# A Fault-tolerance Detection Formulation for Distributed Multisensor Systems

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Abstract—The distributed detection fusion formulation (DDFF) in ideal multisensor systems has been obtained. If some local sensors can not work, the detection performance of system may reduce significantly. It is meaningful to design a fault-tolerant detection fusion rule which can guarantee the performance of system whether the system is good or not. A new distributed detection fusion strategy is thus proposed by minimizing the sum of risk at the fusion center and risks at the local sensors. Under this strategy, a new fault-tolerant distributed detection fusion formulation (FT-DDFF) is derived. Some numerical examples show the performance of the proposed formulation. If the system is perfect, the risks of local sensors would decrease compared to the DDFF, and the risk of fusion center would also decrease a little when an appropriate parameter is selected. If the system is partly destroyed, the FT-DDFF would performance better than the DDFF for both of fusion center and available local sensors. Keywords: Bayesian criterion, fusion strategy, fault-tolerant, multisensor systems, parallel network.

#### I. INTRODUCTION

In the past two decades, multisensor information fusion techniques have attracted significant attentions in practice (see [1], [3], [4], [6], [7], [8], [10], [12], [13]), where observations are processed in a distributed manner and decisions or estimates are made at the individual processors, and processed data are then transmitted to a fusion center where the final global decision or estimate is made. A system with multiple distributed sensors has many advantages over one with a single sensor. These include an increase in the capability, reliability, robustness and survivability of the system.

For a parallel distributed multisensor system, the optimal detection fusion formulation was addressed (see, e.g., [2], [10], [11], [12], [13]). In [10], a distributed detection fusion formulation (DDFF) was provided in ideal conditions. It seeks a detection fusion rule for whole system and local information compression rules for all sensors. Some necessary conditions of an optimal fusion rule and optimal decision rules and are derived using a person-by-person optimization (PBPO) methodology. The PBPO methodology is a method to solve team decision problems. While optimizing one team

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member, it is assumed that the other team members have already been designed and remain fixed. The desired PBPO solution consists of fusion rule and decision rules. An iterative algorithm for distributed detection systems with correlated noises was proposed in [12], [13]. It can provide approximate solutions to the necessary conditions for optimum distributed sensor decision rules under a fixed fusion rule. An algorithm to search for an optimal fusion rule and the corresponding optimal local sensor compression rules simultaneously was derived in [5]. In [9], a computationally efficient iterative algorithm to simultaneously/alternately search for a fusion rule and sensor compression rules was proposed. All of the above work are to minimize the risk at the fusion center under Bayesian criterion. And their solutions which coupled with each other were derived when the system is perfect. As the local sensors are just to serve for the fusion center, there would be redundancy in some case. If the fusion center is destroyed, each local sensor would use their own detection rule to make the final decision for the system. The performance of the local sensors with redundancy would be reduced. If some local sensors are destroyed, or the communications between fusion center and some local sensors are cut off, the fusion center has to make a final global decision with the available local decisions. However, the fusion center and available local sensors would not be coupled anymore, and the performance of fusion center would not be guaranteed. Thus, it is meaningful to design fault-tolerant fusion rule and decision rules.

In this paper, for a general parallel distributed multisensor system, we extend the the idea of existing DDFF by employing a new detection fusion strategy. The goal of this paper is to find a fault-tolerant formulation which could guarantee the performance of fusion center and local sensors whenever the system is perfect or partly destroyed. Under Bayesian criterion, the risks at local sensors (local risks) are defined similar to the risk at fusion center (system risk). We propose a new detection fusion strategy to minimize the total risk, i.e., the sum of system risk and local risks rather than to derive a new algorithm. A new fault-tolerant distributed detection fusion formulation (FT-DDFF) is obtained by the PBPO methodology as the DDFF.

The rest of this paper is organized as follows. A statement of the problem and a briefly review of the DDFF are given in Section II. In Section III, we propose a model under a

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new detection fusion strategy and derive the FT-DDFF by PBPO methodology. Some numerical examples are provided in Section IV, and a conclusion is given in Section V.

### **II. PROBLEM STATEMENT**

Consider the detection fusion problem for a distributed detection system with a fusion center and N local sensors, which is shown in Figure 1.



Fig. 1. Distributed detection system.

Let  $H_0$  and  $H_1$  be the two hypotheses with  $P_0$  and  $P_1$  denoting the associated prior probabilities. All the local sensors observe the same phenomenon. The observations of the local sensors are denoted by  $y_i, i = 1, ..., N$ , and their joint conditional densities  $p(y_1, ..., y_N | H_j), j = 0, 1$ , are assumed to be known. And there are no communication among local sensors. Based on its own observation  $y_i$ , the *i*th local sensor makes a local decision  $u_i, i = 1, ..., N$ , i.e.,

$$u_i = \begin{cases} 0, & H_0 \text{ is declared present,} \\ 1, & H_1 \text{ is declared present.} \end{cases}$$

The local decisions are transmitted over bandlimited channels to the fusion center and are combined to yield a global inference. Each local decision  $u_i$  may take value 0 or 1, depending on which hypothesis is decided at the *i*th sensor.

The fusion center yields the global decision  $u_0$  based on the received decision vector consisting of local decisions, i.e.,  $\mathbf{u}^T = (u_1, \ldots, u_N)$ . The global decision  $u_0$  is dependent only on the local decision vector u. The objective of this distributed detection problem is to obtain the optimal set of decision rules  $\Gamma = \{\gamma_0, \gamma_1, \ldots, \gamma_N\}$ , where the fusion rule is denoted by  $\gamma_0$ and the decision rule at the *i*th sensor is denoted by  $\gamma_i$  for  $i = 1, \ldots, N$ . These rules are mapping from the observation space to the decision space, i.e.,

$$u_0 = \gamma_0(u_1, \dots, u_N), \quad u_i = \gamma_i(y_i), \quad i = 1, \dots, N.$$

For i = 1, ..., N, denote the probabilities of false alarm, miss and detection at the *i*th sensor by  $P_{Fi} = P(u_i = 1|H_0), P_{Mi} = P(u_i = 0|H_1)$  and  $P_{Di} = 1 - P_{Mi}$  respectively, and the probabilities of false alarm, miss and detection the fusion center by  $P_F^0, P_M^0$  and  $P_D^0$  respectively. In [10], the DDFF is given by minimizing the system risk. It can be written as

$$R^{0} = \sum_{i=0}^{1} \sum_{j=0}^{1} C_{ij}^{0} P_{j} P(u_{0} = i | H_{j}),$$
(1)

where,  $C_{ij}^0$  is the cost of global decision being  $H_i$  when  $H_j$  is present, and  $P_j$  is the prior probability of hypothesis  $H_j$ , i, j = 0, 1. Using the PBPO methodology, the decision rules and fusion rule can be obtained from the following DDFF:

$$\frac{p(y_{k}|H_{1})}{p(y_{k}|H_{0})} \stackrel{u_{k} = 1}{\gtrless} \\
\frac{p(y_{k}|H_{0})}{u_{k} = 0} \\
\frac{\sum_{u^{k}} \int_{Y^{k}} A(u^{k})C_{F}P(u^{k}|Y^{k})p(Y^{k}|y_{k},H_{0})dY^{k}}{\sum_{u^{k}} \int_{Y^{k}} A(u^{k})C_{D}P(u^{k}|Y^{k})p(Y^{k}|y_{k},H_{1})dY^{k}}, \quad (2) \\
\frac{P(u^{*}|H_{1})}{P(u^{*}|H_{0})} \stackrel{u_{0} = 1}{\underset{u_{0} = 0}{\overset{C_{F}}{C_{D}}}, \quad (3)$$

where  $u^*$  denotes one out of  $2^N$  possible values of u and

$$\begin{split} C_{P}^{0} &= P_{0}(C_{10}^{0} - C_{00}^{0}), \\ C_{D}^{0} &= (1 - P_{0})(C_{01}^{0} - C_{11}^{0}), \\ C^{0} &= C_{01}^{0}(1 - P_{0}) + C_{00}^{0}P_{0}, \\ \mathbf{u}^{k} &= (u_{1}, \dots, u_{k-1}, u_{k+1}, \dots u_{N})^{T}, \\ \mathbf{Y}^{k} &= (y_{1}, \dots, y_{k-1}, y_{k+1}, \dots y_{N})^{T}, \\ \mathbf{u}^{kj} &= (u_{1}, \dots, u_{k-1}, u_{k} = j, u_{k+1}, \dots u_{N})^{T}, j = 0, 1, \\ \mathbf{4}(\mathbf{u}^{k}) &= P(u_{0} = 1 | \mathbf{u}^{k1}) - P(u_{0} = 1 | \mathbf{u}^{k0}). \end{split}$$

Thus, the DDFF consists of N equations of the form (2) and  $2^N$  equations of the form (3). It provides us a way to find a stable solution to the detection fusion problem. However, it also has some shortcomings:

- The formulation relies on the cost C<sup>0</sup><sub>ij</sub> of final decision. It is nearly impossible to get the exact C<sup>0</sup><sub>ij</sub>. Even to get some approximation of C<sup>0</sup><sub>ij</sub> is also a hard work;
- The DDFF which derived by the PBPO methodology can not ensure to provide an optimal solution;
- 3) The decision rules and fusion rule are coupled with each other. If the system was partly destroyed, for example, some local sensors were destroyed, the decision rules of available sensors and fusion rule would not couple with each other anymore. In this case, the performance of fusion center would become very poor;
- 4) The purpose of Bayesian criterion in [10] is to optimize the performance of fusion center. As the optimal performance of fusion center is received, the performance of some local sensors may be ignored. There must be some redundancy for such local sensors. In fact, it is also important to guarantee the performance of local sensors in many practical applications.

## III. THE FAULT-TOLERANT DETECTION FUSION FORMULATION

The local risk of the kth local sensor is expressed as

$$R^{k} = \sum_{i=0}^{1} \sum_{j=0}^{1} C_{ij}^{k} P_{j} P(u_{0} = i | H_{j}),$$

where  $C_{ij}^k$  denotes the cost of the kth local sensor decision being  $H_i$  when  $H_j$  is present. It is easy to see that the local risk  $R^k$  can be expressed as

$$R^k = C_F^k P_F^k - C_D^k P_D^k + C^k,$$

where

$$\begin{split} C_F^k &= P_0(C_{10}^k - C_{00}^k),\\ C_D^k &= (1 - P_0)(C_{01}^k - C_{11}^k),\\ C^k &= C_{01}^k(1 - P_0) + C_{00}^k P_0. \end{split}$$

In order to design a fault-tolerant formulation which could guarantee the performance of both fusion center and local sensors, we will consider the sum of system risk and local risks (the total risk)

$$R = R^0 + \sum_{k=1}^{N} R^k.$$
 (4)

In next, we seek an optimal fusion rule and local decision rules  $\Gamma$  to minimize the total risk R by adopting the PBPO methodology.

Theorem 1: The solution of the problem to minimize the total risk R by the PBPO methodology can be obtained from the following FT-DDFF:

(a) The formulation for fusion rule at the fusion center as the DDFF:

$$\frac{P(\mathbf{u}^*|H_1)}{P(\mathbf{u}^*|H_0)} \underset{u_0 = 0}{\overset{u_0 = 1}{\gtrless}} \frac{C_F}{C_D}.$$
(5)

(b) The formulation for decision rule at the kth sensor for all k = 1, ..., N alternatively:

$$\frac{p(y_{k}|H_{1})}{p(y_{k}|H_{0})} \approx \frac{1}{\geq} \\
\frac{p(y_{k}|H_{0})}{u_{k}} = 0 \\
\frac{C_{F}^{k} + \sum_{\mathbf{u}^{k}} \int_{\mathbf{Y}^{k}} A(\mathbf{u}^{k}) C_{F}^{0} P(\mathbf{u}^{k}|\mathbf{Y}^{k}) p(\mathbf{Y}^{k}|y_{k}, H_{0}) d\mathbf{Y}^{k}}{C_{D}^{k} + \sum_{\mathbf{u}^{k}} \int_{\mathbf{Y}^{k}} A(\mathbf{u}^{k}) C_{D}^{0} P(\mathbf{u}^{k}|\mathbf{Y}^{k}) p(\mathbf{Y}^{k}|y_{k}, H_{1}) d\mathbf{Y}^{k}}.$$
(6)

*Proof:* First, we consider the fusion rule. By the PBPO methodology, we assume that all the local sensors have been designed and fixed. Thus, the local risks  $R^k$ , k = 1, ..., N are also fixed. It means that we just need to minimize the system risk function  $R^0$ . Therefore, one can obtain the fusion rule of DDFF using the same method in [10].

Next, we deal with the decision rule at the kth sensor by the PBPO methodology for k = 1, ..., N. As the fusion rule and all other local sensors have been designed and fixed, we may express  ${\boldsymbol R}$  as

$$\begin{split} R &= R^{0} + \sum_{i=1}^{N} R^{i} \\ &= C^{0} + C^{k} + C_{F}^{0} \sum_{\mathbf{u}} P(u_{0} = 1 | \mathbf{u}) P(\mathbf{u} | H_{0}) \\ &- C_{D}^{0} \sum_{\mathbf{u}} P(u_{0} = 1 | \mathbf{u}) P(\mathbf{u} | H_{1}) \\ &+ C_{F}^{k} P(u_{k} = 1 | H_{0}) - C_{D}^{k} P(u_{0} = 1 | H_{1}) + \sum_{i \neq k}^{N} R^{i}. \end{split}$$

Since  $C^0, C^k$  and  $\sum_{i \neq k}^N R^i$  are constants, we only need to minimize the remaining items:

$$\overline{R} := C_F^0 \sum_{\mathbf{u}} P(u_0 = 1 | \mathbf{u}) P(\mathbf{u} | H_0) + C_F^k P(u_k = 1 | H_0) - C_D^0 \sum_{\mathbf{u}} P(u_0 = 1 | \mathbf{u}) P(\mathbf{u} | H_1) - C_D^k P(u_0 = 1 | H_1).$$

We expand  $\overline{R}$  in terms of the kth local decision  $u_k$  as follows:

$$\overline{R} = \overline{C} + \sum_{\mathbf{u}^k} \left\{ A(\mathbf{u}^k) [C_F^0 P(\mathbf{u}^{k1} | H_0) - C_D^0 P(\mathbf{u}^{k1} | H_1)] \right\} + C_F^k P(u_k = 1 | H_0) - C_D^k P(u_0 = 1 | H_1),$$

where,

$$\overline{C} = \sum_{\mathbf{u}^k} \left\{ P(u_0 = 1 | \mathbf{u}^{k0}) [C_F^0 P(\mathbf{u}^k | H_0) - C_D^0 P(\mathbf{u}^k | H_1)] \right\},\$$

and the conditional density of u is given by

$$P(\mathbf{u}|H_j) = \int_{\mathbf{Y}} P(\mathbf{u}|\mathbf{Y}) P(\mathbf{Y}|H_j) d\mathbf{Y},$$

where  $\mathbf{Y} = (y_1, \dots, y_N)^T$  and  $\int_{\mathbf{Y}} \cdot$  represents a multifold integral over all components of Y. Since the decision of each local sensor depends only on its own observations, then

$$P(\mathbf{u}|\mathbf{Y}) = \prod_{i=1}^{N} P(u_i|y_i)$$

and

$$P(\mathbf{u}^{ki}|\mathbf{Y}) = P(u_k = i|y_k)P(\mathbf{u}^k|\mathbf{Y}^k), \ i = 0, 1$$

From

$$\begin{split} P(\mathbf{u}^{ki}|H_j) &= \int_{\mathbf{Y}} P(\mathbf{u}^{ki}|\mathbf{Y}) P(\mathbf{Y}|H_j) d\mathbf{Y} \\ &= \int_{\mathbf{Y}} P(u_k = i|y_k) P(\mathbf{u}^k|\mathbf{Y}^k) p(\mathbf{Y}|H_j) d\mathbf{Y}. \end{split}$$

for j = 1, 2, we have

$$\begin{aligned} \overline{R} &= \overline{C} + \int_{y_k} P(u_k = 1|y_k) dy_k \\ &\cdot \left\{ \sum_{\mathbf{u}^k} \int_{\mathbf{Y}^k} A(\mathbf{u}^k) P(\mathbf{u}^k | \mathbf{Y}^k) [C_F^0 p(\mathbf{Y}|H_0) - C_D^0 p(\mathbf{Y}|H_1)] d\mathbf{Y}^k \right. \\ &+ \left[ C_F^k p(y_k|H_0) - C_D^k p(y_k|H_1) \right] \right\}. \end{aligned}$$

As all of other decision rules and fusion rule are fixed and  $\overline{C}$  is a constant, we obtain the following decision rule

$$P(u_k = 1 | y_k) = \begin{cases} 0, & \text{if } D(k) \le 0, \\ 1, & \text{otherwise,} \end{cases}$$

where

$$D(k) = C_F^k p(y_k | H_0) - C_D^k p(y_k | H_1) + \sum_{\mathbf{u}^k} \int_{\mathbf{Y}^k} A(\mathbf{u}^k) P(\mathbf{u}^k | \mathbf{Y}^k) [C_F^0 p(\mathbf{Y} | H_0) - C_D^0 p(\mathbf{Y} | H_1)] d\mathbf{Y}^k$$

Noting that

$$P(\mathbf{Y}|H_j) = P(\mathbf{Y}^k|y_k, H_j)p(y_k|H_j),$$

the decision rule at the *k*th detector can be expressed in an alternate form as (6).  $\Box$ 

*Remark 3.1:* The FT-DDFF also has the shortcoming brought by Bayesian criterion. In the next section, when simulating the performance of FT-DDFF, we introduce an weight vector to measure the differences between the system risk and the local risks rather than to find the appropriate cost coefficients  $C_{ij}^k$ .

*Remark 3.2.*<sup>2</sup> The PBPO methodology is adopted to derive the FT-DDFF just as the DDFF. This method could get the system optimization just under some special conditions. In most case, it may just get the suboptimal solution to the problem. And the fusion rule and decision rules of the new detection formulation are also coupled with each other.

*Remark 3.3:* Though there are defaults just listed as above remarks, the FT-DDFF may have some superiorities of fault-tolerance. As the FT-DDFF partly optimize the local risks, the performance of the local sensors may be better than DDFF. Especially if system is partly destroyed, the FT-DDFF would work efficiently. These will be show in Section IV.

#### **IV. NUMERICAL EXAMPLES**

In this section, some numerical simulations are provided for the binary detection fusion problem in a distributed system with three local sensors (N = 3). The performance of the FT-DDFF is evaluated and compared to the DDFF. The observations at local sensors are assumed to be conditionally independent. Then, the local decision rules (6) reduce to the following threshold tests

$$\frac{p(y_k|H_1)}{p(y_k|H_0)} \stackrel{u_k = 1}{\underset{k = 0}{\gtrless}} \frac{p(y_k|H_0)}{u_k = 0} \frac{C_F^k + \sum_{\mathbf{u}^k} A(\mathbf{u}^k) C_F^0 \prod_{i=1, i \neq k}^N P(u_i|H_0)}{C_D^k + \sum_{\mathbf{u}^k} A(\mathbf{u}^k) C_D^0 \prod_{i=1, i \neq k}^N P(u_i|H_1)}.$$
(7)

As mentioned in Remark 3.1, we will not devote to choosing some appropriate cost  $C_{ij}^k$ . Instead, we introduce a weight vector to treat the difference between the system risk and local risks. Just like [10], the special case occurs when we set  $C_{00}^k = C_{11}^k = 0, C_{10}^k = C_{01}^k = 1, k = 0, 1, \dots, N$ , i.e., the costs of the correct decision and mistake decision are set to zero and unity respectively. In this case, the system risk  $R^0$  and the local risk  $R^k$  are just the probabilities of error. Furthermore, we assume that each local detector has equal importance, then the weight vector is (l, 1, 1, 1) where the value of l > 0 reflects the different importance of the system risk and local risks.

Assume that the observation noises at the three sensors follow the Gaussian distribution. Under  $H_0$ , the conditional probability densities at the three detectors are assumed to be identical with mean zero and variance one. Under  $H_1$ , the mean and variance of observation at *j*th sensor are  $m_j$  and unity respectively for j = 1, 2, 3.

## A. The system is perfect

We will compare the performances of both formulations when the system is perfect. As mentioned before, besides the system risk, we also focus on the local risks. The performance of local sensors and fusion center will be evaluated for both formulations. Table I reports the ratios of risks by the FT-DDFF to those by the DDFF:

$$r_i = \frac{R^i \text{ by the FT-DDFF}}{R^i \text{ by the DDFF}}, \ i = 0, 1, 2, 3.$$

In Table I, for different l, we computed all the risk ratios of the fusion center and local sensors when  $m_1 = 1.1, m_2 =$  $1.5, m_3 = 1.8$ ; and  $m_1 = 1.0, m_2 = 1.25, m_3 = 1.5$ . For example, the values 0.9998, 0.9736, 0.8266, 0.8998 in the sixth row mean that both of the system risk and local risks of the FT-DDFF are reduced in average compared to the DDFF. It is seems that not all the values of l can guarantee this advantages. We just set l = 0.5 simply in this paper. There must be an appropriate selection for l. While how to search an optimal value of l in theory will be considered in future.

As l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ , the comparisons of probabilities of error detections for fusion center and the local sensors under different formulations are shown in Figures 2 and 3. Figure 2 shows the system risks for both formulations. Although the system risk of the FT-DDFF is not less than that of the DDFF uniformly, the average system risk is less indeed. Figure 3 shows that the local risks for the FT-DDFF are less than those of the DDFF uniformly.

Figures 4 and 5 show the evaluations of the performance in terms of the receiver operating characteristic (ROC). Figures 4 and 5 show the comparisons of the ROCs of fusion center and local sensors respectively. It is clear that, by the FT-DDFF, the local sensors have the obviously good performance and system has a little improvement.

When l becomes greater, the improvement of performance both for fusion center and local sensors is less. When l is big enough, the performance of the FT-DDFF is almost same as to the DDFF. Figure 6 shows the system risk and local risks for both formulations for l = 15 and  $m_1 = 1.1, m_2 = 1.5, m_3 =$ 1.8. It is obviously that the performances of the FT-DDFF and DDFF are equal. This is because l reflects to the different importance of fusion center to local detectors. The bigger l is, the more important the system is. When l is big enough, we

	$m_1 =$	$1.1, m_2 =$	$= 1.5, m_3$	= 1.8	$m_1 = 1.0, m_2 = 1.25, m_3 = 1.5$			
l	$r^0$	$r^1$	$r^2$	$r^3$	$r^0$	$r^1$	$r^2$	$r^3$
0.1	1.0050	0.9722	0.8244	0.8985	0.9901	0.9477	0.8728	0.9129
0.2	1.0031	0.9725	0.8251	0.8686	0.9885	0.9479	0.8732	0.8811
0.3	1.0017	0.9727	0.8256	0.8689	0.9875	0.9482	0.8737	0.8813
0.4	1.0004	0.9732	0.8263	0.8693	0.9869	0.9491	0.8742	0.8817
0.5	0.9998	0.9736	0.8266	0.8998	0.9859	0.9498	0.8747	0.9142
0.6	0.9982	0.9742	0.8276	0.9005	0.9851	0.9504	0.8756	0.9148
0.7	0.9983	0.9749	0.8285	0.9012	0.9849	0.9512	0.8763	0.9154
0.8	0.9990	0.9757	0.8294	0.9020	0.9851	0.9518	0.8771	0.9160
0.9	0.9993	0.9765	0.8303	0.9029	0.9862	0.9522	0.8779	0.9169
1.0	1.0002	0.9774	0.8313	0.9038	0.9866	0.9531	0.8788	0.9177

 TABLE I

 Average Risk Ratios of Fusion Center and Local Sensors



Fig. 2. Comparison of the system risks for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .

could just neglect the risks of local detectors. At this time, the two formulations are just equal.

From the simulating examples for perfect system, we would find that no matter what value l takes, the performance of local sensors of the FT-DDFF do have some advantages compare to the DDFF. We could also take some appropriate values of l to make sure the performance of fusion center is not inferior to the DDFF. When the value of l becomes large, the improvement becomes less. When the value of l is large enough, the performance of the FT-DDFF is just the same as the DDFF.

### B. The system is partly destroyed

There are many kinds of "partly destruction" for a distributed system. Next, we consider two kinds of destruction: the fusion center is destroyed or some local sensors are destroyed.



Fig. 3. Comparison of the local risks for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .

If the fusion center is destroyed, each local sensor would use their own detection rule to make the final decision for the system. As there is no fusion center, we would just to compare the performance of the local sensors, which has been shown in Subsection IV-A. If some of local sensors are destroyed or the communications between fusion center and some local sensors are cut off, which means that some local sensors are missing, the fusion center has to make a decision with the available local decisions.

Table II shows the average system risk ratios for l = 0.5 and different settings of  $m_j$ , j = 1, 2, 3 when sensor 2 or sensor 3 is missing. All the six kinds of situations listed in the table show that the average risks of the FT-DDFF are less than those of DDFF. Especially, when  $m_1 = 1.1, m_2 = 1.8, m_3 = 1.8$ , the risk ratio is 0.8704 which means the average system risk is reduced by 12.96% compared to the DDFF. This is because, as  $m_2 = m_3 = 1.8$ , the system redundance is bigger and the



Fig. 4. Comparison of detection probabilities at the fusion center for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .



Fig. 5. Comparison of detection probabilities at the local detectors for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .

space of ascension for the DDFF is greater.

It is worth pointing out that we do not consider the situation which sensor 1 is missing. Because under the assumption, sensor 1 plays a role as main sensor which is most important to the system performance. If sensor 1 is missing, the system performance would not be guaranteed. What we consider in this paper is to reduce system redundant and improve performance by adjusting the "secondary sensors".

The improvement of performance for the FT-DDFF shown in Table II could also be revealed in Figures 7 and 8 where l = 0.5 and  $m_1 = 1.1, m_2 = 1.8, m_3 = 1.8$ .

Figure 7 shows the risks for the fusion center and sensors 1 and 2 when sensor 3 is missing. Figure 8 compares the ROCs of fusion center for both formulations when sensor 3 is missing. From Figure 8 we can hardly distinguish the two ROCs with each other, but in Figure 7, it is clearly show that



Fig. 6. Comparison of the system risks and local risks for l = 15 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .

TABLE II Average rates of System risks to DDFF for l=0.5

$m_1$	$m_2$	$m_3$	Sensor 2 is missing	Sensor 3 is missing
1.0	1.5	1.5	0.9612	0.9612
1.0	1.7	1.8	0.8796	0.8867
1.0	1.6	2.2	0.8584	0.9364
1.1	1.5	1.8	0.9625	0.9390
1.1	1.6	1.9	0.9179	0.9169
1.1	1.8	1.8	0.8704	0.8704

the performance of new detection formulation is better. We do not give out the same situation when sensor 2 is missing. Because sensors 2 and 3 are in the same status as  $m_2 = m_3 = 1.8$ . The similar results can be obtained for different  $m_2$  and  $m_3$ . When l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ , the performance evaluations in terms of the risk and ROC are shown in Figures 9, 10, 11 and 12.

Figure 9 shows the comparison of risks for the fusion center and sensors 1 and 2. Figure 10 shows the comparison of ROCs for fusion center when sensor 3 is missing. Figures 11 and 12 show the corresponding results when sensor 2 is missing. Just as shown in Figure 8, the ROCs in Figures 10 and 12 could hardly be distinguished. From Figures 9 and 11, it is also obviously that the performance of the FT-DDFF is better.

From the simulations, where l = 0.5, if the "secondary sensor" is destroyed, the performance of fusion center and local sensors would be both improved. How much improvement will be taken exactly by the new detection formulation relies on the redundant of the missing "secondary sensor". Therefore, some extension of the FT-DDFF has much more fault-tolerance compared to the DDFF.



Fig. 7. Comparison of system risks and local risks for sensors 1 and 2 when sensor 3 is missing for l = 0.5 and  $m_1 = 1.1, m_2 = 1.8, m_3 = 1.8$ .



Fig. 8. Comparison of detection probabilities of the fusion center when sensor 3 is missing for l = 0.5 and  $m_1 = 1.1, m_2 = 1.8, m_3 = 1.8$ .



Fig. 9. Comparison of system risks and local risks for sensors 1 and 2 when sensor 3 is missing for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .



Fig. 10. Comparison of detection probabilities of the fusion center when sensor 3 is missing for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .



Fig. 11. Comparison of system risks and local risks for sensors 1 and 3 when sensor 2 is missing for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .



Fig. 12. Comparison of detection probabilities of the fusion center when sensor 2 is missing for l = 0.5 and  $m_1 = 1.1, m_2 = 1.5, m_3 = 1.8$ .

## V. CONCLUSION

In this paper, we consider the distributed detection fusion formulation having a fault-tolerance capacity with the destroy of the fusion center or some local sensors. A new detection fusion strategy is provided by minimizing the sum of system risk and local risks in terms of Bayesian criterion. We solve this team decision problem by the PBPO methodology. Although the FT-DDFF could not ensure an optimal solution to the original detection fusion problem, it do have some superiorities compared to the DDFF especially for the case of the local sensors having redundance. The numerical examples confirm the above claims and show that the new detection formulation has more fault-tolerance in some certain situations.

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