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Terminative Joint Sequential Object Detection and Tracking Based on Fused Test Statistics

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***Abstract*—Joint object detection and tracking is a powerful approach to significantly improve the detection of extremely weak targets or phenomena in surveillance systems. Since the Kalman filter is an optimal estimator for object tracking problems under certain conditions and the Wald's sequential probability ratio test (SPRT) requires fewer samples in average than the fixed-sample-size procedure when solving object detection problems, it is beneficial to apply the Kalman filter and the Wald's SPRT to design joint object detection and tracking algorithm. However, the Wald's SPRT cannot be rigorously proved to eventually terminate if the observations are dependent. In this paper, a terminative joint sequential detection and tracking approach is proposed by fusing two test statistics: one is derived in our previous work, and the other is based on independent observations obtained by linearly combining a group of adjacent measurements. The proposed approach takes advantage of both statistics in that it is guaranteed to terminate and it requires on the average a small number of measurements. Numerical results show that the average sample number required by the proposed approach is very small under low signal-to-noise ratio conditions and the actual probabilities of errors are smaller than the nominal probabilities of errors.**

I. Introduction

For most surveillance systems, object detection and tracking are two important problems that need to be solved. The goal of object detection is to determine the presence or absence of a target under uncertainty. In detection algorithms, the presence and absence of targets are usually represented by two hypotheses respectively, under which the knowledge of distributions of measurements are required. Object tracking is to estimate the states of moving targets, which typically consist of their positions and velocities over time. The tracking algorithms usually assume the presence of the target(s). Based on the assumptions of the detection and tracking algorithms, the object detection and tracking are typically implemented separately and object tracking is performed after the target is detected. This two-stage approach works well when the target has a relatively high signal-to-noise ratio (SNR), and it can be reliably detected. But this approach may not detect the weak target reliably with acceptable detection performance using a single sample. This motivates the research on joint object detection and tracking, which has the potential to significantly improve the detection of extremely weak targets or phenomena, such as a weak target that is far away from the radar or the chemical/biological plumes with very low concentration.

There are only a very limited number of publications on joint detection and tracking. One joint detection and tracking approach is along-track integration. The multiple multistage hypothesis test tracking algorithm [1] detects targets by evaluating candidate track hypotheses by using the test statistic in the multistage hypothesis testing algorithm [2]. The premise of along-track integration is that the possible target trajectories are known. Another joint detection and tracking approach is track-before-detect [3], [4], which utilizes a dynamic programming algorithm to evaluate possible target trajectories. Note that these methods work only in discrete state space. In [5], joint detection and tracking of a target was solved by the Bernoulli filter, which models the presence and the state of target as a random set: the posterior probability density function (PDF) of the set's cardinality corresponds to the target's presence, and the posterior PDF of the element in the set corresponds to the target's state given the target's presence. The two posterior PDFs of the random set are updated recursively in a Bayesian framework.

Different from all the joint detection and tracking approaches discussed above, a joint object detection and tracking approach based on the likelihood ratio test (LRT) and the extended Kalman filter (EKF) has been proposed in our previous work [6], which works in continuous state space. However, this approach is a fixed-sample-size (FSS) procedure, where the number of samples has been pre-specified. It is well known that sequential detection on the average requires a smaller number of observations than a detection procedure with a fixed sample size [7]. Therefore, we proposed the joint sequential object detection and tracking approach based on Wald's sequential probability ratio test (SPRT) and the Kalman filter in [8]. However, it is very difficult to rigorously prove that the Wald's SPRT will eventually terminate in this approach as the successive measurements are dependent under hypothesis H_1 which means the existence of a target. Since it was proved in [7] that the SPRT procedure will terminate with probability one if the observations are independent, we propose a terminative joint sequential object detection and tracking algorithm in this paper by constructing a sequence of independent observations based on the sensor measurements. In this new approach, two hypothesis testing statistics are fused to guarantee that the sequential test will not only eventually terminate but also keep the power of our previous approach.

The paper is organized as follows. The joint sequential object detection and tracking algorithm proposed in [8] is summarized in Section II. In Section III, the terminative joint sequential object detection and tracking algorithm is proposed. The simulation results provided in Section IV show

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the performance of proposed algorithm. Finally, the paper is concluded in Section V.

II. Problem Formulation

The joint sequential detection and tracking approach proposed in [8] is summarized in this section. Let us assume that under hypothesis H_1, an object exists and its motion could be modeled by the following discrete-time linear system state equation [9]

$$\mathbf{x}_{k+1} = \mathbf{F}\mathbf{x}_k + \boldsymbol{\Gamma}\mathbf{v}_k \tag{1}$$

where $\mathbf{x}_k$ is $n_x \times 1$ state vector at time k, $\mathbf{F}$ is $n_x \times n_x$ state transition matrix, $\mathbf{v}_k$ is the process noise at time k, and $\boldsymbol{\Gamma}$ is the gain matrix for $\mathbf{v}_k$. Furthermore, $\{\mathbf{v}_k\}$ is a sequence of white Gaussian process noise with $E(\mathbf{v}_k) = \mathbf{0}$ and $E(\mathbf{v}_k\mathbf{v}_k^T) = \mathbf{Q}$ for all $k = 1, 2, \cdots$.

The measurement equation is

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{w}_k \tag{2}$$

where $\mathbf{z}_k$ is the $n_z \times 1$ measurement vector at time k, $\mathbf{H}$ is the $n_z \times n_x$ measurement matrix, and $\mathbf{w}_k$ is the measurement noise at time k. Also, $\{\mathbf{w}_k\}$ is a sequence of white Gaussian measurement noise with $E(\mathbf{w}_k) = \mathbf{0}$ and $E(\mathbf{w}_k\mathbf{w}_k^T) = \mathbf{R}_w$ for $k = 1, 2, \cdots$.

Let us assume that under hypothesis H_0, no object exists and the measurement is purely noise

$$\mathbf{z}_k = \mathbf{u}_k \tag{3}$$

where $\mathbf{u}_k$s are independent and identically distributed (i.i.d.) and follow Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\mathbf{R}_u$. The sequences $\{\mathbf{v}_k\}$, $\{\mathbf{w}_k\}$, and $\{\mathbf{u}_k\}$ are independent with each other.

Since the measurements are independent over time under hypothesis H_0 and the likelihood function $p(\mathbf{z}_{1:K}|H_1)$ can be calculated using chain rule under hypothesis H_1, the log-likelihood ratio for the measurements accumulated up to the Kth step can be written in the following summation form

$$\begin{aligned}\log\Lambda(\mathbf{z}_{1:K}) &= \log\frac{p(\mathbf{z}_{1:K}|H_1)}{p(\mathbf{z}_{1:K}|H_0)} \\ &= \log\frac{p(\mathbf{z}_1|H_1)\prod_{k=1}^{K-1} p(\mathbf{z}_{k+1}|\mathbf{z}_{1:k}, H_1)}{\prod_{k=1}^{K} p(\mathbf{z}_k|H_0)} \\ &= \sum_{k=1}^{K}\log\frac{p(\mathbf{z}_k|\mathbf{z}_{1:k-1}, H_1)}{p(\mathbf{z}_k|H_0)} = \sum_{k=1}^{K}\Theta_k\end{aligned} \tag{4}$$

in which

$$\Theta_k \triangleq \log\frac{p(\mathbf{z}_k|\mathbf{z}_{1:k-1}, H_1)}{p(\mathbf{z}_k|H_0)} \tag{5}$$

and $\mathbf{z}_{1:0}$ is an empty set.

By using the Kalman filter, we can obtain $p(\mathbf{z}_k|\mathbf{z}_{1:k-1}, H_1)$ as follows

$$\begin{aligned}&p(\mathbf{z}_k|\mathbf{z}_{1:k-1}, H_1) \\ &= |2\pi\mathbf{S}_k|^{-\frac{1}{2}} e^{-\frac{(\mathbf{z}_k-\mathbf{H}\hat{\mathbf{x}}_{k|k-1})^T\mathbf{S}_k^{-1}(\mathbf{z}_k-\mathbf{H}\hat{\mathbf{x}}_{k|k-1})}{2}}\end{aligned} \tag{6}$$

where $\mathbf{S}_k$ is the measurement residual covariance, and $\hat{\mathbf{x}}_{k|k-1}$ is the predicted state given the accumulated measurements $\mathbf{z}_{1:k-1}$. Under hypothesis H_0, it is easy to show that

$$p(\mathbf{z}_k|H_0) = |2\pi\mathbf{R}_u|^{-\frac{1}{2}} e^{-\frac{(\mathbf{z}_k-\boldsymbol{\mu})^T\mathbf{R}_u^{-1}(\mathbf{z}_k-\boldsymbol{\mu})}{2}} \tag{7}$$

Substituting (6) and (7) in (5), we have

$$\begin{aligned}\Theta_k =& \frac{1}{2}\log\frac{|\mathbf{R}_u|}{|\mathbf{S}_k|} + \frac{1}{2}(\mathbf{z}_k - \boldsymbol{\mu})^T\mathbf{R}_u^{-1}(\mathbf{z}_k - \boldsymbol{\mu}) \\ &- \frac{1}{2}(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1})^T\mathbf{S}_k^{-1}(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1})\end{aligned} \tag{8}$$

Let $t_a(\mathbf{z}_{1:K}) = 2\sum_{k=1}^{K}\Theta_k$ be the hypothesis testing statistic. According to (4) and (8), we have

$$\begin{aligned}t_a(\mathbf{z}_{1:K}) &= 2\log\Lambda(\mathbf{z}_{1:K}) \\ &= \sum_{k=1}^{K}\Big\{\log\frac{|\mathbf{R}_u|}{|\mathbf{S}_k|} + (\mathbf{z}_k - \boldsymbol{\mu})^T\mathbf{R}_u^{-1}(\mathbf{z}_k - \boldsymbol{\mu}) \\ &\quad - (\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1})^T\mathbf{S}_k^{-1}(\mathbf{z}_k - \mathbf{H}\hat{\mathbf{x}}_{k|k-1})\Big\}\end{aligned} \tag{9}$$

Since the measurements are dependent over time under hypothesis H_1, in this case the optimum detector is in the form of a generalized sequential probability ratio test (GSPRT) [10], [11]. However, the thresholds used in GSPRT are functions of K and the determination of them is still an open problem. Due to the difficulty of implementing the optimal GSPRT, a new joint sequential detection and tracking algorithm by using Wald's SPRT was proposed in [8] and is summarized as follows

$$t_a(\mathbf{z}_{1:K})\begin{cases}\geqslant 2\log A & \text{stop and accept } H_1 \\ \leqslant 2\log B & \text{stop and accept } H_0 \\ \text{otherwise} & \text{continue}\end{cases} \tag{10}$$

where A and B are two thresholds which can be determined by pre-specified probabilities of false alarm and missed detection.

III. Terminative Joint detection and tracking

A. Independent Observations

The Wald's SPRT terminates with probability one on the premise of the observations are independent [7]. Therefore, we construct a sequence of independent observations $\{\mathbf{y}_k\}$ based on the measurements $\{\mathbf{z}_k\}$ to make sure that the Wald's SPRT will eventually terminate.

To construct independent observations $\{\mathbf{y}_k\}$, we only need to consider the measurements under hypothesis H_1. Because the measurements under hypothesis H_0 are independent and identically distributed, and the linear combination of them are still independent and identically distributed. Under hypothesis H_1, we know that the process noise sequence $\{\mathbf{v}_k\}$ and measurement noise sequence $\{\mathbf{w}_k\}$ are independent of each other. So, the measurements $\mathbf{z}_i$ and $\mathbf{z}_j$ are correlated only because they depend on the correlated states $\mathbf{x}_i$ and $\mathbf{x}_j$ respectively. Since the measurements are linear functions of states, the independent observations can be constructed by linear combination of $\mathbf{z}_k$s in which the coefficient corresponding to $\mathbf{x}_k$ should be zero.

Since $\mathbf{z}_{k+m}$ is a linear function of $\mathbf{F}^m\mathbf{x}_k$ for any $m = 0, 1, 2, ...$, we can generate the independent observations $\{\mathbf{y}_k\}$ in the form of linear combination of $\{\mathbf{z}_k\}$ as long as we find a polynomial equation of $\mathbf{F}$. Let us take a look at the characteristic equation of $\mathbf{F}$.

$$\begin{aligned}\kappa(\lambda) &= |\lambda\mathbf{I} - \mathbf{F}| \\ &= \lambda^{n_x} + p_1\lambda^{n_x-1} + \cdots + p_{n_x-1}\lambda + p_{n_x} \\ &= 0\end{aligned} \tag{11}$$

where the p_is determined by the eigenvalues of $\mathbf{F}$. Let us denote the ith eigenvalue of $\mathbf{F}$ as λ_i. The characteristic equation of $\mathbf{F}$ can be rewritten as

$$\kappa(\lambda) = \prod_{i=1}^{n_x}(\lambda - \lambda_i) = 0 \tag{12}$$

According to Cayley-Hamilton theorem, we know that the square matrix $\mathbf{F}$ satisfies its own characteristic equation. Therefore, we have

$$\begin{aligned}\kappa(\mathbf{F}) &= \mathbf{F}^{n_x} + p_1\mathbf{F}^{n_x-1} + \cdots + p_{n_x-1}\mathbf{F} + p_{n_x}\mathbf{I} \\ &= \prod_{i=1}^{n_x}(\mathbf{F} - \lambda_i\mathbf{I}) = \mathbf{0}\end{aligned} \tag{13}$$

According to (13), the independent observations $\{\mathbf{y}_k\}$ is constructed as follows

$$\begin{aligned}\mathbf{y}_k =& \mathbf{z}_{(n_x+1)k} + p_1\mathbf{z}_{(n_x+1)k-1} + \cdots \\ &+ p_{n_x-1}\mathbf{z}_{(n_x+1)k-(n_x-1)} + p_{n_x}\mathbf{z}_{(n_x+1)k-n_x}\end{aligned} \tag{14}$$

By linearly combining $\mathbf{z}_k$s in this way, it can be shown that the terms containing $\mathbf{x}_k$s will be canceled out. This is the general way to construct $\mathbf{y}_k$ for any n_x. To show that $\{\mathbf{y}_k\}$ is a sequence of independent observations clearly and find the distribution of $\mathbf{y}_k$, let us take $n_x = 2$ as an example. In this case, $\mathbf{F}$ is a 2×2 matrix, and the characteristic equation of $\mathbf{F}$ becomes

$$\begin{aligned}\kappa(\mathbf{F}) &= \mathbf{F}^2 + p_1\mathbf{F} + p_2\mathbf{I} \\ &= (\mathbf{F} - \lambda_1\mathbf{I})(\mathbf{F} - \lambda_2\mathbf{I}) \\ &= \mathbf{F}^2 - (\lambda_1 + \lambda_2)\mathbf{F} + \lambda_1\lambda_2\mathbf{I} = \mathbf{0}\end{aligned} \tag{15}$$

where $p_1 = -(\lambda_1 + \lambda_2)$ and $p_2 = \lambda_1\lambda_2$.

According to (15), the observation $\mathbf{y}_k$ in (14) becomes

$$\begin{aligned}\mathbf{y}_k &= \mathbf{z}_{3k} + p_1\mathbf{z}_{3k-1} + p_2\mathbf{z}_{3k-2} \\ &= \mathbf{z}_{3k} - (\lambda_1 + \lambda_2)\mathbf{z}_{3k-1} + \lambda_1\lambda_2\mathbf{z}_{3k-2}\end{aligned} \tag{16}$$

$\mathbf{z}_{3k}$, $\mathbf{z}_{3k-1}$, and $\mathbf{z}_{3k-2}$ are expanded as functions of $\mathbf{x}_{3k-2}$ as follows

$$\begin{aligned}\mathbf{z}_{3k} &= \mathbf{H}\mathbf{x}_{3k} + \mathbf{w}_{3k} \\ &= \mathbf{H}(\mathbf{F}\mathbf{x}_{3k-1} + \mathbf{\Gamma}\mathbf{v}_{3k-1}) + \mathbf{w}_{3k} \\ &= \mathbf{H}(\mathbf{F}^2\mathbf{x}_{3k-2} + \mathbf{F}\mathbf{\Gamma}\mathbf{v}_{3k-2} + \mathbf{\Gamma}\mathbf{v}_{3k-1}) + \mathbf{w}_{3k}\end{aligned} \tag{17}$$

$$\begin{aligned}\mathbf{z}_{3k-1} &= \mathbf{H}\mathbf{x}_{3k-1} + \mathbf{w}_{3k-1} \\ &= \mathbf{H}(\mathbf{F}\mathbf{x}_{3k-2} + \mathbf{\Gamma}\mathbf{v}_{3k-2}) + \mathbf{w}_{3k-1}\end{aligned} \tag{18}$$

and

$$\mathbf{z}_{3k-2} = \mathbf{H}\mathbf{x}_{3k-2} + \mathbf{w}_{3k-2} \tag{19}$$

Substituting (17), (18), and (19) in (16), we have

$$\begin{aligned}\mathbf{y}_k &= \mathbf{z}_{3k} - (\lambda_1 + \lambda_2)\mathbf{z}_{3k-1} + \lambda_1\lambda_2\mathbf{z}_{3k-2} \\ &= \mathbf{H}\left\{\left[\mathbf{F}^2 - (\lambda_1 + \lambda_2)\mathbf{F} + \lambda_1\lambda_2\mathbf{I}\right]\mathbf{x}_{3k-2}\right. \\ &\quad \left. + \left[\mathbf{F} - (\lambda_1 + \lambda_2)\mathbf{I}\right]\mathbf{\Gamma}\mathbf{v}_{3k-2} + \mathbf{\Gamma}\mathbf{v}_{3k-1}\right\} \\ &\quad + \mathbf{w}_{3k} - (\lambda_1 + \lambda_2)\mathbf{w}_{3k-1} + \lambda_1\lambda_2\mathbf{w}_{3k-2}\end{aligned} \tag{20}$$

where the coefficient of $\mathbf{x}_{3k-2}$ is equal to zero as shown in (15). So, the observation $\mathbf{y}_k$ is as follows

$$\begin{aligned}\mathbf{y}_k = \mathbf{H}&\left\{\left[\mathbf{F} - (\lambda_1 + \lambda_2)\mathbf{I}\right]\mathbf{\Gamma}\mathbf{v}_{3k-2} + \mathbf{\Gamma}\mathbf{v}_{3k-1}\right\} \\ &+ \mathbf{w}_{3k} - (\lambda_1 + \lambda_2)\mathbf{w}_{3k-1} + \lambda_1\lambda_2\mathbf{w}_{3k-2}\end{aligned} \tag{21}$$

Obviously, $\{\mathbf{y}_k\}$ follow i.i.d. Gaussian distribution under H_1, because the observation $\mathbf{y}_k$ is linear combination of Gaussian distributed $\{\mathbf{v}_k\}$ and $\{\mathbf{w}_k\}$, and the adjacent observation $\mathbf{y}_{k+1}$ is as follows which has no common items with $\mathbf{y}_k$.

$$\begin{aligned}\mathbf{y}_{k+1} &= \mathbf{z}_{3k+3} - (\lambda_1 + \lambda_2)\mathbf{z}_{3k+2} + \lambda_1\lambda_2\mathbf{z}_{3k+1} \\ &= \mathbf{H}\left\{\left[\mathbf{F} - (\lambda_1 + \lambda_2)\mathbf{I}\right]\mathbf{\Gamma}\mathbf{v}_{3k+1} + \mathbf{\Gamma}\mathbf{v}_{3k+2}\right\} \\ &\quad + \mathbf{w}_{3k+3} - (\lambda_1 + \lambda_2)\mathbf{w}_{3k+2} + \lambda_1\lambda_2\mathbf{w}_{3k+1}\end{aligned} \tag{22}$$

Now, we derive the distribution of i.i.d. observations $\mathbf{y}_k$ under H_1 and H_0, respectively. Let $\mathbf{P}_{H_1}$ and $\mathbf{P}_{H_0}$ denote the covariance matrice under H_1 and H_0, respectively. According to (21), we get $\mathbf{P}_{H_1}$ as follows

$$\begin{aligned}\mathbf{P}_{H_1} = \mathbf{H}&\left\{\left[\mathbf{F} - (\lambda_1 + \lambda_2)\mathbf{I}\right]\mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^T\left[\mathbf{F} - (\lambda_1 + \lambda_2)\mathbf{I}\right]^T\right. \\ &\left. + \mathbf{\Gamma}\mathbf{Q}\mathbf{\Gamma}^T\right\}\mathbf{H}^T + \left[\lambda_1^2\lambda_2^2 + (\lambda_1 + \lambda_2)^2 + 1\right]\mathbf{R}_w\end{aligned} \tag{23}$$

Obviously, the mean of $\mathbf{y}_k$ under H_1 is zero. Therefore, $\mathbf{y}_k|H_1 \sim i.i.d.\,\mathcal{N}(\mathbf{0},\ \mathbf{P}_{H_1})$.

$\mathbf{y}_k$ under H_0 is obtained in the same way as (16) and we have

$$\begin{aligned}\mathbf{y}_k|H_0 &= \mathbf{z}_{3k} - (\lambda_1 + \lambda_2)\mathbf{z}_{3k-1} + \lambda_1\lambda_2\mathbf{z}_{3k-2} \\ &= \mathbf{u}_{3k} - (\lambda_1 + \lambda_2)\mathbf{u}_{3k-1} + \lambda_1\lambda_2\mathbf{u}_{3k-2}\end{aligned} \tag{24}$$

It's easy to show that $\mathbf{y}_k|H_0 \sim i.i.d.\,\mathcal{N}(\mathbf{0},\ \mathbf{P}_{H_0})$ where

$$\mathbf{P}_{H_0} = \left[\lambda_1^2\lambda_2^2 + (\lambda_1 + \lambda_2)^2 + 1\right]\mathbf{R}_u \tag{25}$$

From (23) and (25), we know that the covariance matrices $\mathbf{P}_{H_1}$ and $\mathbf{P}_{H_0}$ are time invariant. So, they can be calculated offline.

B. Fused Hypothesis Testing Statistic

Since we know the distributions of $\mathbf{y}_k$ under both hypotheses, the log-likelihood ratio is adopted to generate the hypothesis testing statistic. For the convenience of description, we still take $n_x = 2$ for example. The hypothesis testing statistic for the observations accumulated up to the Lth step based on the independent observations $\{\mathbf{y}_k\}$ is

$$\begin{aligned}t_b(\mathbf{y}_{1:L}) &= 2\log\frac{p(\mathbf{y}_{1:L}|H_1)}{p(\mathbf{y}_{1:L}|H_0)} = 2\log\frac{\prod_{k=1}^{L}p(\mathbf{y}_k|H_1)}{\prod_{k=1}^{L}p(\mathbf{y}_k|H_0)} \\ &= 2\sum_{k=1}^{L}\log\frac{p(\mathbf{y}_k|H_1)}{p(\mathbf{y}_k|H_0)} \\ &= \sum_{k=1}^{L}\left[\log\frac{\mathbf{P}_{H_0}}{\mathbf{P}_{H_1}} + \mathbf{y}_k^T\mathbf{P}_{H_0}^{-1}\mathbf{y}_k - \mathbf{y}_k^T\mathbf{P}_{H_1}^{-1}\mathbf{y}_k\right]\end{aligned} \tag{26}$$

where $L = 1, 2, \cdots$.

To guarantee that the joint sequential object detection and tracking approach in [8] will eventually terminate with probability one, a terminative approach is constructed as follows by applying a fused hypothesis testing statistic.

1) Hypothesis H_1 will be accepted and the sequential test will terminate if

$$U_K \geqslant 2\log A \tag{27}$$

where

$$U_K = \max\{t_a(\mathbf{z}_{1:K}),\ t_b(\mathbf{y}_{1:\lfloor K/3 \rfloor})\} \tag{28}$$

2) Hypothesis H_0 will be accepted and the sequential test will terminate if

$$L_K \leqslant 2\log B \tag{29}$$

where

$$L_K = \min\{t_a(\mathbf{z}_{1:K}),\ t_b(\mathbf{y}_{1:\lfloor K/3 \rfloor})\} \tag{30}$$

3) Otherwise, the sequential test will continue to take the next sample.

In this procedure, $t_a(\mathbf{z}_{1:K})$ is calculated for each positive integer K, and $t_b(\mathbf{y}_{1:K/3})$ is fused together with $t_a(\mathbf{z}_{1:K})$ when $K = 3r, r = 1, 2, \cdots$. This procedure will terminate if either the larger one between $t_a(\mathbf{z}_{1:K})$ and $t_b(\mathbf{y}_{1:K/3})$ is greater than or equal to $2\log A$ or the smaller one is less than or equal to $2\log B$ when $K = 3r, r = 1, 2, \cdots$. When $K \neq 3r$, equivalently only $t_a(\mathbf{z}_{1:K})$ is used to make decision, since $t_b(\mathbf{y}_{1:\lfloor K/3 \rfloor})$ does not contribute any new information. If we only use $t_b(\mathbf{y}_{1:\lfloor K/3 \rfloor})$ as hypothesis testing statistic in Wald's SPRT, it will eventually terminate as $\mathbf{y}_k$s are independent. It's easy to show that the proposed approach will also terminate with probability one since the Wald's SPRT procedure will terminate as long as either $t_a(\mathbf{z}_{1:K})$ or $t_b(\mathbf{y}_{1:\lfloor K/3 \rfloor})$ crosses one threshold. By applying the same procedure, this terminative joint sequential object detection and tracking approach can be applied to general problems with an arbitrary n_x.

The thresholds $\mathbf{A}$ and $\mathbf{B}$ are set using the same method as in [8]: let α and β be the nominal probabilities of false alarm and miss respectively. Then, $\mathbf{A}$ and $\mathbf{B}$ are obtained by $A = \frac{1-\beta}{\alpha}$ and $B = \frac{\beta}{1-\alpha}$. Regarding the actual probabilities of false alarm α' and miss β', we will investigate their relationship with α and β in our future work.

IV. Simulation Results

The average sample number (ASN), probability of false alarm, and probability of miss of the proposed approach are evaluated under different SNRs in this section.

Let us assume that an object is moving in a 1-dimensional space with its state at time k denoted as $\mathbf{x}_k = [\xi_k\ \dot{\xi}_k]^T$, where ξ_k and $\dot{\xi}_k$ are the object's position and velocity at time k, respectively. The state transition matrix is

$$\mathbf{F} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}$$

where $T_s = 0.5$ seconds is the time interval between two measurements. The eigenvalues of $\mathbf{F}$ are $\lambda_1 = \lambda_2 = 1$, and

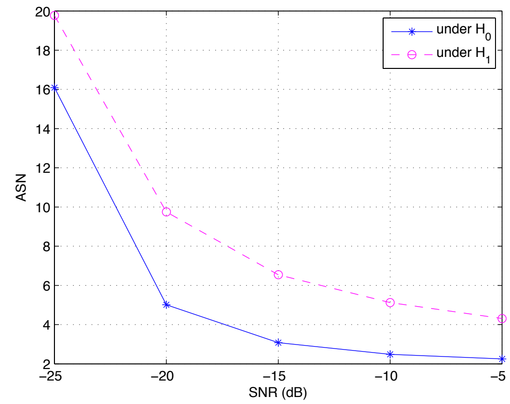

Fig. 1. ASN vs. SNR

we choose $n_x = 2$ in this example. In the joint detection and tracking system, there is a sensor measuring the object's position over time. Therefore, the measurement matrix is $\mathbf{H} = [1\ \ 0]$. The process noise gain matrix $\mathbf{\Gamma}$ in (1) is $[T_s^2/2\ \ T_s]^T$. The variance of state process noise is $Q = 0.01$. The mean of the object's initial state is $\hat{\mathbf{x}}_{0|0} = [0\ \ 1.5]^T$, and its covariance is $\mathbf{P}_{0|0} = diag([1000, 1])$. The mean of measurement noise under H_0 is same as the position mean of $\mathbf{x}_0$, namely $\mu = 0$. The variance of measurement noise under H_0, R_u, is the same as the position variance of $\mathbf{x}_0$, which is $\mathbf{P}_{0|0}(1,1)$. Both probability of false alarm and probability of miss are set to 10^{-3}. All the simulation results are based on 10^5 Monte Carlo simulations. The SNR is defined as Fisher information about the object's position contained in $\mathbf{z}_k$. Therefore, the SNR in decibels is equal to $10\log_{10}(1/R_w)$ in this case.

We evaluate the ASN required by the proposed approach under different SNRs first. The simulation results are shown in Fig. 1, from which we know that ASN is inversely proportional to SNR. This is just as we have expected. We know that when the SNR increases, the distance between distributions of observations under two hypotheses also increases. So, the proposed approach will take less time to terminate under higher SNR. From Fig. 1, we also know that the ASN under low SNR conditions is small, which means that the proposed approach will terminate quickly even when the signal is very weak.

To show the advantage of the proposed fused approach, we compare it with the dependent approach which is our previous work [8] and the independent approach which only uses $t_b(\mathbf{y}_{1:K/3})$ in Wald's SPRT. Under H_0, the ASNs required by each approach under different SNRs are shown in TABLE I, from which we know that the ASN required by the proposed fused approach is the lowest. In this particular example, the fused statistic provides a slight improvement over the dependent approach as in [8] in terms of the ASN. Under H_1, the simulation results are shown in TABLE II, which are similar to those under H_0.

The probabilities of errors made by the proposed approach are tested in this experiment. The probability of false alarm and probability of miss under different SNRs are shown in Fig. 2 and Fig. 3, respectively. Obviously, the actual probabilities of error α' and β' are less than the nominal probabilities of

TABLE I. COMPARE ASN AMONG THREE APPROACHES UNDER H_0

SNR (dB)	-25	-20	-15	-10	-5
Dependent approach	16.1303	5.0296	3.0880	2.4848	2.2477
Independent approach	51.6410	12.3935	6.3170	4.4636	3.7322
Fused approach	16.0878	5.0094	3.0791	2.4810	2.2460

TABLE II. COMPARE ASN AMONG THREE APPROACHES UNDER H_1

SNR (dB)	-25	-20	-15	-10	-5
Dependent approach	19.7756	9.7470	6.5429	5.1206	4.3073
Independent approach	91.5980	31.5792	18.4958	13.3509	10.1639
Fused approach	19.7698	9.7461	6.5424	5.1205	4.3073

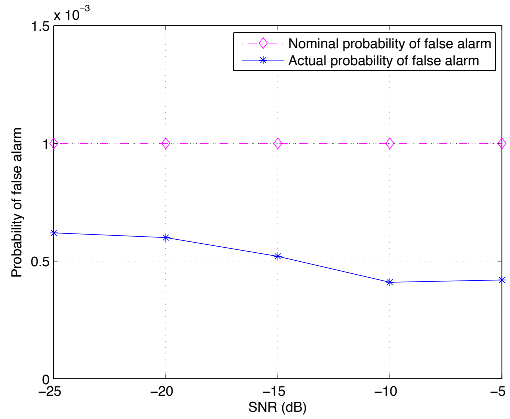

Fig. 2. Probability of False Alarm vs. SNR

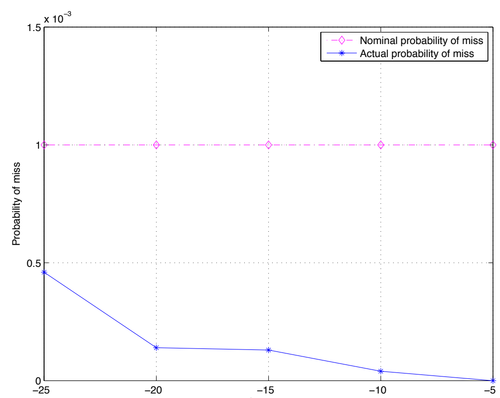

Fig. 3. Probability of Miss vs. SNR

error α and β, respectively. We also know that α' and β' are inversely proportional to SNR from Fig. 2 and Fig. 3. This is because the hypothesis testing statistics will pass the correct threshold with higher probability under lower disturbance of noise.

V. CONCLUSION

We proposed a terminative joint sequential object detection and tracking approach in this paper. In this approach, the adjacent measurements are combined together to generate a sequence of independent and identically distributed observations, and two hypothesis testing statistics cooperate with each other to guarantee that the Wald's SPRT procedure will eventually terminate and keep the good performance of the approach proposed in our previous work [8] in detection and tracking. Numerical results show that the proposed algorithm requires a small ASN with low probabilities of errors under low SNRs and the probabilities of errors are smaller than the nominal probabilities of errors.

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