18th International Conference on Information Fusion
Washington, DC - July 6-9, 2015

Doppler-Only Tracking Under a Minimum Detectable Velocity Constraint

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***Abstract*—This paper addresses the problem of multi-static doppler-only tracking in doppler blind zone (DBZ). In such a problem, target measurements are suppressed when the range-rate (doppler) drops below a specified threshold in magnitude (the minimum detectable velocity, MDV). Moreover, tracking using doppler-only measurements is not an easy problem due to its weak observability. In order to improve the estimation performance, a novel three-step method is proposed. In the first step, a standard Extended Kalman Filter (EKF) is applied using obtained measurements. In the second step, preliminary estimates of mean and covariance are obtained conditioned on the MDV constraints, to which we refer as the coarse-step since it needs a linearization step. In the third step, Monte Carlo (MC) truncation technique for enforcing the constraints is used for further process, to which we refer as the fine-step. Simulation results show that the proposed method is more effective with moderate computational cost.**

I. Introduction

Driven by applications, such as passive surveillance and the technology improvements in wireless network, target tracking using measurements of doppler-shift frequencies has attracted much more attentions recently [1]-[3]. For example, in the radar context, GSM-based passive radar systems, have attracted tremendous research interest [4]-[6]. As a passive radar system, it provides crucial advantages over active systems such as no frequency allocation problem, improved anti-jamming performance, energy saving and much lower costs. Although GSM waveform has poor range resolution, it can achieve good Doppler resolution, which makes GSM-based passive radar suitable for Doppler detection and tracking.

Target tracking using doppler-only shift measurements has been studied in different contexts for several decades [7]-[10]. Some of the studies in the literature mainly concentrate on the static estimation solutions, observability analysis, and optimal positioning of the passive system [11]-[15]. Recently, tracking moving targets using doppler-only measurements is mostly considered in multi-static passive radar framework [16]-[19]. However, tracking using doppler-only measurements is not an easy problem due to several reasons. Among all these reasons, weak observability due to the uninformative doppler-shift measurements is the key difficult problem. More specifically, target state remains unobservable unless more than three doppler measurements are obtained. Since target tracking using Doppler-only measurements strongly depends on the number of doppler-shift measurements obtained, it's highly demanded to incorporate the DBZ into the tracker design [20][21].

Doppler blind zone arises from preprocessing of sensor outputs to suppress low range-rate measurements and remove heavy static clutters, that is, the measurements are deliberately suppressed once the magnitude of the range rate drops below a specific threshold (the Minimum Detectable Velocity MDV) [22]-[25]. For such a set-up, the occurrence, or non-occurrence, of a measurement in itself can still provide information about target motion. The remaining question in multi-static doppler-only tracker design is how to exploit this 'negative' information. Extensive research about doppler blind zone (DBZ) has been done on Ground Moving Target Indicator (GMTI) sensors, which can be classified mainly into two categories based on the different modelling of DBZ. The first is proposed by Koch *et* al.[26], their algorithm accommodates the loss of a measurement within the model of the measurement process. DBZ is accounted by constructing a suitable state-dependent detection probability, which takes low values inside the zone. The tracker takes the form of a Gaussian mixture Kalman filter, in which negative weights may possibly arise. This idea is generalized by [27], which approximates detection probability by arbitrary Gaussian mixture. In contrast to [26], an extra approximation step is introduced in order to replace the resulting 'negative' Gaussian mixture with one of strictly positive mixture weights, thus improve algorithmic stability. The second category is proposed by Clark et al., DBZ is formulated as constraints on state estimation. Different from the blind doppler mixture filter (BDMF) proposed in reference [28] and its modified version in reference [29], the algorithm proposed in reference [31] is better matched to the practical data gathering process since the decision to suppress a measurement is made on the basis of the noise-corrupted, rather than the exact as the former one. In reference [31], it makes use of formulae for the conditional mean and covariance of a random variable x, given that a scalar measurement m lies in a specified interval A. In constructing the algorithm, they interpret '$m \in A$' as a 'measurement is suppressed'. In order to fit for such a linear formula, a linearization procedure is applied since the measurement equation is nonlinear, which is relatively rough to some extent.

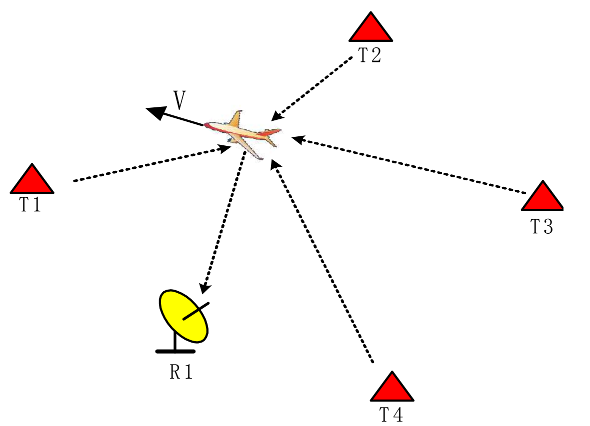

Fig. 1. Multi-static Doppler-only survelliance network. Transmitted signal from an illuminaor Tx is reflected from a detected moving target and received by the Doppler-shift measuring sensors.

In this paper, we focus on the problem of target tracking using doppler-only measurements under a minimum detectable velocity constraint. Following the idea of reference [31], a novel three-step tracking algorithm is proposed. In the first step, we perform standard EKF to update state using available doppler-shift measurements. In the second step, a modified method suitable for our problem based on reference [31] is applied, which we also denote as the coarse-step. In the third step, the truncation approach for enforcing the constraint is chosen, which we refer to as the fine-step. The simulation results demonstrate the effectiveness of our method.

The remainder of this paper is organized as follows: in the next section, the tracking problem with DBZ is formulated and the necessity of incorporating DBZ into the tracker design is fully explained. In Section III, we give out the detailed algorithm design process including two techniques on which our method is based. Simulation results are discussed in Section IV. Section V provides conclusions and some directions for future work.

II. Problem Formulation

A. State and Observation Model

The scenario considered in this paper is as follows: Four transmitters of opportunity, one doppler-shift measuring receiver, as in Fig.1. The locations of the transmitters and receiver are assumed to be known. The state of the moving target in the 2D scenario at time t_k is represented by the state vector

$$x_k = [\ x_k \quad \dot{x_k} \quad y_k \quad \dot{y_k}\ ]^T \tag{1}$$

where superscript T denotes the matrix transpose.

Target motion is modelled by a CV model

$$\mathbf{x}_{k+1} = F_k \mathbf{x}_k + \mathbf{u}_k \tag{2}$$

where F_k is the transition matrix and $\mathbf{u}_k \sim N(\mathbf{u}; 0, Q_k)$ is zero-mean white Gaussian process noise with covariance Q_k, We adopt

$$F_k = I_2 \otimes \begin{bmatrix} 1 & T_k \\ 0 & 1 \end{bmatrix} \tag{3}$$

$$Q_k = I_2 \otimes q \begin{bmatrix} \frac{T_k^3}{3} & \frac{T_k^2}{2} \\ \frac{T_k^2}{2} & T_k \end{bmatrix} \tag{4}$$

where $\otimes$ is Kronecker product, T_k is the sampling interval and q is the level of power spectral density of the corresponding continuous process noise.

Bi-static doppler-shift measurements are collected by the receiver located at $\mathbf{r} = [x_r, y_r]$ from four transmitters with position $\mathbf{t}_i = [x_t^i, y_t^i], i = 1, 2, 3, 4$. The target state at time k is denoted as $\mathbf{x}_k$, and the doppler-shift measurement received from transmitter T_i is modelled as follows

$$z_k^i = h_k^i(\mathbf{x}_k) + w_k^i \tag{5}$$

where

$$h_k^i(\mathbf{x}_k) = -\frac{\mathbf{v}_k^T}{\lambda}\left[\frac{\mathbf{p}_k - \mathbf{t}_i}{\|\mathbf{p}_k - \mathbf{t}_i\|} + \frac{\mathbf{p}_k - \mathbf{r}}{\|\mathbf{p}_k - \mathbf{r}\|}\right] \tag{6}$$

is the true doppler frequency shift, λ is the wavelength of the transmitted signal and w_k^i is the measurement noise ($w_k^i \sim N(\mathbf{0}, R_k^i)$). $\mathbf{p}_k$ denotes the location of the illuminated target. In addition, we do not distinguish between doppler-shift measurement and bi-static range rate in this paper.

B. Doppler Blind Zone

As mentioned above, for the purpose of separating out moving targets of interest from heavy, static clutter, 'Doppler Blind Zone' (the region of the state space in which the range rate magnitude is small) is introduced artificially as a sensor data preprocessing stage. As we can see from Fig.2, when the plane moves in the red rectangular region, due to the bi-static range keeps almost unchanged. Consequently, the bi-static range rate (i.e. Doppler shift) is close to zero, which will surely be suppressed during the sensor data preprocessing stage. Thus, receiver will not obtain doppler-shift measurement from this transmitter.

However, as reference [7] says, the number of measurements obtained in the fusion center directly determines the tracker performance for multi-static doppler-only tracking due to its inherent poor observability. So, it is necessary to incorporate the doppler blind zone into the tracker design.

For simplicity, detection probability P_d is considered to be unity in this paper. In other words, a missed detection means $z_k^i \in A$, where z_k^i is the expected but not detected measurement and A is the interval

$$A = (-mdv, +mdv) \tag{7}$$

The constant mdv is termed the minimum detectable velocity threshold.

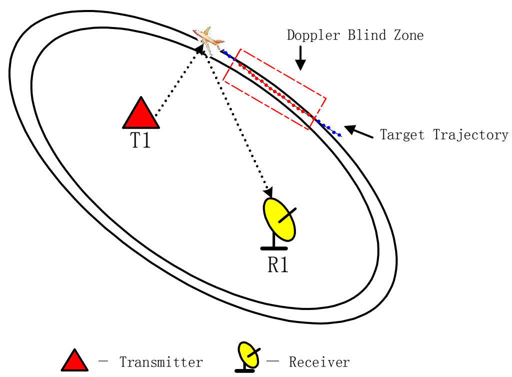

Fig. 2. Doppler Blind Zone (DBZ). When the target moves in the red rectangular, the bi-static range keeps almost unchanged. Consequently, the signal reflected will be easily drown by noises.

III. Doppler-Only Tracking Incorporating the Doppler Blind Zone

A. Preliminary Analysis

First, we present a proposition which is the theoretical basis of reference [31].

Proposition 1: Take independent random variables $x \sim N(\hat{x}_0, P_0)$ and $w \sim N(0, \sigma^2)$ and an interval $A = [a, b]$. Define the scalar random variable m to be

$$m = q^T \mathbf{x} + w \tag{8}$$

Here $\hat{x}_0, q$ are given n-vectors, P_0 is a given $n \times n$ covariance matrix, and $\sigma^2 > 0$. Write

$$\begin{aligned} K &= P_0 q (q^T P_0 q + \sigma^2)^{-1} \\ P &= P_0 - K q^T P_0 \end{aligned} \tag{9}$$

Then

$$\begin{aligned} E[\mathbf{x}|m &\in A] = \hat{\mathbf{x}}_0 + K[\hat{m}_A - \mu] \\ cov[\mathbf{x}|m &\in A] = P + K V_A K^T \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mu &= q^T \hat{\mathbf{x}}_0 \\ \hat{m}_A &= E[m|m \in A] \\ V_A &= cov[m|m \in A] \end{aligned} \tag{11}$$

Furthermore

$$\hat{m}_A = c^{-1} \bar{\sigma}^2 [N(a; \mu, \bar{\sigma}^2)] + \mu \tag{12}$$

$$V_A = c^{-1} \bar{\sigma}^2 [(a + \mu) N(a; \mu, \bar{\sigma}^2) - (b + \mu) N(b; \mu, \bar{\sigma}^2)] + (\mu^2 + \bar{\sigma}^2) - \hat{m}_A^2 \tag{13}$$

where

$$\bar{\sigma}^2 = q^T P_0 q + \sigma^2 \tag{14}$$

c denotes a normalizing constant ensuring that a probability density integrates to unity. In this case

$$c = \int_a^b N(m; \mu, \bar{\sigma}^2) dm \tag{15}$$

B. Monte Carlo truncation technique

Multi-static doppler-only tracking with minimum detectable velocity can be formulated as a constrained estimation problem. However, most constrained estimation problems proposed such as optimization based filters moving horizon estimation (MHE) [33] and unscented kalman filter (UKF) [34] realizations are based on linear (in)equality constraints, and their computational cost are relatively high. A computationally efficient constrained nonlinear filter is proposed [32], which is based on the Monte Carlo truncation method.

Suppose unconstrained estimate is given as

$$p(\mathbf{x}_k|\mathbf{z}^k) = N(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k}, \mathbf{P}_{k|k}) \tag{16}$$

Let ς_k be a set of all states satisfying the inequality constraints:

$$\varsigma_k = \{\mathbf{x}_k : -mdv < h(\mathbf{x}_k) < mdv\} \tag{17}$$

Then, the truncated pdf (probability density function) is:

$$p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k) \tag{18}$$

which can be further expressed as follows

$$p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k) = \begin{cases} \xi_k^{-1} p(\mathbf{x}_k|\mathbf{z}^k), & if\ \mathbf{x}_k \in \varsigma_k \\ 0 & otherwise, \end{cases} \tag{19}$$

where

$$\xi_k = prob\{\mathbf{x}_k \in \varsigma_k | \mathbf{z}^k\} = \int_{\varsigma_k} p(\mathbf{x}_k|\mathbf{z}^k) d\mathbf{x}_k \tag{20}$$

In order to complete the recursive loop, the truncation pdf must be approximated by a Gaussian pdf:

$$p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k) \approx N(\mathbf{x}_k; \hat{\mathbf{x}}_{k|k}^c, \mathbf{P}_{k|k}^c) \tag{21}$$

The state estimate and the corresponding covariance are given by

$$\hat{\mathbf{x}}_{k|k}^c = \int \mathbf{x}_k p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k) d\mathbf{x}_k \tag{22}$$

$$P_{k|k}^c = \int (\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}^c)(\mathbf{x}_k - \hat{\mathbf{x}}_{k|k}^c)^T p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k) d\mathbf{x}_k \tag{23}$$

However, except for some special cases, the mean $\hat{\mathbf{x}}_{k|k}^c$ and covariance $P_{k|k}^c$ of the truncated pdf $p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k)$ cannot be expressed analytically. In reference [32], the Monte Carlo (MC) techniques are used to approximate the moment of the truncated pdf for its relatively simple implementation and moderate computational cost, not significantly depending on the dimension of the state [35].

In this paper, the perfect Monte Carlo technique [36] is chosen. In order to approximate the moments of the truncated pdf derived above, first suppose that N samples $x_k^{(i)}, i = 1, 2, ...N$ are drawn from $p(x_k|z^k)$, and the samples satisfying the constraints ς_k are denoted as $x_k^{c,(j)}, j = 1, 2..., N^c$.

Then, the truncated pdf can be approximated by the samples $\mathbf{x}_k^{c,(j)}$ as follows

$$p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k) \approx \frac{1}{N^c}\sum_{j=1}^{N^c} \delta(\mathbf{x}_k - \mathbf{x}_k^{c,(j)}) \tag{24}$$

where δ is the Dirac delta function.

Next, the mean and the covariance matrix of the truncation distribution can be approximated as

$$\hat{\mathbf{x}}_{k|k}^c \approx \frac{1}{N^c}\sum_{j=1}^{N^c} \mathbf{x}_k^{c,(j)} \tag{25}$$

$$P_{k|k}^c \approx \frac{1}{N^c - 1}\sum_{j=1}^{N^c} (\mathbf{x}_k^{c,(j)} - \hat{\mathbf{x}}_{k|k}^c)(\mathbf{x}_k^{c,(j)} - \hat{\mathbf{x}}_{k|k}^c)^T \tag{26}$$

Nevertheless, we wish to emphasize that the quality of truncation technique is affected by the mass of the unconstrained filtering pdf within the constrained region. That is to say, the effectiveness of the proposed MC truncation techniques may degrade if significant mass of the unconstrained filtering pdf is outside the constrained region. A direct solution is to increase the number of samples at the cost of the time consumption. In this work, in order to improve the efficiency of the MC truncation technique, we propose a modified method which adds a preprocessing step before the MC truncation step. The purpose is to increase the mass of the unconstrained filtering pdf within the constrained region, which is also the motivation of our method.

C. A Novel Three-step Algorithm

As we can see from Fig.3, when the target moves in the red rectangular region, $R1$ will receive only three true Doppler-shift measurements (from $T2, T3, T4$ respectively) due to the bi-static range of $TX1$ and $R1$ keeps almost unchanged, the expected but missed measurement is denoted as z_k^1.

Proposition 1 is the theoretical basis of reference [31] to model doppler blind zone in GMTI applications, while it is not accurate enough since the linearity of reference (8) in $\mathbf{x}$ is not satisfied. What's more, the method proposed in reference [31] is for single sensor, while what we have to deal with here is multi-sensor target tracking. Nevertheless, the Proposition itself can yet be regarded as a good preprocessing method, which will be used in the coarse step of our method.

Based on the theoretical basis mentioned above, we propose a three-step method incorporating the doppler blind zone information into multi-static doppler-only tracker design.

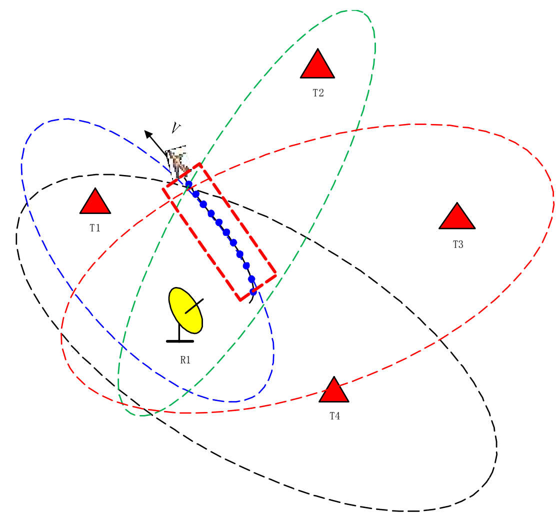

Fig. 3. Multi-static doppler only tracking scenario. The receiver will only receive the signals come from T2-T4, and the measurement should be received from T1 is hidden in DBZ.

1) Unconstrained Estimation: In this step, we derive the unconstrained state estimate using recorded doppler-shift measurements. Suppose $\mathbf{x}_{k-1} \sim N(\hat{\mathbf{x}}_{k-1|k-1}, P_{k-1})$. It follows

$$\hat{\mathbf{x}}_{k:k-1}^u = F\hat{\mathbf{x}}_{k:-1k-1} \tag{27}$$

$$P_{k|k-1}^u = FP_{k-1|k-1}F^T + Q_{k-1} \tag{28}$$

$$\hat{\mathbf{x}}_{k|k}^u = \hat{\mathbf{x}}_{k|k-1} + K_k(\mathbf{z}_k - H_k\hat{\mathbf{x}}_{k|k-1}) \tag{29}$$

$$P_{k|k}^u = P_{k|k-1}^u - K_kS_kK_k^T \tag{30}$$

where

$$S_k = H_kP_{k|k-1}^uH_k^T + R_k \tag{31}$$

is the covariance of the innovation term $\upsilon_k = \mathbf{z}_k - h_k^i(\hat{\mathbf{x}}_{k|k-1})$,and the Kalman gain K_k is as follows

$$K_k = P_{k|k-1}^uH_k^TS_k^{-1} \tag{32}$$

where

$$H_k = \begin{bmatrix} \cdots & \nabla h^i(\hat{x}_{k|k-1}) & \cdots \end{bmatrix}^T \tag{33}$$

is the Jacobian matrix with $i \in \{\ i \mid z_k^i \notin [-mdv, mdv]\ \}$.

2) Coarse Step: Suppose z_k^i is not recorded, i.e. $z_k^i \in A, A = [-mdv, mdv]$. In order to apply Proposition 1, we should perform a linearization procedure at first, by a first order Taylor approximation around the predicted state $\hat{x}_{k|k-1}$

$$z_k^i = h_i(\hat{x}_{k|k-1}) + H^i(x_k - \hat{x}_{k|k-1}^i) + n_{k,i} \approx H^ix_k + n_{k,i} \tag{34}$$

with noise term $n_{t,4} \sim N(0, \sigma_i^2)$.

$$H^i = \nabla h^i(\hat{x}_{k|k-1}) \tag{35}$$

Let

$$\hat{\mathbf{x}}_{k|k-1} = \hat{\mathbf{x}}^u_{k|k} \tag{36}$$

$$P_{k|k-1} = P^u_{k|k} \tag{37}$$

Now, according to Proposition 1, we can obtain:

$$\mu = h_i(\hat{\mathbf{x}}^u_{k|k}) \tag{38}$$

$$q^T = H^i \tag{39}$$

$$m = z^i_k \tag{40}$$

So,

$$\hat{x}^-_{k|k} = \hat{\mathbf{x}}^u_{k|k} + K_t\left[\hat{m}_A - \mu\right] \tag{41}$$

$$K_k = P^u_{k|k} q (q^T P^u_{k|k} q + \sigma^2)^{-1} \tag{42}$$

$$P^-_{k|k} = P^u_{k|k} - K_k q^T P^u_{k|k} + K_k V_A K^T_k \tag{43}$$

and

$$\hat{m}_A = \gamma_k^{-1}\bar{\sigma}^2[N(a;\mu,\bar{\sigma}^2) - N(b;\mu,\bar{\sigma}^2)] + \mu \tag{44}$$

$$\begin{aligned} V_A \quad = \quad & \gamma_k^{-1}\bar{\sigma}^2[(a+\mu)N(a;\mu,\bar{\sigma}^2) - \\ & (b+\mu)N(b;\mu,\bar{\sigma}^2)] + (\mu^2+\bar{\sigma}^2) - \hat{m}_A^2 \end{aligned} \tag{45}$$

$$\bar{\sigma}^2 = q^T P^u_{k|k} q + \sigma^2 \tag{46}$$

$$\gamma_k = \int_{-mdv}^{mdv} N(m;\mu,\bar{\sigma}^2)dm \tag{47}$$

3) Fine Step: Suppose we have obtained the rough estimate $\mathbf{x}^-_k, P^-_{k|k}$ from Step2. Obviously, $N(\mathbf{x};\mathbf{x}^-_k, P^-_{k|k})$ is preferred here since it has been preprocessed by Step 2, which is relatively close to the pdf $p(\mathbf{x}_k|\mathbf{z}^k, \mathbf{x}_k \in \varsigma_k)$. Consequently, the quality of MC truncation techniques will be hopefully improved.

So, let N samples $\mathbf{x}^i_k$, $i = 1,2,...,N$ drawn from $N(\mathbf{x};\mathbf{x}^-_k, P^-_{k|k})$. Following the MC truncation technique mentioned above, we can approximate the moments of the truncated pdf as follows

$$\hat{\mathbf{x}}_{k|k} \approx \frac{1}{N^c}\sum_{j=1}^{N^c}\mathbf{x}^{c,(j)}_k \tag{48}$$

$$P_{k|k} \approx \frac{1}{N^c-1}\sum_{j=1}^{N^c}(\mathbf{x}^{c,(j)}_k - \hat{\mathbf{x}}^c_{k|k})(\mathbf{x}^{c,(j)}_k - \hat{\mathbf{x}}^c_{k|k})^T \tag{49}$$

where $\mathbf{x}^{c,(j)}_k, j = 1,2...N_c$ are the samples within the constraint region ς_k.

The outline of the proposed method is summarized in Table.I.

TABLE I
ALGORITHM OUTLINE

Input: $\hat{x}_{k-1\|k-1}, P_{k-1\|k-1}$.
—if all measurements are obtained(No measurement is in DBZ)
Perform standard Kalman Filter process
1) Predict: use the equations (27)-(28) to evaluate $\hat{\mathbf{x}}^u_{k:k-1}$ and $P^u_{k\|k-1}$.
2) Update: use the equations (29)-(32) to evaluate $\hat{\mathbf{x}}^u_{k\|k}$ and $P^u_{k\|k}$.
—if some measurement, for example, z^i_k is not recorded due to DBZ
Unconstrained Estimation Step:
1) Use equation (33) to evaluate H_k
2) Predict: use the equations (27)-(28) to evaluate $\hat{\mathbf{x}}^u_{k:k-1}$ and $P^u_{k\|k-1}$.
3) Update: use the equations (29)-(32) to evaluate $\hat{\mathbf{x}}^u_{k\|k}$ and $P^u_{k\|k}$.
Coarse Step: (add constraints on estimate based on Proposition 1)
1) Calculate H^i using (35)
2) Calculate $\hat{x}^-_{k\|k}$ and $P^-_{k\|k}$ from equations (41)-(47)
Fine Step: (Further Process using MC truncation)
1) Draw N samples $\mathbf{x}^i_k$, $i = 1,2,...,N$ from $N(\mathbf{x};\mathbf{x}^-_k, P^-_{k\|k})$
2) Approximate the moments of the truncated pdf, that is $\hat{\mathbf{x}}_{k\|k}, P_{k\|k}$ using (48)-(49)
Recursion: $k = k+1$.

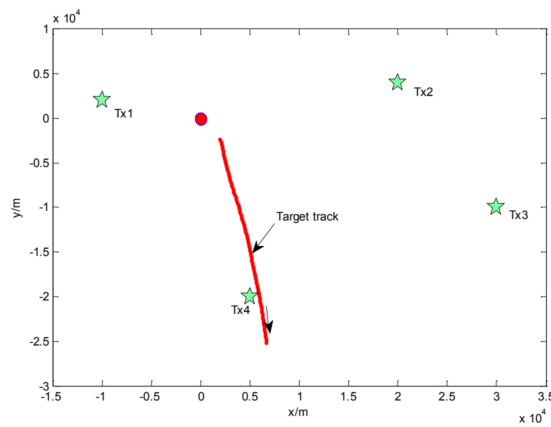

Fig. 4. Sensors and target trajectories

IV. Simulation Results

A. Scenario

A problem of tracking a moving vehicle in a scenario with 4 transmitters and 1 receiver is considered. The locations of transmitters are known with $Tx1 = [-10,2]km, Tx2 = [20,4]km, Tx3 = [30,-10]km, Tx4 = [5,-20]km$. The receiver is located at $[0,0]km$.The initial state of the target $\mathbf{x}_0 = [2km, -2.4km, 50m/s, 200m/s]$. The target is supposed to follow the continuous white noise acceleration motion model with $T = 1s, \sigma = 10m/s^2$. The minimum detectable velocity threshold MDV is set to $3m/s$ as [31] .The receiver inaccuracy is characterized by standard deviation of $\sigma_z = 2m/s$. The scenario and the simulated measurements are given in Fig.4 and Fig.5, respectively.

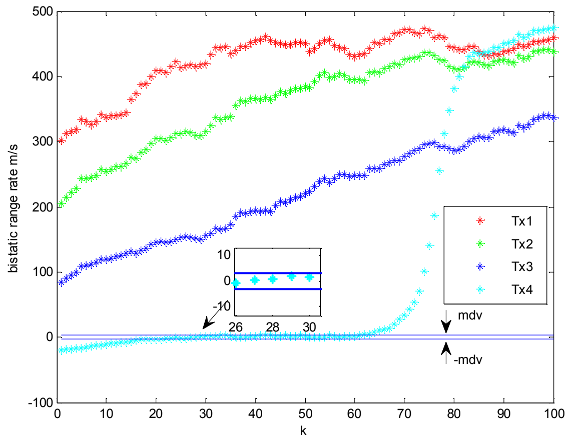

Fig. 5. Bistatic range-rate measurment

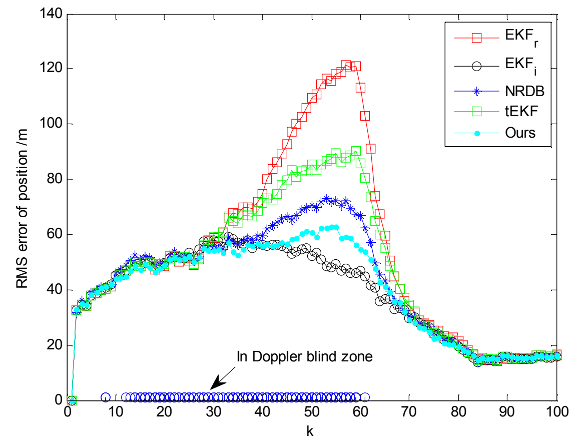

Fig. 6. RMSE of position Estimation

TABLE II
ESTIMATION PERFORMANCE OF FILTERS

	EKF_i	EKF_r	$NRDB$	$tEKF$	$Ours$
$rmse_{pos}$	33.40	52.65	43.16	44.87	35.71
$rmse_v$	1.7011	1.7439	1.7184	1.7239	1.7035
$Time(s)$	0.2236	0.2025	0.2543	1.4409	1.4817

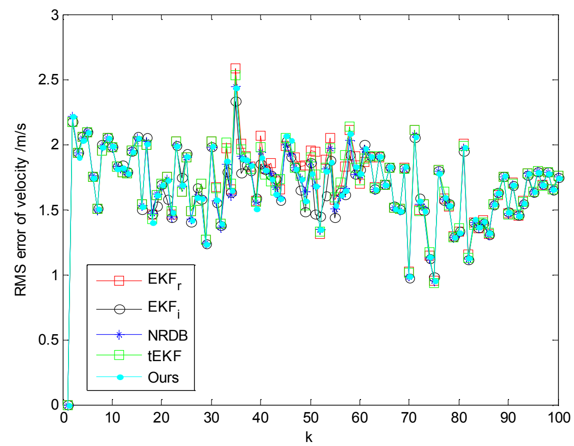

Fig. 7. RMSE of velocity estimation

In order to demonstrate the efficiency of our algorithm, four comparative filters are chosen. The first is named EKF_i assuming all measurements are received ideally, which is taken as the lower bound (benchmark). The second is named EKF_r, which is also EKF-based. In contrast to the first one, when some measurement drops below the DBZ, it only uses the remaining measurements to update target state without incorporating the DBZ into the tracker design. The third is realized using the modified method based on [31]. In order to keep consistent, we also refer to the third method as the noise related doppler blind mixture filter (NRDB). The difference is we need extend NRDB to the case of multi-static. The forth is perfect MC-based truncation using $N = 500$ samples[32]. RMS error curves of the position and velocity are shown in Fig.6 and Fig.7 respectively. The time averaged 100-trial RMS error for position and velocity is also summarized in Table II, the average computational costs for one MC simulation are presented as well (Pentium(R) Dual-Core CPU E5400 @2.70GHz).

As illustrated in Fig.4, the bi-static distance of $Tx4$ and the receiver is almost unchanged during $k = 10$ to 60. So, from Fig.5 we can easily find that the doppler-shift measurement falls below the MDV, that is to say, the target is hidden in the doppler blind zone.

Fig.6 and Fig.7 illustrate the comparison results of the proposed method with aforementioned methods. Firstly, it is found that by incorporating doppler blind zone information, filters such as $NRDB$, $tEKF$, and our algorithm provide much higher estimation performance in comparison with EKF_r. Secondly, in comparison with the other filters ($NRDB, tEKF$), our algorithm still exhibits uniform performance improvements as expected. The reason is the first algorithm $NRDB$ we compared with is based on the idea of paper [31]. It is exact only for linear constraints, rather than the nonlinear case here. The second algorithm $tEKF$ we compared with is based on the idea of paper [32]. As mentioned before, the truncation quality is easy to be affected by the mass of the unconstrained filtering pdf in the constrained region, so it may suffer from performance degradation in cases that samples are not enough. Our algorithm, hopefully, outperforms these two methods in both position and velocity estimation since the nonlinearity of the DBZ constraint is considered together with the possible inefficiency of MC truncation technique. The coarse step is essential for higher efficiency of MC truncation technique used in the fine step, and the fine step is also meaningful to further improve the estimation performance, which can be seen in Fig.8. In addition, as listed in Table.II,

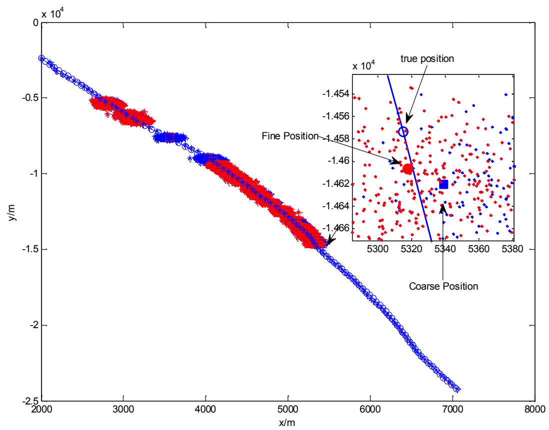

Fig. 8. Tracking results. The blue points in the local figure denote the unconstrained sample set, and the red points are the samples that meet DBZ constraints. The position estimate after Coarse step is the blue square and the position estimation after the Fine step is plotted as the red circle.

the proposed algorithm obtained an improvement in the order of 32% with respect to EKF_r, 17% with respect to $NRDB$, and 20% with respect to $tEKF$. The time consumption is also increased due to the MC technique adopted. Fortunately, it should not be a difficult problem nowadays.

V. Conclusion

In this paper, we studied the problem of multi-static doppler-only tracking with minimum detectable velocity constraints (or doppler blind zone). For this challenging problem, a novel three-step method is proposed by incorporating the DBZ constraints into the tracker design. Two techniques including proposition 1 and MC truncation are used to improve the estimation performance. Such a set-up is motivated by the nonlinearity of doppler blind zone constraints and the possible inefficiency of truncation quality induced by MC techniques. Experimental results reveal that the proposed method with three steps is more effective with moderate computational cost. A future direction would be to extend this result to the case of multi-target tracking.

Acknowledgments

This work was supported by the National Science Foundation Council of China under Grant 61135001, 61374159, and 61374023, and the Graduate Innovation Foundation of the Northwestern Polytechnical University Z2015123.

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