Parameter estimation for multistatic active sonar using extended fixed points

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Abstract—The main task of multistatic active sonar is the localisation and tracking of objects of interest (targets). Therefore, a precise knowledge of the parameters of the multistatic scenario is mandatory. These are the positions of the acoustic sources, the times of transmission, as well as the position and heading of the own sonar sensor. Reflections from so called "fixed points" can be used to improve knowledge about these parameters. A fixed point can be a wreck or geographical feature (e.g. a small island) with known position. In general these fixed points consist of multiple scattering points, thus, the assumption of a point-like target is not fulfilled.

In this paper we discuss the impact that an extension of the fixed point has on the estimation process and derive a method within the Kalman filter measurement update to incorporate knowledge about the extension in the estimation approach. Results will be discussed for simulated data.

I. INTRODUCTION

Multistatic sonar tracking based on stationary receivers aims to the surveillance of a given area and has been studied in detail e.g. by the members of the ISIF Multistatic Tracking Working Group (MSTWG), see e.g. [1]. At the Centre for Maritime Research and Experimentation (CMRE) in La Spezia (Italy) several experiments have been conducted [2], [3]. In cooperation with CMRE we developed a multi hypothesis tracker (MHT) for multistatic active sonar, which is described and discussed in [4].

The use of Autonomous Underwater Vehicles (AUV) operating cooperatively in a multistatic network for Anti Submarine Warfare (ASW) surveillance has been topic of further research, e.g. at CMRE [5]–[9]. Each AUV has to localise targets (submarines) robustly and precisely by evaluating the target's sonar echoes. Avoiding detection, i.e. a covert operation, requires a minimisation of own emissions, like communication or navigation signals. For target localisation and tracking, this is realised by a multistatic sonar configuration: The AUV is receiving the signals transmitted by acoustic sources located elsewhere.

In [10] and [11] we presented an extension of the algorithm in [4] and demonstrated that we are able to increase tracking accuracy by estimating inaccuracies in knowledge of the receiving array (position, heading). This is realised by exploiting knowledge on the operation area. It utilises sonar echoes of stationary objects ("clutter", "fixed points") with known positions. We found that by exploitation of multistatic sonar measurements we can even aid navigation of the AUV, if navigational data alone is not sufficient.

In [12] and [13] this technique was applied to noncooperative bistatic active sonar. Instead of improving the self-localization of the AUV the focus lies on estimation of the foreign source position and time of transmissions. Rough estimates, which can be gathered at the signal processing stage, are used as prior knowledge. The multihypothesis estimation algorithm derived in [12] assumes fixed points to be stationary point-like targets. As discussed in [13] a more realistic model of the structure of the fixed points is needed. Thus, we assume here, fixed points to consist of multiple reflection (scattering) points. The position of scattering points is uncorrelated in time, but is spatially restricted by the extension of the fixed point.

The model of an extended fixed point is associated with an additional source of uncertainty involving our estimation approach. We discuss the impact on estimation performance and show how it can be incorporated in the Kalman filter update.

Results are discussed for simulated data. For the first data set a model match between simulation and estimation is applied. This gives us the possibility to compare the estimation performance of the derived algorithms with theoretical estimation bounds (given by the Cramér Rao Lower Bound (CRLB) [14]) and thereby check the efficiency of our approach. To test also the robustness of our approach a more realistic simulation is conducted. The exact model is not known to the estimation algorithm, instead fixed standard values are used.

The paper has the following structure: After this introduction, the multistatic setup and adapted models are introduced in Sec. II. Formulas for calculating the CRLB under the model of an extended fixed point are given in Sec. III. Adaptation of the estimation technique is derived in Sec. IV. In Sec. V we introduce our simulation scenarios and discuss the results in Sec. VI.

II. PROBLEM DESCRIPTION

A. Bistatic setup

For simplification, we consider here a bistatic (one receiver, one source) system. The bistatic measurement setup (see Fig. 1) consists of the following parameters:

- the target position $\boldsymbol{p} = (x^p, y^p)^T$
- the source position $\boldsymbol{t}_{\boldsymbol{x}} = (x^t, y^t)^T$
- the receiver position $\mathbf{r_x} = (x^r, y^r)^T$ and velocity $\dot{\mathbf{r_x}} = (\dot{x}^r, \dot{y}^r)^T$
- the antenna orientation (heading) ϑ
- time relative to direct signal τ_0
- the propagation speed of sound in water $c_{\rm S}$.
- signal frequency f



Fig. 1. Bistatic Setup: Sound from source at t_x is reflected by the target at p and received at r_x . ϑ is the heading of the receiver relative to North.

For a stationary target and stationary source the measurement equation for the azimuth angle φ and time of arrival (ToA) τ and Doppler frequency d is given by

$$\varphi = \arctan\left(\frac{x^p - x^r}{y^p - y^r}\right) - \vartheta,$$

$$\tau = \frac{||\boldsymbol{p} - \boldsymbol{t}_{\boldsymbol{x}}|| + ||\boldsymbol{p} - \boldsymbol{r}_{\boldsymbol{x}}|| - ||\boldsymbol{t}_{\boldsymbol{x}} - \boldsymbol{r}_{\boldsymbol{x}}||}{c_{\mathrm{S}}} + \tau_0 \qquad (1)$$

$$d = \frac{f}{c_{\mathrm{S}}} \cdot \frac{(\boldsymbol{p} - \boldsymbol{r}_{\boldsymbol{x}})^T}{||\boldsymbol{p} - \boldsymbol{r}_{\boldsymbol{x}}||} \cdot \dot{\boldsymbol{r}}_{\boldsymbol{x}}$$

where $|| \cdot ||$ denotes the Euclidian norm. The time of arrival is given relative to the first signal that arrives at the receiver (=0). This typically corresponds to the direct blast (direct signal from the transmitter) and is therefore depending on the distance between source and receiver $||t_x - r_x||$. To capture inaccuracies in the measurement of the direct blast, we introduce the parameter τ_0 , which is described by the time difference between the direct blast and the first available signal. The measurement equation of the direct blast is given by:

$$\varphi^{DB} = \arctan\left(\frac{x^t - x^r}{y^t - y^r}\right) - \vartheta,$$

$$\tau^{DB} = \tau_0$$
(2)

$$d^{DB} = \frac{f}{c_{\rm S}} \cdot \left(\frac{(\boldsymbol{t_x} - \boldsymbol{r_x})^T}{||\boldsymbol{t_x} - \boldsymbol{r_x}||} \cdot \dot{\boldsymbol{r}_x}\right)$$

Generally, only estimates of the parameters of the bistatic setup are available. The precision of these estimates is affected by the variability of the underwater sound channel (probabilistic features) and the modelling precisions. The available (prior) knowledge about these parameters is modelled by Gaussian random variables, see also [4].

Aim of this study is the estimation of parameters which are relevant for application of multistatic target tracking. Thus, we define $\boldsymbol{x} = (\boldsymbol{t}_{\boldsymbol{x}}, \boldsymbol{r}_{\boldsymbol{x}}, \boldsymbol{c}_{\mathrm{S}}, \vartheta, \tau_0)$, which contains the multistatic parameters. The position of fixed point \boldsymbol{p} is exploited for estimation, but not relevant for the tracking application.

The measurement function respective a given fixed point is given by

$$\boldsymbol{z} = \mathbf{h}(\boldsymbol{x}, \boldsymbol{p}). \tag{3}$$

Fixed points are assumed to consist of multiple potential scattering points, which define a elliptical extension modelled by $\mathcal{N}(\boldsymbol{p}; \hat{\boldsymbol{p}}, \mathbf{C}_p)$.

For calculation of the CRLB and the update formulas of the extended Kalman filter the Jacobian matrix needs to be calculated. In the following we use the abbreviations $\mathbf{H}_x = \frac{d\mathbf{h}}{dx}$ and $\mathbf{H}_p = \frac{d\mathbf{h}}{dp}$.

III. CALCULATION OF THE CRLB

For calculation of the CRLB [14] the position of the fixed point is included in the parameter state vector. Under the assumption of Gaussian measurement noise the Fischer information matrix of the full state vector is given by

$$\begin{split} \mathbf{FIM} &= \begin{pmatrix} \mathbf{H}_x \\ \mathbf{H}_p \end{pmatrix} \mathbf{R}^{-1} \begin{pmatrix} \mathbf{H}'_x & \mathbf{H}'_p \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_p^{-1} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\mathbf{H}_x \mathbf{R}^{-1} \mathbf{H}'_x & | \mathbf{H}_x \mathbf{R}^{-1} \mathbf{H}'_p \\ \overline{\mathbf{H}_p \mathbf{R}^{-1} \mathbf{H}'_x & | \mathbf{H}_p \mathbf{R}^{-1} \mathbf{H}'_p + \mathbf{C}_p^{-1} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\mathbf{X} & | \mathbf{B}_1 \\ \overline{\mathbf{B}'_1 & | \mathbf{A}_1} \end{pmatrix}, \end{split}$$

where \mathbf{R} is the measurement covariance matrix. \mathbf{R} is assumed here to have diagonal form. \mathbf{C}_p models the extension of the fixed point and goes into the formula as prior knowledge. Since we assume the position of the scattering point to be uncorrelated in time the state vector increases for each measurement which is processed. The resulting information matrix has block matrix form

$$\mathbf{FIM} = \begin{pmatrix} \mathbf{X} & \left(\begin{array}{cccc} \mathbf{B}_1 & \mathbf{B}_2 & \dots & \mathbf{B}_n \end{array}\right) \\ \hline \begin{pmatrix} \mathbf{B}_1' \\ \mathbf{B}_2' \\ \vdots \\ \mathbf{B}_n' \end{array} & \left(\begin{array}{cccc} \mathbf{A}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{A}_n \end{array}\right) \\ \end{array} \right).$$

When calulating the CRLB as the inverse of the FIM, we are only interested in the entries with respect to \mathbf{X} . By application of the matrix inversion lemma we get

$$CRLB = (\mathbf{X} - \mathbf{B}\mathbf{A}^{-1}\mathbf{B}')^{-1} = (\mathbf{X} - \sum_{i} (\mathbf{B}_{i}\mathbf{A}_{i}^{-1}\mathbf{B}_{i}'))^{-1}$$

Thus, we can prevent storing and inverting of the increasing FIM.

IV. PARAMETER ESTIMATION USING EXTENDED FIXED POINTS

Implementation of the MHT for parameter estimation has been described in [12]. The extension of the fixed points is modelled as an additional uncertainty in the position of the scattering point. This can be incorporated in the initialisation and Kalman filter measurement update.

A. Initialisation

The measurement function (3) is generally not invertable. However we can find a function

$$\boldsymbol{t}_{\boldsymbol{x}} = \mathbf{g}((\varphi^{\mathrm{DB}}, \tau^{\boldsymbol{p}}), (\boldsymbol{r}_{\boldsymbol{x}}, \dot{\boldsymbol{r}}_{\boldsymbol{x}}, c_{\mathrm{S}}, \vartheta, \tau_{0}), \boldsymbol{p}),$$

where τ^{p} is the ToA measurement of a fixed point and φ^{DB} the azimuth of the direct blast, see [12]. According to prior and measurement assumptions (mean and covariances) the initial estimate of the source position can be derived by application of unscented transform [15] or linearization of g.

The remaining elements of the state vector are initialized according to the prior assumptions.

Note that the initial estimate of the source position is not dependent on the prior knowledge in source position.

B. Kalman filter update

As shown in [4] (formula (19)) in more detail we can incorporate uncertainties about additional parameters by increasing the estimation vector by these additional parameters. Here the augmented state vector is $\boldsymbol{x}^{(p)} = (\boldsymbol{x}, \boldsymbol{p})$. The measurement update according to a measurement at time t_k follows from evaluation of

$$p\left(\boldsymbol{z}|\mathbf{h}(\boldsymbol{x}^{(p)}),\mathbf{R}\right)\cdot p\left(\boldsymbol{x}^{(p)};\begin{pmatrix}\hat{\boldsymbol{x}}_{k|k-1}\\\hat{\boldsymbol{p}}\end{pmatrix},\begin{pmatrix}\hat{\mathbf{P}}_{k|k-1}&\mathbf{0}\\\mathbf{0}&\mathbf{C}_{p}\end{pmatrix}
ight),$$

where $(\hat{x}_{k|k-1}, \hat{\mathbf{P}}_{k|k-1})$ describes the predicted state estimate. From this the standard UKF or EKF update formulas can be derived. This delivers an additional update of the scattering point, which is subsequently discarded.

The EKF update of the parameter state x is derived here: The innovation matrix is given by

$$\mathbf{S} = \begin{pmatrix} \mathbf{H}_{x} & \mathbf{H}_{p} \end{pmatrix} \begin{pmatrix} \hat{\mathbf{P}}_{k|k-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_{p} \end{pmatrix} \begin{pmatrix} \mathbf{H}_{x}^{T} \\ \mathbf{H}_{p}^{T} \end{pmatrix} + \mathbf{R}$$
$$= \mathbf{H}_{x} \hat{\mathbf{P}}_{k|k-1} \mathbf{H}_{x}^{T} + \mathbf{H}_{p} \mathbf{C}_{p} \mathbf{H}_{p}^{T} + \mathbf{R}.$$
(4)

Thus, the innovation matrix differs from the standard case only by addition of $\mathbf{H}_p \mathbf{C}_p \mathbf{H}_p^T$. The Kalman gain matrix is

$$\begin{pmatrix} \mathbf{W}_x \\ \mathbf{W}_p \end{pmatrix} = \begin{pmatrix} \hat{\mathbf{P}}_{k|k-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{C}_p \end{pmatrix} \begin{pmatrix} \mathbf{H}_x \\ \mathbf{H}_p \end{pmatrix}^T \mathbf{S}^{-1}$$

From this follows $\mathbf{W}_x = \hat{\mathbf{P}}_{k|k-1} \mathbf{H}_x^T \mathbf{S}^{-1}$. Thus, the EKF update formulas are given by

$$\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{W}_x(\boldsymbol{z} - \mathbf{h}(\hat{\boldsymbol{x}}_{k|k-1}, \hat{\boldsymbol{p}})) \\ \hat{\mathbf{P}}_{k|k} = \hat{\mathbf{P}}_{k|k-1} - \mathbf{W}_x \mathbf{S} \mathbf{W}_x^T.$$

Calculation shows that the extension of the fixed point has only influence on the innovation matrix **S**. Clearly it has the same effect as an increase of the measurement noise by $\mathbf{H}_p \mathbf{C}_p \mathbf{H}_p^T$.



Fig. 2. Geometry of scenario A. Triangles show the position of two acoustic sources, red circles show the trajectory of the AUV. Fixed targets (clutter) are shown by a star.

V. SIMULATION SCENARIOS

In simulation estimates of the multistatic setup are sampled for 100 Monte Carlo runs. These estimates are input of the MHT estimation algorithm and used as prior information. Samples are generated from a Gaussian distribution with a standard deviations given by:

- source position: $\sigma_{t_x} = 2000 \text{ m}$
- receiver position / velocity: $\sigma_{r_x} = 3 \text{ m} / \sigma_{\dot{r}_x} = 1 \text{ m/s}$
- antenna heading: $\sigma_{\vartheta} = 1^{\circ}$
- speed of sound: $\sigma_{c_s} = 20$ m/s

A. Simulation scenario A

The first simulation scenario follows the setup of a real data experiment (GLINT'10) which was conducted by CMRE in 2010 in the Mediterranean Sea [5]. The setup is shown in Fig. 2. Two AUVs as well as two acoustic sources took part. The trajectory of one AUV (receiver Rx) is shown by the red circles. During a time window of two hours it is moving around at a triangle. This trajectory will serve as the receiver path in our simulations.

The positions of the three clutter objects are shown by blue stars. These clutter objects were extracted from multistatic sonar data, see [11], thus, describe reflection points with good detection rate.

The measurements are generated according to a Gaussian distribution with accuracy in azimuth given by $\sigma_{\varphi} = 3^{\circ}$, $\sigma_{\tau} = 1/50$ sec. in ToA and $\sigma_D = 5$ Hz in Doppler.

For the three fixed points the position of the scattering points is sampled from $\mathcal{N}(\hat{p}, \mathbf{C}_p)$. We define three scenarios by

Scenario A1 :
$$\mathbf{C}_{p} = \begin{pmatrix} (50m)^{2} & 0m^{2} \\ 0m^{2} & (50m)^{2} \end{pmatrix}$$

Scenario A2 : $\mathbf{C}_{p} = \begin{pmatrix} (100m)^{2} & 0m^{2} \\ 0m^{2} & (100m)^{2} \end{pmatrix}$
Scenario A3 : $\mathbf{C}_{p} = \begin{pmatrix} (50m)^{2} & (40m)^{2} \\ (40m)^{2} & (50m)^{2} \end{pmatrix}$

The probability of detection is chosen by $P_D = 0.8$. A mean number of 40 false alarms was generated in each time scan. The time interval between two pings is 60 sec.

B. Simulation scenario B

The contact data set of the scenario B is created with a hydrophone signal simulator (HSS) for active sonar, which contains both a hydrophone signal simulation and signal processing for generating contacts. The properties of the simulator are closely related to those of the TNO SIMONA simulation software described in [16]. A short summary of parameters is given below:

• The HSS basically implements the bistatic active sonar equation

$$SNR = SL - TL_1 - TL_2 + TS - NL + AG, \quad (5)$$

where SL is the signal level of the source, TL_1 and TL_2 is the transmission loss from source to target and target to receiver respectively, TS is the target strength, NL is the noise level and AG is the array gain of the receiving hydrophone array, while this parameter occurs in the contact generation.

- The impulse response of the underwater channel is modelled by the expected reverberation and an additional stochastic process (see [16] for further details) and is assumed to be a slow-fading Rayleigh channel.
- The intensity of the ambient (background) noise depends on the simulated sea state. It is modelled as coloured noise with respect to ocean turbulence, shipping noise and wave noise [17].
- Two different clutter types are generated as reverberation: (1) Uniformly distributed in the field of view with a certain reflection strength (comparable with a target strength), (2) extended clutter points with a predefined area which generates a certain number of reflection each ping.
- Targets are point-like with a predefined target strength which is omni directional.

The contact generation procedure contains four steps, which are commonly used in active sonar [18]:

- 1. A broadband filter and sum beamformer with an optional left-right distinguishing procedure if the array works with two hydrophone lines,
- 2. a matched filter for each signal transmitted by an acoustical source,
- a normalization with an integration length of 1000m for removing fluctuations in the matched filter output and a Page test [18], [19] as detector,
- 4. a contact generator which generates contacts with the range and azimuth information. If two hyperbolic frequency modulated (HFM) waveforms were transmitted, one with a positive gradient (HFM up) and the other with a negative gradient (HFM down), pairs of these HFM up and HFM down contacts are identified and fused to a HFM contact with an additional Doppler information [20].

Figure 3 shows the setup of scenario B



Fig. 3. Geometry of Scenario B. The stars show the fixed clutter points, where C1 and C2 mark extended fixed points, the dashed lines show the trajectories of moving targets, Rx and Tx mark the positions/movements of receivers and the acoustical source respectively. Rx1 is working with a single-line hydrophone array; Rx2 is working with a twin-line array.

The setup contains one acoustical source (Tx1) and two receivers (Rx1 and Rx2). Four fixed clutter points (C3-C6) and two extended clutter points (C1 and C2) are simulated. The areas of the extended clutter points are approximated with five Gaussian mixtures and each extended clutter point generates a mean number of 10 reflections (Poisson distributed) with uniformly distributed reflection strength between 10 and 15 dB. The extended clutter points both have a spreading to approximately [200m x 200m]. The two targets (T1 and T2) move with a nearly constant velocity of $8\frac{m}{s}$ (T1) and $6\frac{m}{s}$ (T2), T1 with a varying target strength between 15dB and 1dB, T2 with a TS defined by the Metron Type 2 target strength. The acoustical source transmits two waveforms: First a HFM up pulse with centre frequency $f_0 = 1600$ Hz, second a HFM down pulse with a centre frequency of $f_0 = 1900$ Hz. Both pulses have a bandwidth of B = 200 Hz and a duration of $T_s = 0.5$ s. The source works with a signal level of 180 dB and transmits with a ping period of $T_p = 50$ s. Rx1 works with 128 hydrophones in a single line hydrophone array and generates a mean of 170 fused HFM contacts in each ping. Rx2 works with a twin-line antenna with 128 hydrophones in each line and generates a mean of 190 fused HFM contacts in each ping. The transmission loss in the scenario is calculated with $TL = 17 \cdot \log_{10}(r)$ and the sea state is four.

VI. SIMULATION RESULTS

A. Measures of performance

The estimation algorithm generates improved estimates of the full bistatic setup. For our application the source position is the crucial parameter, i.e. the parameter for which only little prior information is available. The estimation performance of source position is analysed by means of Monte-Carlo simulations. We compare the root-mean-squared-error (RMSE) and the root-trace-CRLB (RTC). Let \hat{x}_i be the estimate from Monte-Carlo run i and the truth be given by x, so the RMSE is defined by

$$\sqrt{\frac{1}{N}\sum_{i}(\hat{\boldsymbol{x}}_{i}-\boldsymbol{x})^{T}(\hat{\boldsymbol{x}}_{i}-\boldsymbol{x})},$$
(6)

with N the number of Monte-Carlo runs. The RTC is given by the square root of the trace of the CRLB. If the entries 1 and 2 describe the x and y position of the source, the respective RTC is e.g. given by

$$\sqrt{\mathbf{CRLB}(1,1) + \mathbf{CRLB}(2,2)}.$$
 (7)

During the analysis we found outliers to have a significant effect on the results in terms of RMSE values. Therefore, outlier detection according to the Grubbs test [21] is applied. A track is classified as an outlier track, if a significant deviation is detected for half the scenario time. The number of outlier tracks is used as an additional criterion for discussion of the estimation performance.

For scenario A the measurements can be uniquely identified with respective fixed points. This knowledge can be used to evaluate the association performance of the MHT estimation algorithm. Therefore we calculate the frequency of associating the true measurements to a fixed point or the direct blast. This value is limited by the probability of detection. At each time only the association regarding to the best hypothesis is used for the evaluation. Since the MHT is able to correct its decision at a later time scan the association statistic provides only a pessimistic criterion of association performance.

B. Impact of the extension of fixed points on the estimation performance

Additional uncertainty in terms of the extension of fixed point has impact on the source position estimate. This is visualized for scenario A in Fig. 4. The CRLB of the source position (RTC value) is displayed for potential positions of the acoustic source. In Fig. 4(a) the fixed points are assumed to be point-like targets. The assumption of extended fixed points (Fig. 4(b) and (c)) results in significantly worse estimation performance.

As shown in section IV-B the extension of the fixed point has a similar effect to an increase of the measurement error. For scenario A this is visualized for the combination of Tx1 and Rx in Fig. 5. The variation of the measurement errors is plotted over the scenario time (receiver moving once around the triangle) and different variations of the extensions of the fixed points.

Whilst the impact on the Doppler accuracy seems to be negligible, we observe a significant impact for ToA and azimuth measurement errors. These additional errors are dependent on the structure of the fixed points and the bistatic geometry.

C. Estimation results for simulation scenario A

For analysis of the tracking results 100 Monte Carlo runs are evaluated for scenario A1 and A2 (only the first hour is shown). Since the results of the UKF and EKF approach show no significant difference, for reasons of clarification



(c) Scenario A2, extended fixed points

Fig. 4. CRLB in m of source position estimation (single ping). Values larger than 500 m are replaced by this maximal value. Circle marks the position of the receiver and stars the position of the fixed points.



Fig. 5. Visualization of the impact of the extension of fixed point as an increase of measurement errors. Results of the three fixed points are represented by different colors.

only results of the UKF are shown in Fig. 6. The 'standard' approach (red colour) stands for the results of the estimation algorithm, when ignoring the extension of the fixed points. This is compared to the approach, when knowledge about the extension is incorporated (blue colour, 'with knowledge').

The total error increases with the extension of the fixed points. Using the knowledge of the extension of the fixed points results in a significant improvement in terms of RMSE. The results show a good match with the CRLB, however, for scenario A2 and Tx2 a gap between RMSE and RTC shows that the approach is not totally efficient.

The association statistic is shown in Tab. I. Exploiting knowledge about the extension results in a significant better association of the measurements of the fixed point (maximum is 80% since $P_D = 0.8$).

Only few tracks have been classified as outliers, we note a slight improvement when using the model of the extension.

D. Estimation results of scenario B

As in scenario A we run 100 Monte Carlo runs by varying the estimates of the bistatic setup as input to our estimation algorithm. In scenario B six fixed points have been simulated (C1 and C2 have an extended structure). For the two combinations of source and receivers (Rx1Tx1 and Rx2Tx1) we generate two scenarios by varying the number of fixed points as input to the estimation algorithm. In the first scenario ("4 fixed points") we use the fixed points C1 - C4, and in the second scenario ("three fixed points") we use the fixed points C1-C3. The simulation scenario is thereby not modified, measurements of the remaining fixed points and the moving targets are still contained in the data set. The positions of these unknown targets are not estimated and are not exploited for the parameter estimation. However, the measurements of unknown targets increase the association difficulty that has to be solved by the MHT estimation algorithm. For the more realistic scenario the measurement errors and P_D are dependent on

the bistatic geometry. But, as this is also the case for real data, we assume here that the estimation algorithm has no knowledge about the exact values, it uses fixed values instead. One example for a mismatch between estimation model and simulation is the detection rate of fixed point C2 by sensor pair Rx2Tx1. Whilst the estimation algorithm assumes $P_D = 0.8$, in simulation no measurements are generated, because C2 is located within the bistatic ellipse of the direct blast. In particular this means that for Rx2Tx1 effectively only three out of four (resp. two out of three) fixed points are available. In simulation extended fixed points can generate more than one measurement, this is not accounted for in the estimation algorithm yet. The MHT chooses the best fit.

The extension of fixed points C1 and C2 inside the estimation algorithm is modelled by $\mathbf{C}_p = \begin{pmatrix} (200m)^2 & \mathbf{0} \\ \mathbf{0} & (100m)^2 \end{pmatrix}$.

Results by comparison of the RMSE are shown in Fig. 7 and Tab. II. Generally we note an improvement in RMSE and/or number of outliers when the model of extension is used.

Since the measurements cannot be uniquely identified with fixed points the association statistic only reflects if any measurement is associated with a fixed point. For fixed points C1 and C2 the number of associations significantly increases when the model of extension is used. The total errors and number of outliers is relatively high for Rx2Tx1. This demonstrates that the bistatic geometry and the availability of fixed points has a significant impact on the estimation performance. For better understanding the association statistic for "true tracks" and "outlier tracks" is compared in Tab. III for Rx2Tx1 ("3 fixed points"). We notice an increased number of associations with fixed point C2 for which no measurements were available. The estimation algorithm suffers from associating wrong measurements to these fixed points. This effect is alleviated when using the model of extension (reduced number of outliers), but is still significant.

	Frequency of correct association to				Number of
	Direct Blast	Fixed Point C1	Fixed Point C2	Fixed Point C3	outliers
Scenario A1, Tx1	99.3% / 99.7%	52.8% / 77.3%	54.2% / 76.2%	49.4% / 76.6%	4.00% / 1.00%
Scenario A2, Tx1	96.7% / 99.7%	32.9% / 76.6%	34.3% / 75.2%	31.2% / 75.8%	5.00% / 2.00%
Scenario A1, Tx2	99.84% / 99.86%	50.1% / 77.2%	56.0% / 76.8%	49.7% / 76.5%	1.00% / 0.00%
Scenario A1, Tx2	99.46% / 99.88%	33.4% / 76.0%	38.3% / 76.7%	33.2% / 76.8%	0.00% / 0.00%

TABLE I

ASSOCIATION AND OUTLIER STATISTIC FOR SCENARIO A: (BLACK) STANDARD APPROACH, (BLUE) WITH KNOWLEDGE OF EXTENSION

Frequency of association to Number of Direct Blast Fixed Point C1 Fixed Point C3 Fixed Point C4 Fixed Point C2 outliers Rx1Tx1 (4 fixed points) 32.2% / 53.1% 44.3% / 86.7 60.7% / 64.4% 19% / 2% 71.2% / 71.1% 54.0% / 88.29 Rx1Tx1 (3 fixed points) 70.9% / 70.7% 41.5% / 84.9% 53.5% / 86.3% 60.4% / 65.4% 5% / 0% Rx2Tx1 (4 fixed points) 51.8% / 55.9% 51.9% / 93.5% 0.3% / 0.2% 43.5% / 46.6% 50.0% / 49.2% 18% / 10% 54.1% / 56.3% 55.8% / 92.6% 0.53% / 0.2% 48.3% / 49.8% 50% / 12% Rx2Tx1 (3 fixed points) -/ -

TABLE II

ASSOCIATION AND OUTLIER STATISTIC FOR SCENARIO B: (BLACK) STANDARD APPROACH, (BLUE) WITH KNOWLEDGE OF EXTENSION

	Free			
Rx2Tx1	Direct Blast	Fixed Point 1	Fixed Point 2	Fixed Point 3
true tracks	54.1% / 56.3%	0.53% / 0.2%	55.8% / 92.6%	48.3% / 49.8%
outlier tracks	43.7% / 50.0%	7.37% / 8.8%	63.1% / 92.1%	22.8% / 37.3%

TABLE III

ASSOCIATION STATISTIC FOR SCENARIO B, Rx2Tx1, 3 FIXED POINTS: COMPARISON FOR "TRUE" AND "OUTLIER" TRACKS

VII. CONCLUSIONS

Incorporating knowledge about the extension of a clutter object increases the estimation accuracy and robustness of the estimation algorithm. It is necessary to correctly associate measurements to fixed points. However, the overall performance is crucially dependent on the number of available fixed points. The MHT structure of our estimation algorithm allows the incorporation of multiple association hypotheses. But, the need for hypotheses reduction techniques can provoke a wrong decision resulting in divergence of the estimation process. As implemented currently the estimation process is not able to recover. This could be solved by the implementation of a likelihood ratio test that restarts the estimation at a subsequent time step. This procedure would be similar to standard target tracking, where a diverging track is closed and a new track is initialized.

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Fig. 6. Estimation results of scenario A. Performance of our algorithm (RMSE) is compared to the CRLB (RTC).



Fig. 7. Estimation results of scenario B: Incorporation of the extension of the fixed points results in three out of four cases in improved RMSE values. For scenario Rx1Tx1 using 3 fixed points (b) we note a slightly worse RMSE but a reduced number of outlier tracks (Tab. II). Please note that the CRLB has been calculated for mean P_D and measurements errors values, it is used here to evaluate the estimation geometry, but provides not an absolute error bound.