variables:

$$x_k = [x_k, \dot{x}_k, y_k, \dot{y}_k, z_k, \dot{z}_k, c\delta t, c\dot{\delta t}] \tag{44}$$

[x,y,z], $[\dot{x}, \dot{y}, \dot{z}]$: represent respectively the 3D position and the 3D velocity of the vehicle in the Earth Centered Earth Fixed (ECEF) frame.

$\delta t, \dot{\delta t}$: represent respectively the clock range and the clock drift of the receiver.

The measurement (observartion) represents the pseudo-range between the satellite and receiver.

To test the performance of the proposed architecture, the residual tests are calculated, and the trajectory before and after FDE are shown. The reference trajectory is given in figure 2.

Figure 3 shows the GOMI before FDE. As one can remark, it converges exponentially to the 0 value when no faults. The different surges of information contribution observed in this figure reflect the divergence of the IF due to the occurrence of faulty measurements. By observing figure 4, we notice the appearance of simultaneous faulty satellites in the highlited epochs *136* to *144*,*153* to *190*, *303* to *323*, *351* to *375* and *399* to *416* which is coherent with results obtained is figure 3.

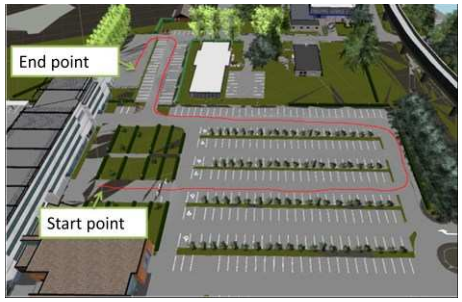

Figure2. Our experimental test area

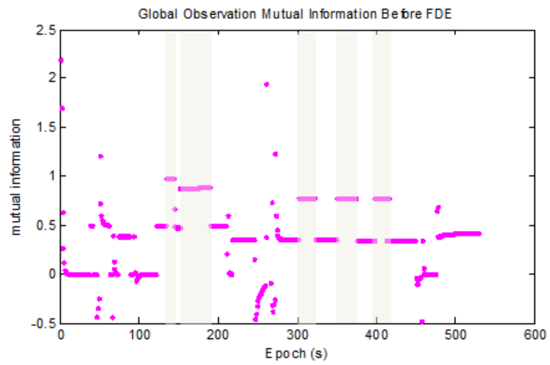

Figure 3. Global Observation Mutual Information Before detection

Figure 4 shows the POMI of different satellites in order to exclude the erroneous measurements. The two faulty satellites 14 and 31 appears obviously in epochs 136 to 144, moreover the simultaneous faulty satellites 4 and 14 are remarkable in the epochs 153 to *190.* Same to the epochs *303* to *323*, *351* to *375* and *399* to *416* the two faulty satellites are 2 and 4.

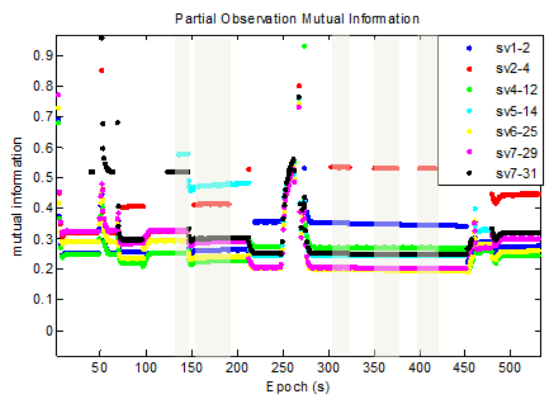

Figure 4. Partial Observation Mutual Information Before detection

Figure 5 shows the GOMI after FDE. Without faulty satellites, the GOMI converge exponentially to zero value. Note for the epochs 263 to 275, the persistence of errors is due to the insufficient in number of the observed satellites caused by trees. Indeed, a minimum of 5 visible satellites is expected to achieve an accurate positioning.

These results are coherent with the theoretical study presented in sections 2 and 3.

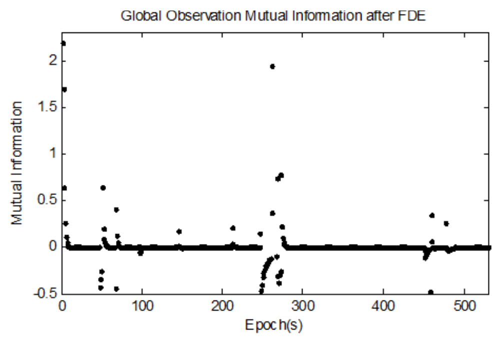

Figure 5. Global Observation Mutual Information After detection

Figure 6 shows the performance of the proposed approach after exclusion of the faulty satellites from the fusion procedure. As one can remark, improvement in the trajectory is noticed especially according to the z axis, representing the ellipsoidal height (h).

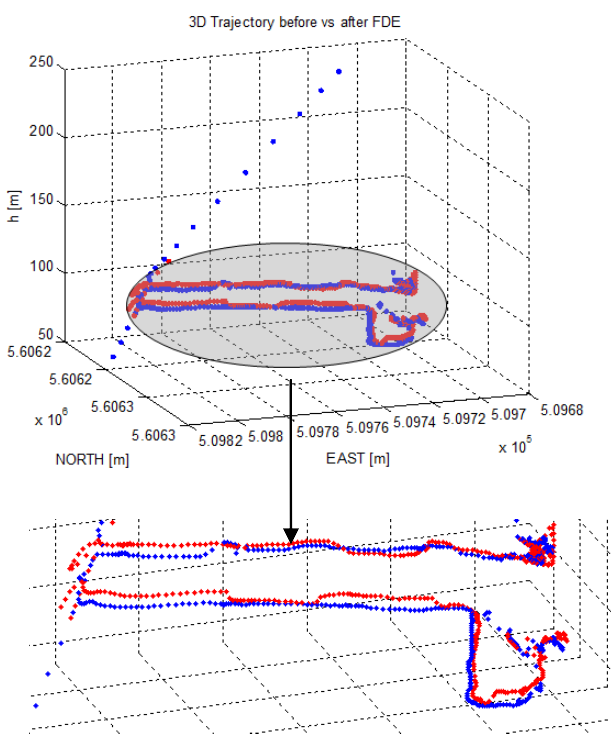

Figure 6 . 3D trajectory before FDE in red vs after FDE in blue

5 Conclusion

In this paper, we propose a fault tolerant data fusion method for GNSS localization based on fault detection and exclusion using Information theory, in order to guarantee high integrity positioning.

After a study of the IF convergence, a coherence test, directly linked to the convergence of the filter is generated. The importance of this method lies by its efficiency in detecting multiple faulty measurements with less computation complexity.

The relation between the GCS and the proposed test shows that GCS is a simplified approximation of the GOMI. Moreover, the proposed approach uses covariance matrix presenting the uncertainty of measurements, instead of working with the mean values as in the GCS test.

The performance of the proposed framework applied to GNSS localization is tested using real data of GPS measurements. This method is able to exclude multiple simultaneous faulty satellites from the fusion procedure with low computational costs.

Future work aims to find a generalized test applied in generalized conditions with application on the supervision of Multi-Robot system.

References

[1] A. Mirabad, N. Mort, and F. Schmid, “A fault tolerant train navigation system using multisensor, multifilter integration techniques,” in *Proceedings of Fusion-98, First International Conference on Multisource-Multisensor Information Fusion*, 1998.

[2] H. Durrant-Whyte, “Introduction to decentralised data fusion,” *Aust. Cent. Field Robot. Univ. Syd. Aust.*, 2004.

[3] M. R. Walter, R. M. Eustice, and J. J. Leonard, “Exactly sparse extended information filters for feature-based SLAM,” *Int. J. Robot. Res.*, vol. 26, no. 4, pp. 335–359, 2007.

[4] A. J. Krener, “The convergence of the extended Kalman filter,” in *Directions in mathematical systems theory and optimization*, Springer, 2003, pp. 173–182.

[5] L. Cao and H. M Schwartz, “Exponential convergence of the Kalman filter based parameter estimation algorithm,” *Int. J. Adapt. Control Signal Process.*, vol. 17, no. 10, pp. 763–783, 2003.

[6] M. B. Rhudy and Y. Gu, “Online Stochastic Convergence Analysis of the Kalman Filter,” *Int. J. Stoch. Anal.*, 2013.

[7] A. Giremus, J.-Y. Tourneret, and V. Calmettes, “A Particle Filtering Approach for Joint Detection/Estimation

of Multipath Effects on GPS Measurements," *IEEE Trans. Signal Process.*, vol. 55, no. 4, pp. 1275–1285, Apr. 2007.

[8] R. A. Horn and C. R. Johnson, "Topics in Matrix Analysis". Cambridge: Cambridge University Press, 1991.

[9] A. Sayyareh, "A new upper bound for Kullback-Leibler divergence," *Appl Math Sci*, vol. 67, pp. 3303–3317, 2011.

[10]T. M. Cover and J. A. Thomas, "Elements of information theory", John Wiley & Sons, New York, NY, Inc., Chapter 9 1991.

[11] M. O Parkash, "Relation Between Information Measures And Chi-Square Statistic," *Int. J. Pure Appl. Math.*, vol. 84, no. 5, pp. 517–524, 2013.

[12] J. Hoey, "The Two-Way Likelihood Ratio (G) Test and Comparison to Two-Way Chi Squared Test," *Stat*, Jun. 2012.

[13] K. Petersen and M. Pedersen, "The matrix cookbook', nov 2012.

[14] N. A. Tmazirte, M. E. E. Najjar, C. Smaili, and D. Pomorski, "Multi-sensor data fusion based on information theory. Application to GNSS positionning and integrity monitoring," in *Information Fusion (FUSION), 2012 15th International Conference on information fusion*, 2012, pp. 743–749.

[15] S. Cooper and H. Durrant-Whyte, "A Kalman filter model for GPS navigation of land vehicles," in *Proceedings of the IEEE/RSJ/GI International Conference on Intelligent Robots and Systems '94. "Advanced Robotic Systems and the Real World", IROS '94*, 1994, vol. 1, pp. 157–163 vol.1.