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Generalizations to the Track-Oriented MHT Recursion

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***Abstract*—This paper addresses some aspects of multi-target tracking (MTT) with a specific focus on track-oriented multiple-hypothesis tracking (TO-MHT). First, we address the time-discretization of birth-death statistics, and propose an aggregation approach that is useful in low detection probability settings. A target stationarity assumption is required for use of aggregated statistics as part of the MTT solution. Second, we generalize the TO-MHT recursion to allow for redundant target measurements, and suggest a two-stage processing approach that can exploit the recursion while maintaining computational feasibility.**

***Keywords*—multi-target tracking (MTT); multiple-hypothesis tracking (MHT); track-oriented MHT (TO-MHT); undetected target births; redundant measurements.**

I. Introduction

Multi-target tracking (MTT) poses significant technical challenges principally due to the unknown time-varying number of targets as well as to measurement provenance uncertainty, i.e. which measurement originates from which target, and which measurements are false alarms [1]. These challenges are generally not found in classical detection, estimation, and nonlinear filtering problems. Many approaches have emerged over the years; among labelled-tracking approaches, track-oriented *multiple-hypothesis tracking* (MHT) is generally acknowledged as the most powerful paradigm [2]. Recently, we have extended the MHT formulation to allow for undetected target births. Crucially, this is achieved without increasing the number of track hypotheses and shows promising performance improvements in some multi-sensor settings [3-4].

Allowing for undetected target births enables more relevant track hypotheses to be considered, by identifying the *maximum a posteriori* (MAP) birth and death interval for each track. More generally, we may consider an aggregate track hypothesis that accounts for all possible birth and death intervals for a sequence of associated detections. Doing so requires that we first determine aggregate birth and death statistics; this is the topic of Section II, with further discussion in Section III.

The use of aggregate birth-death statistics as part of the MTT solution requires an assumption of target statistical stationarity. For example, stable motion models based on the *Ornstein Uhlenbeck* (OU) process admit stationary statistics; a more detailed discussion of 1st and 2nd order OU models may be found in [5-6].

MTT with redundant measurements poses a significant challenge. For simplicity, most paradigms adapt a Bernoulli measurement model. There are some exceptions, e.g. the *probabilistic MHT* (PMHT) and its *non-generative* sensor model, for which a nice discussion may be found in [7]. A complementary difficulty – merged measurements due to more than one target – also is not considered in most MTT treatments.

Redundant measurements induced by multipath phenomena or multiple emissions have been analyzed; see [8-10] and references therein. While these papers are of interest, they do not address the same problem that we consider here, where all redundant measurements are characterized by the *same* measurement equation. A recent treatment of redundant measurement in the context of *probability hypothesis density* (PHD) research is discussed in [11]. Both merged and redundant measurements are addressed using a *Markov Chain Monte Carlo* (MCMC) approach in [12], and in [13] in the context of multi-dimensional assignment algorithms; however, neither work appears to introduce likelihood adjustments consistent with an explicit measurement model for these phenomena.

In Sections IV-V, we provide an explicit derivation from first principles of the MHT recursion with a Poisson measurement model. The recursion for the general case is in Section VI. The Poisson case in particular takes a simple and appealing form that has significant similarities with the Bernoulli case.

We suggests a practical means to exploit the generalized MHT recursion via two-stage tracking whereby the computational complexity associated with redundant-measurement updates are deferred to the second, track-fusion stage, where the number of association hypotheses is greatly reduced. Conclusions and directions for future work are in Section VII.

II. Generalized Birth and Death Statistics

Our starting point is a continuous time birth-death process with exponentially distributed target inter-arrival (birth) times with parameter λ_b, and exponentially distributed target lifetime with parameter λ_χ. Discrete-time statistics may be readily obtained, leading to a Poisson distributed number of births with mean $\mu_b(t)$ and death probability $p_\chi(t)$ over an interval of duration t. The expressions are given in equations (1-2).

$$\mu_b(t) = \frac{\lambda_b}{\lambda_\chi}\left(1 - e^{-\lambda_\chi t}\right), \quad (1)$$

$$p_\chi(t) = 1 - e^{-\lambda_\chi t}. \quad (2)$$

Letting t_0 denote the initial time at which no targets are present, and assuming sensors scans at time $(t_i, i \geq 1)$ with detection probability p_d, we have the following aggregate birth rate $\hat{\mu}_b[k]$ that accounts for all previously unobserved targets for the scan at time t_k.

$$\hat{\mu}_b[k] = \sum_{i=1}^{k} \mu_b(t_i - t_{i-1})(1 - p_d)^{k-i} \cdot \left(1 - p_\chi(t_k - t_i)\right), k = 1, \dots, N. \quad (3)$$

In the time-invariant case, for which all inter-scan times are given by Δt, the aggregate birth rate takes a simpler form.

$$\hat{\mu}_b[k] = \textstyle\sum_{i=1}^{k} \mu_b(\Delta t)\,(1 - p_d)^{k-i}\left(e^{-\lambda_\chi \Delta t}\right)^{k-i}, \quad (4)$$

$$\alpha = (1 - p_d)e^{-\lambda_\chi \Delta t}, \quad (5)$$

$$\hat{\mu}_b[k] = \mu_b(\Delta t)\frac{(1-\alpha^k)}{1-\alpha}. \quad (6)$$

There are several limiting cases of interest, including those noted below. Perhaps the most useful is given by equation (12), as this captures the customary setting for many MTT applications.

1. $\Delta t \ll \lambda_\chi^{-1}$:

$$\hat{\mu}_b[k] \longrightarrow \lambda_b \Delta t \frac{1}{p_d}\left(1 - (1 - p_d)^k\right). \quad (7)$$

2. $p_d = 0$:

$$\hat{\mu}_b[k] = \frac{\lambda_b}{\lambda_\chi}\left(1 - e^{-k\lambda_\chi \Delta t}\right). \quad (8)$$

3. $p_d = 0, \Delta t \ll \lambda_\chi^{-1}$:

$$\hat{\mu}_b[k] \longrightarrow k\lambda_b \Delta t. \quad (9)$$

4. $p_d = 1$:

$$\hat{\mu}_b[k] = \mu_b(\Delta t). \quad (10)$$

5. $k \longrightarrow \infty$ (steady-state limit):

$$\hat{\mu}_b[k] \longrightarrow \mu_b(\Delta t)\frac{1}{1-\alpha}. \quad (11)$$

6. $k \longrightarrow \infty$ and $\Delta t \ll \lambda_\chi^{-1}$:

$$\boxed{\hat{\mu}_b[k] \longrightarrow \bar{\mu}_b = \frac{\lambda_b \Delta t}{p_d}.} \quad (12)$$

Likewise, the aggregate death probability $\hat{p}_\chi[k]$ accounts for the probability of target death at time t_k or subsequently and with no further detections.

$$\hat{p}_\chi[k] = \sum_{i=k}^{N}\left(1 - p_\chi(t_{i-1} - t_{k-1})\right)(1 - p_d)^{i-k} \cdot p_\chi(t_i - t_{i-1}), k = 2, \dots, N. \quad (13)$$

Once more, time-invariant case takes a simpler form.

$$\hat{p}_\chi[k] = \textstyle\sum_{i=k}^{N}\left(e^{-\lambda_\chi \Delta t}\right)^{i-k}(1 - p_d)^{i-k}\,p_\chi(\Delta t), \quad (14)$$

$$\hat{p}_\chi[k] = p_\chi(\Delta t)\frac{1-\alpha^{N-k+1}}{1-\alpha}. \quad (15)$$

Here too there are limiting cases of interest, including the following. The case of interest given by equation (21) is of interest for many MTT settings.

1. $\Delta t \ll \lambda_\chi^{-1}$:

$$\hat{p}_\chi[k] \longrightarrow \lambda_\chi \Delta t \frac{1-(1-p_d)^{N-k+1}}{p_d}. \quad (16)$$

2. $p_d = 0$:

$$\hat{p}_\chi[k] = 1 - \left(1 - p_\chi(\Delta t)\right)^{N-k+1}. \quad (17)$$

3. $p_d = 0, \Delta t \ll \lambda_\chi^{-1}$:

$$\hat{p}_\chi[k] \longrightarrow (N - k + 1)\lambda_\chi \Delta t. \quad (18)$$

4. $p_d = 1$:

$$\hat{p}_\chi[k] = p_\chi(\Delta t). \quad (19)$$

5. $N \longrightarrow \infty$ (steady-state or infinite-horizon limit):

$$\hat{p}_\chi[k] \longrightarrow p_\chi(\Delta t)\frac{1}{1-\alpha}. \quad (20)$$

6. $N \longrightarrow \infty$ and $\Delta t \ll \lambda_\chi^{-1}$:

$$\boxed{\hat{p}_\chi[k] \longrightarrow \bar{p}_\chi = \frac{\lambda_\chi \Delta t}{p_d}.} \quad (21)$$

To our knowledge, the use of aggregate birth and death statistics has not been considered in the MTT community. Rather, only the current time interval is considered, consistent with equations (1-2). These differ noticeably from the aggregate statistics in settings where the detection probability is low.

A first demonstration of the potential benefits of revised birth/death statistics in the context of MTT processing is described in [4] and illustrated in Figure 1.

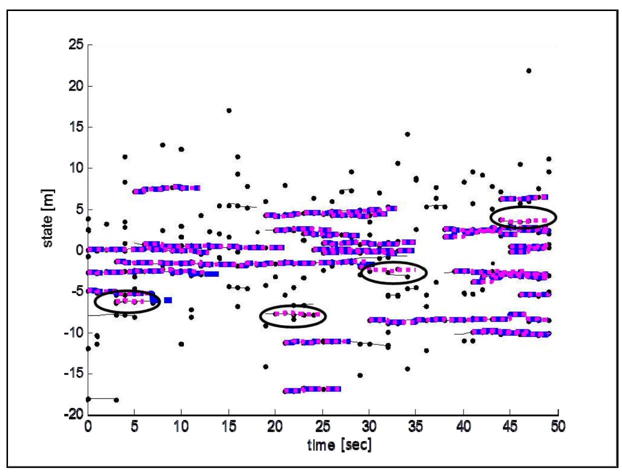

Figure 1. Enhanced MHT processing extracts short-duration targets from the data that classical MHT cannot find.

The approach taken in [4] identifies the likeliest intervals of birth and deaths for each track hypothesis; an illustration of the benefits of doing so is illustrated in Figure 1. This can be viewed as a first step towards the more powerful aggregated-statistics approach described in this paper.

Interestingly, the use of an aggregated birth rate within the MTT solution does rely on the subtle assumption that the time of birth does not impact the likelihood function associated with the first sensor measurement. This is not true in general, but is true under the assumption of statistical stationarity.

III. Further Considerations

In principle, MTT solutions must account for those targets that are never detected. In some cases e.g. high birth rate, short expected lifetime, and low detection probability, this may have a nontrivial impact on the expected number of targets identified by the MTT solution. There is a principled way for identifying the *maximum a posteriori* (MAP) set of ghost targets within the enhanced MHT paradigm. The reader will find details in [4].

As with the detected-target solution, aggregated statistics can improve the estimate of the number of ghost targets as well, at the cost of no precise birth-death structure as we obtain with the MAP solution. The death and birth statistics of ghost targets are given below.

$$\tilde{p}_\chi(\Delta t) = 1 - \left(1 - p_\chi(\Delta t)\right)(1 - p_d), \quad (22)$$

$$\tilde{\mu}_b(\Delta t) = \frac{(1-p_d)\mu_b(\Delta t)}{\tilde{p}_\chi(\Delta t)}. \quad (23)$$

Limiting case:

- $\Delta t \ll \lambda_\chi^{-1}$:

$$\tilde{\mu}_b(\Delta t) \longrightarrow \tilde{\mu}_b = \frac{(1-p_d)\lambda_b \Delta t}{p_d}. \quad (24)$$

Note:

$$\tilde{\mu}_b = (1 - p_d)\bar{\mu}_b. \quad (25)$$

Aside from those targets that are never detected, it is worth noting that there are also unaccounted for targets resulting from the discretization of birth-death statistics. Specifically, targets that are born and die within the same discretization interval are not accounted for in the discretized birth-death statistics. Hence, there is a slight approximation introduced by time discretization. This may seem like an obvious point, but note that no approximation is introduced when, for instance, discretizing a (linear) continuous-time motion model via time integration.

The discrepancy between continuous time and discrete time birth rates is noted below. Not surprisingly, the discrepancy is small for small discretization intervals.

$$\tilde{\mu}_a = \lambda_b(t_N - t_0) - \textstyle\sum_{i=1}^N \mu_b(t_i - t_{i-1}). \quad (26)$$

Time-invariant case:

$$\tilde{\mu}_a = N\big(\lambda_b \Delta t - \mu_b(\Delta t)\big). \quad (27)$$

Limiting case:

- $\Delta t \longrightarrow 0$:

$$\tilde{\mu}_a \longrightarrow N\left(\lambda_b \Delta t - \frac{\lambda_b}{\lambda_\chi}\left(1 - \left(1 - \lambda_\chi \Delta t + \frac{\left(\lambda_\chi \Delta t\right)^2}{2}\right)\right)\right)$$
$$= \frac{N\lambda_b\lambda_\chi\Delta t^2}{2}. \quad (28)$$

Note that, in the absence of filter innovation scores, measurement association hypotheses would rely on the following condition to be satisfied for a single-target hypothesis to exceed the probability of a two-target hypothesis.

$$\mu_b(\Delta t)\frac{1}{1-\alpha} \cdot p_\chi(\Delta t)\frac{1}{1-\alpha} \le 1 - p_\chi(\Delta t), \quad (29)$$

Limiting cases:

1. LHS upper bound for arbitrary p_d (equality for $p_d = 0$):

$$\mu_b(\Delta t)\frac{1}{1-\alpha} \cdot p_\chi(\Delta t)\frac{1}{1-\alpha} \le \frac{\lambda_b}{\lambda_\chi}. \quad (30)$$

2. Necessary condition for arbitrary p_d and Δt:

$$\lambda_b \le \lambda_\chi. \quad (31)$$

As a final consideration, it is helpful for some applications to consider a single observation time that differs from an otherwise regular sensor scan time with interval Δt. For instance, we may have a passive emission at an arbitrary time that must be combined with active sensor data with regular observation times.

In particular, for $\Delta t \ll \lambda_\chi^{-1}$ and with t the time since last sensor scan, we have:

$$\boxed{\mu_b(t, \Delta t) = (1 - p_d)\bar{\mu}_b + \lambda_b t.} \quad (32)$$

Simlilarly, for $\Delta t \ll \lambda_\chi^{-1}$ and with t the time until next sensor scan, we have:

$$\boxed{p_\chi(t, \Delta t) = \lambda_\chi t + (1 - p_d)\bar{p}_\chi.} \quad (33)$$

The expressions given in equations (32-33) are closely related to the time-invariant case discussed previously. In particular, we have the following relations.

$$\mu_b(t, \Delta t) \le \bar{\mu}_b, \quad (34)$$

$$\mu_b(\Delta t, \Delta t) = \bar{\mu}_b. \quad (35)$$

$$p_\chi(t, \Delta t) \le \bar{p}_\chi, \quad (36)$$

$$p_\chi(\Delta t, \Delta t) = \bar{p}_\chi. \quad (37)$$

IV. MHT Recursion With Redundant Measurements

We proceed with a hybrid-state decomposition approach with the usual notation [2,4]. The key innovation is that we consider an arbitrary measurement cardinality model.

$$\boxed{p(q_k|Z^{k-1}, q^{k-1}) = p(\psi_k|Z^{k-1}, q^{k-1}) \cdot p(q_k|Z^{k-1}, q^{k-1}, \psi_k).} \quad (38)$$

$$p(\psi_k|Z^{k-1}, q^{k-1}) = \left\{\binom{\tau}{\chi} p_\chi^\chi (1 - p_\chi)^{\tau-\chi}\right\}$$

$$\cdot\left\{\frac{(\tau-\chi)!}{\prod_i d_i!}\prod_i p(i)^{d_i}\right\}\cdot\left\{\frac{\Lambda^{n_1}e^{-\Lambda}}{n_1!}\right\}$$

$$\cdot\left\{\prod_i \frac{e^{-p(i)\mu_b}p(i)^{b_i}\mu_b^{b_i}}{b_i!}\right\}, \tag{39}$$

$$p(q_k|Z^{k-1},q^{k-1},\psi_k)=\frac{1}{\left\{\begin{array}{c}\binom{\tau}{\chi}\left(\frac{(\tau-\chi)!}{\prod_i d_i!}\right)\left(\prod_i\frac{(d_i+b_i+n_i)!}{(b_i+n_i)!}\right)\\ \cdot\left(\frac{r!}{\prod_i (d_i+b_i+n_i)!(i!)^{d_i+b_i}}\right)\\ \cdot\prod_i\binom{b_i+n_i}{b_i}\end{array}\right\}}, \tag{40}$$

$$\prod_i\binom{b_i+n_i}{b_i}=\prod_i\frac{(b_i+n_i)!}{b_i!n_i!}, \tag{41}$$

$$n_1+\sum_i ib_i+\sum_i id_i=r, \tag{42}$$

$$\sum_i d_i=\tau-\chi. \tag{43}$$

Notes:

- The factors in (39) represent, respectively: the probability of χ deaths among τ tracks; the probability of measurement cardinalities according to d_i for surviving tracks, using the multinomial distribution; the probability of n_1 false alarms; and the probability of birth cardinalities according to b_i, using Poisson sifting.
- The denominator factors in (40) represent, respectively: the number of ways to select track terminations; the number of ways of selecting tracks for specific cardinality updates; the number of ways of assigning measurement clusters to tracks; the number of ways of assigning measurements to clusters (order does not matter); and the number of ways of selecting birth clusters.
- The total number of returns is r according to (42); the total number of update clusters is $\tau-\chi$ according to (43).
- All products and summations are over $i=0,\ldots$
- Undetected births and terminal missed detections are accounted for via generalized birth and death statistics. Thus w.l.o.g. we have $b_0=0$.
- There are n false alarms, and $n_1=n$.
- The measurement-cardinality distribution is $p(\cdot)$.

V. MHT Recursion With Redundant Measurements: The Poisson Case

$$p(i)=\frac{\lambda^i}{i!}e^{-\lambda}. \tag{44}$$

Substitution of (44) into (39):

$$p(\psi_k|Z^{k-1},q^{k-1})=\left\{\binom{\tau}{\chi}p_\chi^\chi(1-p_\chi)^{\tau-\chi}\right\}$$

$$\cdot\left\{\frac{(\tau-\chi)!}{\prod_i d_i!}\prod_i\left(\frac{\lambda^{id_i}e^{-\lambda d_i}}{(i!)^{d_i}}\right)\right\}\cdot\left\{\frac{\Lambda^{n_1}e^{-\Lambda}}{n_1!}\right\}$$

$$\cdot\left\{e^{-\mu_b}\prod_i\frac{\lambda^{ib_i}e^{-\lambda b_i}\mu_b^{b_i}}{(i!)^{b_i}b_i!}\right\}, \tag{45}$$

Reciprocal factors among (40-41) and (44) are noted in red; cancellations within (40-41) are noted in blue. Combining (40-41) and (44) according to (38) results in the following:

$$p(q_k|Z^{k-1},q^{k-1})=\left\{\frac{\Lambda^{n_1}e^{-\Lambda}e^{-\mu_b}}{r!}\right\}$$

$$\cdot p_\chi^\chi(1-p_\chi)^{\tau-\chi}\prod_i(\lambda^{id_i}e^{-\lambda d_i})\prod_i(\lambda^{ib_i}e^{-\lambda b_i}\mu_b^{b_i}). \tag{46}$$

Further manipulation and use of (42-43) yields the following:

$$p(q_k|Z^{k-1},q^{k-1})=\left\{\frac{\Lambda^r e^{-\Lambda}e^{-\mu_b}}{r!}\right\}$$

$$\cdot p_\chi^\chi\prod_i\left(\frac{\lambda^i}{\Lambda^i}e^{-\lambda}(1-p_\chi)\right)^{d_i}\prod_i\left(\frac{\lambda^i}{\Lambda^i}e^{-\lambda}\mu_b\right)^{b_i}. \tag{47}$$

Notes:

- The generalized MHT recursion with Poisson-distributed measurement cardinality is such that: (i) for track birth and update hypotheses, the denominator factorial in the Poisson distribution is absent due to hypothesis aggregation over indistinguishable ordering of the measurements; (ii) conditioned on all other assignments, a measurement is equally likely to be associated with any track, save for the impact of filter innovation scores.
- Use of the generalized MHT recursion is potentially problematic due to the large number of track hypotheses. This can be addressed via two-stage processing whereby redundant-measurement hypotheses are only considered in the second (track-fusion) stage.
- We have $1-p_d=e^{-\lambda}$. Substitution into (47) yields a form that is similar to the classical MHT recursion based on the Bernoulli distribution for measurement cardinality.

$$p(q_k|Z^{k-1},q^{k-1})$$

$$=\left\{\frac{\Lambda^r e^{-\Lambda}e^{-\mu_b}}{r!}\right\}p_\chi^\chi\left((1-p_d)(1-p_\chi)\right)^{d_0}$$

$$\cdot\prod_{i>0}\left(\frac{\lambda^i}{\Lambda^i}e^{-\lambda}(1-p_\chi)\right)^{d_i}\prod_i\left(\frac{\lambda^i}{\Lambda^i}e^{-\lambda}\mu_b\right)^{b_i}. \tag{48}$$

VI. MHT Recursion With Redundant Measurements: The General Case

The MHT recursion in the general case can still be expressed in factored form – enabling track-oriented MHT – and the expression is only slightly more complex than (47). It is obtained by combining (39-40) with many cancellations as in the Poisson case. The final form is given below.

$$p(q_k|Z^{k-1},q^{k-1})=\left\{\frac{\Lambda^r e^{-\Lambda}e^{-\mu_b}}{r!}\right\}$$

$$\cdot\, p_\chi^\chi \prod_i \left(\frac{p(i)i!}{\Lambda^i}\left(1-p_\chi\right)\right)^{d_i} \prod_i \left(\frac{p(i)i!}{\Lambda^i}\mu_b\right)^{b_i}. \qquad (49)$$

Notes:

- Global hypothesis factorization is achieved.
- The number of measurements in a cluster impacts explicitly the global hypothesis score. The benefit of the principled derivation is to establish the exact dependence, which would be difficult otherwise to infer. The explicit dependence on measurement-cluster cardinality vanishes in the Poisson case (47).

VII. CONCLUSIONS

This paper proposes the use of generalized birth-death statics for MTT, and a generalized TO-MHT that handles arbitrary measurement cardinalities. Special cases of interest include the Bernoulli case – the well-known approach that has been treated in the literature – and the Poisson case studied here. Use of the generalized MHT recursion in practical setting will likely rely on two-stage processing whereby redundant updates are considered only for track-level data.

It is interesting to compare the Poisson MHT recursion with the Bernoulli MHT recursion. In the Poisson recursion, we have a factor of $e^{-\lambda}$ in all track initiation, update, and missed detection hypotheses. This is analogous to what is done in the Bernoulli case only for missed detection hypotheses – indeed, recall that $1-p_d=e^{-\lambda}$. Also, in the Poisson recursion each measurement requires a factor $\lambda/_{\Lambda}$. Thus, in single-measurement birth and update hypotheses, we have an overall factor of $\lambda e^{-\lambda}$ that plays the same role as p_d in the Bernoulli case (though they are not equal).

Interestingly, since use of a redundant measurement incurs the same factor $\lambda/_{\Lambda}$ regardless of how many other measurements are used in a given track hypothesis, there is no penalty associated with redundant updates in the Poisson case.

In ongoing research, we are exploring the effectiveness of generalized MHT processing using the two innovations discussed in this paper – aggregate birth-death statistics and redundant measurement processing – in challenging and practical settings where multi-stage MHT continues to be the leading paradigm for high-performance MTT.

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