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A Revised Method for Ranking Generalized Fuzzy Numbers

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Abstract*—A number of fuzzy number ranking methods have been proposed by the researchers in recent years. However, most of them have exhibited several shortcomings associated with non-discriminative and counter-intuitive problems, especially, when the fuzzy numbers are symmetric fuzzy numbers or crisp numbers. In this paper, we propose a new method for ranking generalized fuzzy numbers where the weight of centroid points, degrees of fuzziness and the spreads of fuzzy numbers are taken into consideration, which can overcome the drawbacks of exiting methods and is efficient for evaluating symmetric fuzzy numbers and crisp numbers. At last, several numerical examples are provided to illustrate the superiority of the proposed method.

Keywords—generalized fuzzy numbers, the degree of fuzziness, centroid point, spread, weight.

1 Introduction

In recent years, a lot of achievements about the theory and method of ranking fuzzy numbers are obtained. Since Jain [1] proposed the first fuzzy number ranking method, various methods have been devised for ranking fuzzy numbers [2–19]. Yager (1978) [2] proposed the centroid index ranking method with a weighting function. Cheng (1998) [3] presented an approach for ranking fuzzy numbers by using the distance method, where the distance represents the original point to the centroid point. Chu and Tsao (2002) [4] proposed an approach for ranking fuzzy numbers with the area between the centroid point and original point. Chen and Chen (2007) [6] presented a method for ranking generalized fuzzy numbers based on the centroid points and the standard deviations of generalized fuzzy numbers. Wang et al. (2009) [8] presented a method for ranking fuzzy numbers by combining the transfer coefficient and LR deviation degree of a fuzzy number. Chen and Chen (2009) [9] presented a method for ranking generalized fuzzy numbers by considering the defuzzified values, the heights and the spreads of the generalized fuzzy numbers. Nejad and Mashinchi (2011) [11] proposed a method for ranking fuzzy numbers based on the areas of the LR sides of the fuzzy numbers. Chen and Sanguansat (2011) [12] proposed a method by considering the areas on the positive side, the areas on the negative side and the heights of the generalized fuzzy numbers to evaluate the ranking scores of the generalized fuzzy numbers. Chen (2012) [14] presented a method for ranking generalized fuzzy numbers with different left heights and right heights. Emrah et al. (2013) [15] presented a new method for ranking generalized trapezoidal fuzzy numbers based on the incenter and inradius of a triangle, which rank crisp numbers and fuzzy numbers with the same centroid point. Madhuri et al. (2014) [17] proposed a new method for ranking generalized trapezoidal fuzzy numbers based on the Circumcentre points. Bakar and Gegoy (2014) [18] proposed a novel method for ranking fuzzy numbers which integrated the centroid point and the spread approaches and overcomes the limitations and weaknesses of some existed methods. Wang (2015) [19] first proposed a fuzzy preference relation with membership function representing preference degree to compare two fuzzy numbers. Then a relative preference relation was constructed on the fuzzy preference relation to rank a set of fuzzy numbers.

The ranking methods stated above are commonly used approaches, which are highly cited and have widely applied. However, they still have some drawbacks such as non-discriminative and counter-intuitive problems. Therefore, in this paper, we present a new method for ranking generalized fuzzy numbers based on centroid points, degrees of fuzziness and spreads of fuzzy numbers, which can overcome the drawbacks of existing methods. Meanwhile, the proposed method considers that the weights of all factors which influence the ranking result should be different. Finally, we make a comparison of the calculation results with the existing methods to illustrate the superiority of the proposed method.

The rest of this paper is organized as follows. In Section 2, we briefly review the basic concepts of generalized fuzzy numbers. In Section 3, we present a new method for ranking generalized fuzzy numbers based on centroid points, degrees of fuzziness and spreads of fuzzy numbers. In Section 4, we make a comparison of the ranking results of the proposed fuzzy ranking method with the existing methods. In Section 5, the conclusions are discussed.

2 Preliminaries

In this section, we briefly review some basic concepts of generalized fuzzy numbers.

2.1 Generalized fuzzy numbers

Chen (2012) [14] proposed the concept of generalized fuzzy numbers with the different left height μ_L and right height μ_R. Let $\tilde{A}$ be a generalized fuzzy number with the different left height and right height, $\tilde{A}=(a_1,a_2,a_3,a_4;\mu_L,\mu_R)$, where a_1,a_2,a_3,a_4 are real values, μ_L is called the left height of the generalized fuzzy number $\tilde{A}$, μ_R is called the right height of the generalized fuzzy number, $\mu_L\in[0,1]$ and $\mu_R\in[0,1]$. If $0\le a_1\le a_2\le a_3\le a_4\le 1$, then $\tilde{A}$ is called a standardized generalized fuzzy number. If $\mu_L=\mu_R=1$, then $\tilde{A}$ becomes a normal trapezoidal fuzzy number. If $a_2=a_3$, $\tilde{A}$ is called a generalized triangular fuzzy number. If $a_1=a_2=a_3=a_4$, $\tilde{A}$ is called a crisp number. The membership function $f_{\tilde{A}}(x)$ of a generalized fuzzy number $\tilde{A}$ and the membership function $f_{\tilde{F}}(x)$ of a fuzzy number $\tilde{F}$ are shown in Figure1.

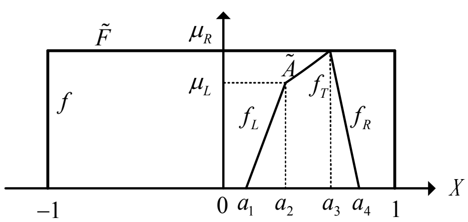

Figure1. Membership functions of generalized fuzzy numbers $\tilde{A}$ and $\tilde{F}$

2.2 The degree of fuzziness

DeLuca and Termini proposed Shannon entropy as the measure of degree of fuzziness in 1972 [20]. Let A be a fuzzy set in the universe of U and all the fuzzy sets on U are denoted by $F(U)$. The membership function of A is denoted by $A(x)$, the degree of fuzziness of A is denoted by $d(A)$. For $A\in F(U)$, the degree of fuzziness $d(A)$ or $d(B)$ satisfies the following properties:

(1)If $A\in F(U)$, $d(A)=0$.

(2)If $A(x)\equiv 0.5$, $d(A)=1$.

(3)If $A(x)\le B(x)\le 0.5$ or $A(x)\ge B(x)\ge 0.5$, $d(A)\le d(B)$.

GuoChun Tang [21] proposed a revised degree of fuzziness which called area degree of fuzziness in 1999. Simultaneously, he also presented the Minkowski degree of fuzziness on the continuous fuzzy set. Now we briefly review the area degree of fuzziness and Minkowski degree of fuzziness on the continuous fuzzy set. If $U=[\alpha,\beta]$, fuzzy number $\tilde{A}=(a_1,a_2,a_3,a_4;w)\in U$, then the area degree of fuzziness of $\tilde{A}$ is defined as follows：

$$d(\tilde{A})=\frac{2}{\beta-a}\left[\int_{A(x)\le 0.5}A(x)dx+\int_{A(x)\ge 0.5}(1-A(x))dx\right], \quad (1)$$

where $d(\tilde{A})$ is the area degree of fuzziness of $\tilde{A}$.

The Minkowski degree of fuzziness of $\tilde{A}$ is defined as follow：

$$d(\tilde{A})=2\left[\frac{1}{\beta-a}\int_{\alpha}^{\beta}\left|A(x)-A_{0.5}(x)\right|^{p}dx\right]^{\frac{1}{p}}. \quad (2)$$

If $p=1$, the Minkowski degree of fuzziness of $\tilde{A}$ becomes area degree of fuzziness.

3 A new method for ranking generali - zed fuzzy numbers

In this section, we proposed a new method for ranking generalized fuzzy numbers based on centroid points, degrees of fuzziness and spreads of fuzzy numbers. The proposed method indicates that the value of the centroid point on the horizontal axis($x_{\tilde{A}_i}$) is the most important index for ranking generalized fuzzy numbers, whereas the value of the centroid point on the vertical axis($y_{\tilde{A}_i}$), the spread and the degree of fuzziness are the aid indexes. Therefore, all the indexes have different weight on the process of ranking generalized fuzzy numbers.

3.1 The proposed method

Assume that there are n generalized fuzzy numbers $\tilde{A}_1,\tilde{A}_2,\ldots,\tilde{A}_n$ to be ranked, where $\tilde{A}_i=(a_{i1},a_{i2},a_{i3},a_{i4};\mu_{iL},\mu_{iR})$, $-1\le a_{i1}\le a_{i2}\le a_{i3}\le a_{i4}\le 1$; $\mu_{iL}\in[0,1]$, $\mu_{iR}\in[0,1]$, μ_{iL} denotes the left height of fuzzy number $\tilde{A}_i$, μ_{iR} denotes the right height of fuzzy number $\tilde{A}_i$ and $1\le i\le n$.The proposed method is shown as follows:

Step 1: According to the method proposed by Wang et al. (2006) [22], calculate the each centroid point $(x_{\tilde{A}_i},y_{\tilde{A}_i})$ of generalized fuzzy numbers, shown as follows.

Case 1: If $\tilde{A}_i=(a_{i1},a_{i2},a_{i3},a_{i4};\mu_{iL},\mu_{iR})$ and $\mu_{iL}=\mu_{iR}=\mu_i$, then the centroid point is shown as follows:

$$x_{\tilde{A}_i}=\frac{\int_{a_{i1}}^{a_{i2}}(xf_{\tilde{A}_i}^{L})dx+\int_{a_{i2}}^{a_{i3}}xdx+\int_{a_{i3}}^{a_{i4}}(xf_{\tilde{A}_i}^{R})dx}{\int_{a_{i1}}^{a_{i2}}(f_{\tilde{A}_i}^{L})dx+\int_{a_{i2}}^{a_{i3}}dx+\int_{a_{i3}}^{a_{i4}}(f_{\tilde{A}_i}^{R})dx}=\frac{1}{3}\left(a_{i1}+a_{i2}+a_{i3}+a_{i4}-\frac{a_{i3}a_{i4}-a_{i1}a_{i2}}{(a_{i3}+a_{i4})-(a_{i1}+a_{i2})}\right), \quad (3)$$

$$y_{\tilde{A}_i}=\frac{\int_0^{\mu_i}(yg_{\tilde{A}_i}^{R}-yg_{\tilde{A}_i}^{L})dy}{\int_0^{\mu_i}(g_{\tilde{A}_i}^{R}-g_{\tilde{A}_i}^{L})dy}=\frac{\mu_i}{3}\left(1+\frac{a_{i3}-a_{i2}}{(a_{i3}+a_{i4})-(a_{i1}+a_{i2})}\right) \quad (4)$$

where $1 \le i \le n$, $g_{\tilde{A}_i}^R$ and $g_{\tilde{A}_i}^L$ are the inverse functions of $f_{\tilde{A}_i}^R$ and $f_{\tilde{A}_i}^L$ respectively.

Case 2: If $\tilde{A}_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; \mu_{iL}, \mu_{iR})$ and $\mu_{iL} < \mu_{iR}$, then the centroid point is shown as follows:

$$x_{\tilde{A}_i} = \frac{\int_{a_{i1}}^{a_{i2}} (x f_{\tilde{A}_i}^L) dx + \int_{a_{i2}}^{a_{i3}} x dx + \int_{a_{i3}}^{a_{i4}} (x f_{\tilde{A}_i}^R) dx}{\int_{a_{i1}}^{a_{i2}} (f_{\tilde{A}_i}^L) dx + \int_{a_{i2}}^{a_{i3}} dx + \int_{a_{i3}}^{a_{i4}} (f_{\tilde{A}_i}^R) dx}, \tag{5}$$

$$y_{\tilde{A}_i} = \frac{\int_0^{\mu_{iL}} (y g_{\tilde{A}_i}^R - y g_{\tilde{A}_i}^L) dy + \int_{\mu_{iL}}^{\mu_{iR}} (y g_{\tilde{A}_i}^R - y g_{\tilde{A}_i}^T) dy}{\int_0^{\mu_{iL}} (g_{\tilde{A}_i}^R - g_{\tilde{A}_i}^L) dy + \int_{\mu_{iL}}^{\mu_{iR}} (g_{\tilde{A}_i}^R - g_{\tilde{A}_i}^T) dy}, \tag{6}$$

where $1 \le i \le n$, $g_{\tilde{A}_i}^R$, $g_{\tilde{A}_i}^L$ and $g_{\tilde{A}_i}^T$ are the inverse functions of $f_{\tilde{A}_i}^R$, $f_{\tilde{A}_i}^L$ and $f_{\tilde{A}_i}^T$ respectively.

Case 3: If $\mu_{iL} > \mu_{iR}$, then the centroid point is shown as follows:

$$x_{\tilde{A}_i} = \frac{\int_{a_{i1}}^{a_{i2}} (x f_{\tilde{A}_i}^L) dx + \int_{a_{i2}}^{a_{i3}} x dx + \int_{a_{i3}}^{a_{i4}} (x f_{\tilde{A}_i}^R) dx}{\int_{a_{i1}}^{a_{i2}} (f_{\tilde{A}_i}^L) dx + \int_{a_{i2}}^{a_{i3}} dx + \int_{a_{i3}}^{a_{i4}} (f_{\tilde{A}_i}^R) dx}, \tag{7}$$

$$y_{\tilde{A}_i} = \frac{\int_0^{\mu_{iR}} (y g_{\tilde{A}_i}^R - y g_{\tilde{A}_i}^L) dy + \int_{\mu_{iR}}^{\mu_{iL}} (y g_{\tilde{A}_i}^T - y g_{\tilde{A}_i}^L) dy}{\int_0^{\mu_{iR}} (g_{\tilde{A}_i}^R - g_{\tilde{A}_i}^L) dy + \int_{\mu_{iR}}^{\mu_{iL}} (g_{\tilde{A}_i}^T - g_{\tilde{A}_i}^L) dy}, \tag{8}$$

where $1 \le i \le n$, $g_{\tilde{A}_i}^R$, $g_{\tilde{A}_i}^L$ and $g_{\tilde{A}_i}^T$ are the inverse functions of $f_{\tilde{A}_i}^R$, $f_{\tilde{A}_i}^L$ and $f_{\tilde{A}_i}^T$ respectively.

Step 2: Calculate the Minkowski degree of fuzziness of each generalized fuzzy numbers when $p = 2$ according to the method proposed by Guo-Chun Tang, shown as follows:

$$\begin{aligned} d(\tilde{A}_i) &= 2[\frac{1}{\beta - a} \int_\alpha^\beta \left| f_{\tilde{A}_i}(x) - f_{0.5}(x) \right|^2 dx]^{\frac{1}{2}} \\ &= 2[\frac{1}{\beta - a} (\int_{a_{i1}}^{a_{i2}} \left| 0.5 - \left| f_{\tilde{A}_i}^L(x) - 0.5 \right| \right|^2 dx \\ &+ \int_{a_{i2}}^{a_{i3}} \left| 0.5 - \left| f_{\tilde{A}_i}^T(x) - 0.5 \right| \right|^2 dx + \int_{a_{i3}}^{a_{i4}} \left| 0.5 - \left| f_{\tilde{A}_i}^R(x) - 0.5 \right| \right|^2 dx)]^{\frac{1}{2}}, \end{aligned} \tag{9}$$

where $\tilde{A}_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4}; \mu_{iL}, \mu_{iR}) \in [-1, 1]$ and $\beta = 1, \alpha = -1$.

Step 3: Calculate the spread of each generalized fuzzy number, shown as follows:

$$STD_{\tilde{A}_i} = \sqrt{\frac{\sum_{j=1}^{4} (a_{ij} - \bar{x}_{\tilde{A}_i})^2}{4 - 1}}, \tag{10}$$

where $1 \le i \le n$ and $\bar{x}_{\tilde{A}_i} = \frac{a_{i1} + a_{i2} + a_{i3} + a_{i4}}{4}$.

Step 4: Calculate the $score(\tilde{A}_i)$ of each generalized fuzzy number, shown as follows:

$$score(\tilde{A}_i) = \frac{\mu(x_{\tilde{A}_i})[\left| x_{\tilde{A}_i} \right| + \omega_1 \sqrt{2(x_{\tilde{A}_i}^2 + y_{\tilde{A}_i}^2)}]}{1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_i} + \omega_3 d(\tilde{A}_i)}, \tag{11}$$

where $0 \le \omega_2 \le \omega_1 < \omega_3 < 1$ and they were given by the decision-maker. $\mu(x)$ is shown as follows:

$$\mu(x) = \begin{cases} 1, & x \in [0,1]; \\ -1, & x \in [-1,0). \end{cases}$$

If $\tilde{A}_i = (a_i, a_i, a_i, a_i; \mu_i, \mu_i)$ is a crisp number, the centroid point is shown as follows:

$$x_{\tilde{A}_i} = a_i, \tag{12}$$

$$y_{\tilde{A}_i} = \mu_i. \tag{13}$$

The Minkowski degree of fuzziness of crisp number $\tilde{A}_i$ is shown as follows:

$$d(\tilde{A}_i) = 0. \tag{14}$$

The spread of crisp number $\tilde{A}_i$ is shown as follows:

$$STD_{\tilde{A}_i} = 0. \tag{15}$$

The score of crisp number $\tilde{A}_i$ is shown as follows:

$$score(\tilde{A}_i) = \frac{\mu(a_i)[\left| a_i \right| + \omega_1 \sqrt{2(a_i + \mu_i)}]}{1 + 2\omega_1}, \tag{16}$$

where $1 \le i \le n$

3.2 The properties of proposed fuzzy score function

In the following, we present four properties of the proposed fuzzy score function.

Property1 (Normalization property): If $\tilde{A}_1$ be a generalized fuzzy number, where $\tilde{A}_1 = (a_1, a_2, a_3, a_4; \mu_1, \mu_2)$, $-1 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ and $\mu_1, \mu_2 \in [0,1]$, then $score(\tilde{A}_1) \in [-1,1]$

Proof: If $\tilde{A}_1 = (a_1, a_2, a_3, a_4; \mu_1, \mu_2)$, where $-1 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ and $\mu_1, \mu_2 \in [0,1]$, then based on (3)-(10), we can know

$\left| x_{\tilde{A}_1} \right| \in [0,1]$, $\sqrt{2(x_{\tilde{A}_1}^2 + y_{\tilde{A}_1}^2)} \in [0,2]$, $STD_{\tilde{A}_1} \in [0,1]$, $d(\tilde{A}_1) \in [0,1]$. So

$\left| x_{\tilde{A}_1} \right| + \omega_1 \sqrt{2(x_{\tilde{A}_1}^2 + y_{\tilde{A}_1}^2)} \le 1 + 2\omega_1 \le 1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_1} + \omega_3 d(\tilde{A}_1)$,

then

$$\frac{\left| x_{\tilde{A}_1} \right| + \omega_1 \sqrt{2(x_{\tilde{A}_1}^2 + y_{\tilde{A}_1}^2)}]}{1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_1} + \omega_3 d(\tilde{A}_1)} \in [0,1].$$

Therefore, we can get

$$score(\tilde{A}_1) = \frac{\mu(x)[\left| x_{\tilde{A}_1} \right| + \omega_1 \sqrt{2(x_{\tilde{A}_1}^2 + y_{\tilde{A}_1}^2)}]}{1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_1} + \omega_3 d(\tilde{A}_1)} \in [-1,1].$$

Property 2(symmetric property) : If $\tilde{A}_1 = (a_1, a_2, a_3, a_4; \mu_L, \mu_R)$ and $\tilde{A}_2 = (-a_4, -a_3, -a_2, -a_1; \mu_R, \mu_L)$ be generalized fuzzy numbers, where $-1 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ and $\mu_L, \mu_R \in [0,1]$, then $Score(\tilde{A}_1) = -Score(\tilde{A}_2)$.

Proof: If $\tilde{A}_1 = (a_1, a_2, a_3, a_4; \mu_L, \mu_R)$ and $\tilde{A}_2 = (-a_4, -a_3, -a_2, -a_1; \mu_R, \mu_L)$ where $-1 \le a_1 \le a_2 \le a_3 \le a_4 \le 1$ and $\mu_L, \mu_R \in [0,1]$, then based on (3)-(8), we can see that

$x_{\tilde{A}_1} = -x_{\tilde{A}_2}$, $y_{\tilde{A}_1} = y_{\tilde{A}_2}$.

Based on (9) and (10), we can see that

$STD_{\tilde{A}_1} = STD_{\tilde{A}_2}$ $d(\tilde{A}_1) = d(\tilde{A}_2)$.

Based on (11), we can see that

$$\frac{\left|x_{\tilde{A}_1}\right| + \omega_1\sqrt{2(x_{\tilde{A}_1}^2 + y_{\tilde{A}_1}^2)}}{1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_1} + \omega_3 d(\tilde{A}_1)} = \frac{\left|x_{\tilde{A}_2}\right| + \omega_1\sqrt{2(x_{\tilde{A}_2}^2 + y_{\tilde{A}_2}^2)}}{1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_2} + \omega_3 d(\tilde{A}_2)}$$

$\mu(x_{\tilde{A}_1}) = -\mu(x_{\tilde{A}_2})$

Therefore, we can get $Score(\tilde{A}_1) = -Score(\tilde{A}_2)$.

Property3. If $\tilde{A}_1 = (a, a, a, a; a, a)$ where $-1 \le a \le 1$, then $Score(\tilde{A}_1) = a$

Proof: If $\tilde{A}_1 = (a, a, a, a; a, a)$ where $-1 \le a \le 1$, then based on (12), (13), (14)and(15), we can see that:

$x_{\tilde{A}_1} = a,\ y_{\tilde{A}_1} = a,\ STD_{\tilde{A}_1} = 0,\ d(\tilde{A}_1) = 0$.

So

$$score(\tilde{A}_1) = \frac{\mu(a)(\left|x_{\tilde{A}_1}\right| + \omega_1\sqrt{2(x_{\tilde{A}_1}^2 + y_{\tilde{A}_1}^2)}}{1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_1} + \omega_3 d(\tilde{A}_1)} = \frac{\mu(a)(|a| + \omega_1\sqrt{4a^2})}{1 + 2\omega_1}$$

If $a \ge 0$, $\mu(a) = 1$, then

$$score(\tilde{A}_1) = \frac{\mu(a)(|a| + \omega_1\sqrt{4a^2})}{1 + 2\omega_1} = \frac{a + 2\omega_1 a}{1 + 2\omega_1} = a$$

If $a < 0$, $\mu(a) = -1$, then

$$score(\tilde{A}_1) = \frac{\mu(a)(|a| + \omega_1\sqrt{4a^2})}{1 + 2\omega_1} = \frac{-[(-a) + \omega_1(-2a)]}{1 + 2\omega_1} = \frac{a + 2\omega_1 a}{1 + 2\omega_1} = a$$

Therefore, if $\tilde{A}_1 = (a, a, a, a; a, a)$, where $-1 \le a \le 1$, then $score(\tilde{A}_1) = a.$

Property4: $\tilde{A}_1 = (-1, -1, -1, -1; 1, 1)$, then $Score(\tilde{A}_1) = -1.$

Proof: If $\tilde{A}_1 = (-1, -1, -1, -1; 1, 1)$, based on (12), (13), (14)and(15), we can see that:

$x_{\tilde{A}_1} = -1,\ \ y_{\tilde{A}_1} = 1,\ STD_{\tilde{A}_1} = 0,\ \ d(\tilde{A}_1) = 0$, then

$$score(\tilde{A}_1) = \frac{\mu(a)(\left|x_{\tilde{A}_1}\right| + \omega_1\sqrt{2(x_{\tilde{A}_1}^2 + y_{\tilde{A}_1}^2)}}{1 + 2\omega_1 + \omega_2 STD_{\tilde{A}_1} + \omega_3 d(\tilde{A}_1)} = \frac{-(|-1| + \omega_1\sqrt{4})}{1 + 2\omega_1} = -1.$$

4 A comparison of the proposed ranking method with existing methods.

The eight sets of generalized fuzzy numbers shown in the paper of Deng and Liu (2005) are classical examples for the fuzzy number and usually are made as the benchmarks by the other papers such as the paper of Chen et al. (2012) and Chen & Chen (2009). So, we will use them to compare the proposed method with the existing methods in this paper. Besides, the other four sets of generalized fuzzy numbers are added to the paper to supplement the situation which has not been considered by Deng and Liu (2005). Therefore, the 12 sets of generalized fuzzy numbers are shown in Figure.2 can comprehensively represent the generalized fuzzy numbers in different situations and the ranking results of them can be extend to the other fuzzy numbers in the uncertain situation.

Though the weight of factors is given by the decision-maker, they should satisfy $0 \le \omega_2 \le \omega_1 < \omega_3 < 1$. $\omega_1 = 0.1$, $\omega_2 = 0.1$ and $\omega_3 = 0.8$ are suggested because the ranking result coincides with the analytical geometry and intuition of human beings better than the others at this time . The centroid point on X-axis is the most important factor, so its weight is always 1. The results shown in Table1 are obtained when $\omega_1 = 0.1$, $\omega_2 = 0.1$ and $\omega_3 = 0.8$. From Table 1, we can see that

(1)For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 1 of Figure. 2, the existing ranking methods and the proposed method get the same ranking order, which coincides with the intuition of human beings.

(2)For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 2 of Figure. 2, Bakar and Gegoy's method (2014), Madhuri et al.'s method (2014) and the proposed method get a reasonable ranking order, i.e., $\tilde{B} < \tilde{A}$, which coincides with the intuition of human beings due to the fact that the centroid point of $\tilde{A}$ is the larger than the centroid point of $\tilde{B}$ on the Y-axis and the degree of fuzziness of $\tilde{B}$ is larger than $\tilde{A}$ .Yager's method (1978) , Cheng's method (1998), Chu and Tsao's method (2002), Chen's method (2009), Chen and Sanguansat's method (2011), Chen et al.'s method (2012), Emrah's method (2013) and Wang's method (2015) get an unreasonable ranking order, i.e., $\tilde{B} = \tilde{A}$ or $\tilde{B} > \tilde{A}$.

(3) For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 3 of Figure. 2, Yager's method (1978) , Cheng's method (1998), Chu and Tsao's method (2002), Chen and Sanguansat's method (2011), Chen et al.'s method (2012) and Wang's method (2015) get an unreasonable ranking order, i.e., $\tilde{B} = \tilde{A}$, whereas Chen and Chen's method (2009), Emrah's method (2013), Bakar and Gegoy's method (2014), Madhuri et al.'s method (2014) and the proposed method get the same ranking order, i.e., $\tilde{B} > \tilde{A}$, which coincides with the intuition of human beings .

(4)For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 4 of Figure. 2, Yager's method (1978) get an incorrect ranking order, i.e., $\tilde{B}=\tilde{A}$ and Wang's method (2015) can not calculate the generalized fuzzy numbers , whereas Cheng's method (1998), Chu and Tsao's method (2002), Chen's method (2009), Chen and Sanguansat's method (2011), Chen et al.'s method (2012), Emrah's method (2013), Bakar and Gegoy's method (2014), Madhuri et al.'s method (2014) and the proposed method get the same ranking order, i.e., $\tilde{B}>\tilde{A}$, which coincides with the intuition of human beings.

(5)For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 5 of Figure. 2, Yager's method (1978), Cheng's method (1998) , Chu and Tsao's method (2002) and Madhuri et al.'s method (2014) cannot calculate the crisp number, whereas Chen and Chen's method (2009), Chen and Sanguansat's method (2011), Chen et al.'s method (2012), Emrah's method (2013), Bakar and Gegoy's method (2014), Wang's method (2015) and the proposed method get the same ranking order, i.e., $\tilde{B}>\tilde{A}$, which coincides with the intuition of human beings.

(6)For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 6 of Figure. 2, Yager's method (1978), Chu and Tsao's method (2002), Chen and Chen's method (2009), Chen and Sanguansat's method (2011), Chen et al.'s method (2012), Emrah's method (2013)，Bakar and Gegoy's method (2014), Wang's method (2015) and the proposed method get the same ranking order, i.e., $\tilde{B}>\tilde{A}$, which coincides with the intuition of human beings due to the fact that the centroid point of $\tilde{B}$ on the X-axis is larger than the centroid point of $\tilde{A}$ on the X-axis , whereas Cheng's method (1998) and Madhuri et al.'s method (2014) get an incorrect ranking order, i.e., $\tilde{B}=\tilde{A}$.

(7) For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 7 of Figure. 2, all the ranking methods except Madhuri et al.'s method (2014) get the same ranking order, i.e., $\tilde{B}<\tilde{A}$, which coincides with the intuition of human beings.

(8)For the fuzzy numbers $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ shown in Set 8 of Figure 2, Cheng's method (1998), Chu and Tsao's method (2002), Chen and Chen's method (2009), Chen and Sanguansat's method (2011), Chen et al.'s method (2012), Bakar and Gegoy's method (2014), Madhuri et al.'s method (2014), Wang's method (2015) and the proposed method get the same ranking order, i.e., $\tilde{C}>\tilde{B}>\tilde{A}$, which coincides with the intuition of human beings. However, Yager's method (1978) and Emrah's method (2013) get the incorrect ranking order, i.e., $\tilde{B}>\tilde{C}>\tilde{A}$.

(9)For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 9 of Figure 2, Cheng's method (1998) and Wang's method (2015) cannot calculate the generalized fuzzy numbers $\tilde{A}$ and $\tilde{B}$ when they themselves are symmetrical about the Y-axis. Yager's method (1978), Chu and Tsao's method (2002), Chen and Chen's method (2009), Chen and Sanguansat's method (2011), Chen et al.'s method (2012) and Bakar and Gegoy's method (2014) get the incorrect ranking order, i.e., $\tilde{B}=\tilde{A}$, whereas Emrah's method (2013), Madhuri et al.'s method (2014) and the proposed method get the reasonable ranking order, i.e., $\tilde{B}<\tilde{A}$, which coincides with the intuition of human beings due to the fact that the centroid point of $\tilde{B}$ on Y-axis is larger than the centroid point of $\tilde{A}$ on Y-axis.

(10)For the fuzzy numbers $\tilde{A}$ and $\tilde{B}$ shown in Set 10 of Figure 2, Cheng's method (1998) cannot calculate the generalized fuzzy numbers $\tilde{A}$ and $\tilde{B}$ when they themselves are symmetrical about the Y-axis. Yager's method (1978), Chu and Tsao's method (2002), Chen and Chen's method (2009), Chen and Sanguansat's method (2011) , Chen et al.'s method (2012), Emrah's method (2013), Bakar and Gegoy's method (2014), Madhuri et al.'s method (2014) and Wang's method (2015) get the incorrect ranking order, i.e., $\tilde{B}=\tilde{A}$ or $\tilde{B}>\tilde{A}$, whereas the proposed method get the ranking order, i.e., $\tilde{B}<\tilde{A}$, which is reasonable due to the fact that the centroid point of $\tilde{A}$ on Y-axis is larger than the centroid point of $\tilde{B}$ on Y-axis and the degrees of fuzziness of $\tilde{A}$ and $\tilde{B}$ are identical.

(11)For the fuzzy numbers $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ shown in Set 11 of Figure 2, Cheng's method (1998) and Wang's method (2015) cannot calculate the generalized fuzzy numbers $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ when they are symmetrical about the Y-axis. Yager's method (1978), Chu and Tsao's method (2002), Chen and Chen's method (2009), Chen and Sanguansat's method (2011) and Chen et al.'s method (2012) Emrah's method (2013) Bakar and Gegoy's method (2014) and Madhuri et al.'s method (2014) get the incorrect ranking order, i.e., $\tilde{A}=\tilde{B}=\tilde{C}$ or $\tilde{B}>\tilde{A}>\tilde{C}$, whereas the proposed method get the reasonable ranking order, i.e., $\tilde{A}>\tilde{B}>\tilde{C}$, which coincides with the intuition of human beings due to the fact that the centroid point of on Y-axis is $\tilde{A}>\tilde{B}>\tilde{C}$ and the degrees of fuzziness of $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ is $\tilde{A}<\tilde{B}<\tilde{C}$

(12)For the fuzzy numbers $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ shown in Set 12 of Figure 2, only Chen et al.'s method (2012), Bakar and Gegoy's method (2014) and the proposed method can calculate the generalized fuzzy numbers with different left heights and right heights correctly and they get the same ranking order, i.e., $\tilde{B}>\tilde{A}>\tilde{C}$, which coincides with the intuition of human beings.

In summary, from Figure 2 and Table 1, we can see that the proposed method gives the reasonable ranking order and overcomes the drawbacks of the existing ranking methods. Especially, the proposed method has advantage when the generalized fuzzy numbers are symmetrical about the Y-axis.

Table1 A comparison of the ranking results of the proposed method with existing methods shown in Figure2.

Methods	Set 1		Set 2		Set 3		Set 4	
	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$
Yager's method (1978)	0.3000	0.5000	0.3000	0.3000	0.3000	0.3000	0.3000	0.3000
Cheng's method(1998)	0.5831	0.7071	0.5831	0.5831	0.5831	0.5831	0.4610	0.5831
Chu and Tsao's method (2002)	0.1500	0.2500	0.1500	0.1500	0.1500	0.1500	0.1200	0.1500
Chen and Chen's method(2009)	0.2579	0.4298	0.2537	0.2579	0.2579	0.2774	0.2063	0.2579
Chen and Sanguansat's method(2011)	0.3000	0.5000	0.3000	0.3000	0.3000	0.3000	0.2824	0.3000
Chen et al.'s method(2012)	0.2553	0.4444	0.2553	0.2553	0.2553	0.2553	0.2462	0.2553
Emrah et al.'s method(2013)	0.2787	0.4788	0.2622	0.2787	0.2787	0.2866	0.2250	0.2787
Bakar and Gegoy's method(2014)	0.0867	0.1444	0.1096	0.0867	0.0867	0.0933	0.0715	0.0867
Madhuri et al.'s method(2014)	0.5774	0.7024	0.5885	0.5774	0.5774	0.5817	0.4934	0.5774
Wang's method(2015)	02500	0.7500	0.5000	0.5000	0.5000	0.5000	N	N
The proposed method	0.2567	0.4131	0.2770	0.2567	0.2567	0.2690	0.2483	0.2567

Methods	Set5		Set6		Set7		Set8		
	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$
Yager's method (1978)	0.3000	N	-0.3000	0.3000	0.6000	0.5000	0.4400	0.5333	0.5250
Cheng's method(1998)	0.4243	N	0.5831	0.5831	0.7673	0.7241	0.6800	0.7257	0.7462
Chu and Tsao's method (2002)	0.1500	N	-0.1500	0.1500	0.2870	0.2619	0.2281	0.2624	0.2784
Chen and Chen's method(2009)	0.2537	1.0000	-0.2579	0.2579	0.4428	0.4043	0.3354	0.4079	0.4196
Chen and Sanguansat's method(2011)	0.3000	1.0000	-0.3000	0.3000	0.5750	0.5350	0.4500	0.5250	0.5500
Chen et al.'s method(2012)	0.2553	1.0000	-0.2553	0.2553	0.5111	0.4773	0.4000	0.4667	0.5057
Emrah et al.'s method(2013)	0.2622	1.0000	-0.3213	0.2787	0.5684	0.4837	0.4013	0.5063	0.4947
Bakar and Gegoy's method(2014)	0.1096	0.3333	-0.0867	0.0867	0.1533	0.1278	0.1197	0.1363	0.1452
Madhuri et al.'s method(2014)	0.5885	N	0.5774	0.5774	0.7322	0.7322	0.6794	0.7052	0.7684
Wang's method(2015)	0.0000	1.0000	0.0000	1.0000	0.5385	0.4615	0.4615	0.5119	0.5275
The proposed method	0.2770	1.0000	-0.2567	0.2567	0.4677	0.3925	0.3559	0.4177	0.4189

Methods	Set9		• Set10		Set11			Set12		
	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$
Yager's method (1978)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3000	N	N
Cheng's method(1998)	N	N	N	N	N	N	N	0.5831	N	N
Chu and Tsao's method (2002)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1500	N	N
Chen and Chen's method(2009)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2537	N	N
Chen and Sanguansat's method(2011)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.3000	N	N
Chen et al.'s method(2012)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.2533	0.2687	0.2420
Emrah et al.'s method(2013)	-0.0568	-0.1045	-0.0568	-0.0378	-0.0568	-0.0244	-0.0721	0.2662	N	N
Bakar and Gegoy's method(2014)	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1096	0.1175	0.1099
Madhuri et al.'s method(2014)	0.4632	0.3542	0.4632	0.4861	0.4632	0.4850	0.3813	0.5885	N	N
Wang's method(2015)	N	N	0.5000	0.5000	N	N	N	0.5000	0.5000	0.5000
The proposed method	0.0484	0.0369	0.0484	0.0463	0.0484	0.0321	0.0253	0.2770	0.2815	0.2689

Note: "N" denotes cannot be calculated; "gray background" denotes unreasonable results.

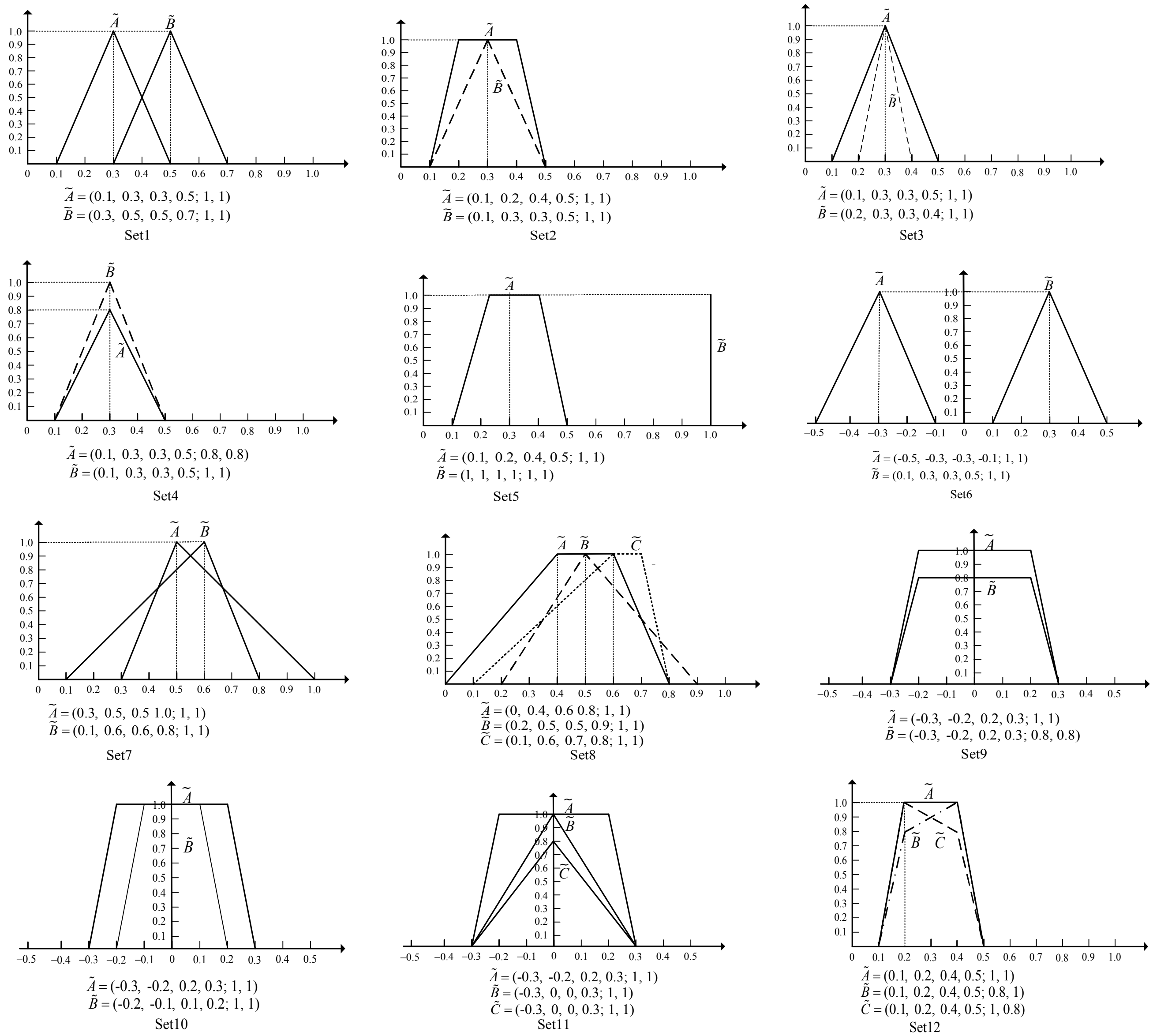

Figure2. Twelve sets of generalized fuzzy numbers.

5 Conclusions

In the data fusion processes or systems, no matter whatever the form of data is, it can be converted to the fuzzy number. Fuzzy number has become an excellent math tool for the problem of data fusion. Especially, nearly all the problems of decision-making can be solved by ranking fuzzy numbers. So, ranking fuzzy numbers properly is a key and necessary step in the fusion processes or systems. In this paper, we present a new method for ranking generalized fuzzy numbers based on centroid points, degrees of fuzziness and spreads of fuzzy numbers. The centroid point reflects the integrative character of the generalized fuzzy number. The degree of fuzziness and spread of fuzzy number reflect how fuzzy a generalized fuzzy number is. The larger the values of the centroid point on X-axis and Y-axis are, the better the ranking order. The larger the degree of fuzziness and spread are, the worse the ranking order. Although the centroid point, degrees of fuzziness and spreads are all considered by the proposed method, they should have different status when ranking generalized fuzzy numbers. As the main factor, the weight of the centroid point should be larger than the degrees of fuzziness and spreads. This coincides with the analytical geometry and intuition of human beings.

Comparing with the existing methods, the proposed method has introduced both degree of fuzziness and spread of fuzzy numbers which can reflect how fuzzy a generalized fuzzy number is and should be considered carefully in decision making. The proposed method has considers different factors of generalized fuzzy numbers should have different weight, which can overcome some drawbacks of the existing methods and has obvious advantage when the generalized fuzzy numbers such as Set9, Set10 and set11 in Figure2 are symmetrical about the Y-axis.

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References

[1] R. Jain (1976). Decision making in the presence of fuzzy variables, IEEE. Trans. Syst. Man Cybern, 6(1976), pp.698–703.

[2] R.R. Yager (1978). Ranking fuzzy subsets over the unit interval, Decision and Control including the 17th Symposium on Adaptive Processes, 1978 IEEE Conference, pp.1435–1437.

[3] C.H. Cheng. (1998). A new approach for ranking fuzzy numbers by distance method, Fuzzy Sets and Systems 95(3), pp.307–317.

[4] T.C. Chu, C.T. Tsao. (2002), Ranking fuzzy numbers with an area between the centroid point and original point, Comput. Math. Appl, 43 (2002), pp.111–117.

[5] Y. Deng, Q. Liu, (2005). A TOPSIS-based centroid-index ranking method of fuzzy numbers and it's application in decision-making, Cybernetics and Systems, 36(7), pp.581–595.

[6] S.J. Chen, S.M. Chen (2007). Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers, Applied Intelligence, 26(1), pp.1–11.

[7] B. Asady, A. Zendehnam. (2007). Ranking fuzzy numbers by distance minimization, Appl. Math. Modell. 31 (2007), pp. 2589–2598.

[8] Z.X. Wang, Y.J. Liu, Z.P. Fan, B. Feng, Ranking L–R fuzzy number based on deviation degree, Inf. Sci. 179 (2009), pp. 2070–2077.

[9] S.M. Chen, J.H. Chen (2009). Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Expert Systems with Applications, 36(3), pp.6833–6842.

[10] Y.M. Wang (2009). Centroid defuzzification and the maximizing set and minimizing set ranking based on alpha level sets. Computers & Industrial Engineering, 57, pp.228–236.

[11] A.M. Nejad, M. Mashinchi. (2011). Ranking fuzzy numbers based on the areas on the left and the right sides of fuzzy number, Comput. Math. Appl. 61 (2011), pp.431–442.

[12] S.M. Chen, K. Sanguansat. (2011). Analyzing fuzzy risk based on a new fuzzy ranking method between generalized fuzzy numbers, Expert Systems with Applications, 38 (2011), pp.2163–2171.

[13] S.Y. Chou, L.Q. Dat, V.F. Yu. (2011). A revised method for ranking fuzzy numbers using maximizing set and minimizing set, Computers & Industrial Engineering , 61 (4), pp.1342-1348.

[14] S.M. Chen, A. Munif, G.S. Chen, H.C. Liu, B.C. Kuo. (2012). Fuzzy risk analysis based on ranking generalized fuzzy numbers with different left heights and right heights, Expert Systems with Applications, 39 (2012), pp.6320–6334.

[15] EmrahAkyar, H. Akyar, S. A. D¨uzce. (2013). Fuzzy risk analysis based on a geometric ranking method for generalized trapezoidal fuzzy numbers, Journal of Intelligent & Fuzzy Systems, 25 (2013), pp. 209–217.

[16] V.F. Yu, H. Thi, X. Chi. (2013). Ranking generalized fuzzy numbers in fuzzy decision making based on the left and right transfer coefficients and areas, Applied Mathematical Modelling, 37 (2013) pp.8106–8117

[17] K.U. Madhuri, S.S. Babu , N.R. Shankar (2014). Fuzzy risk analysis based on the novel fuzzy ranking with new arithmetic operations of linguistic fuzzy numbers, Journal of Intelligent & Fuzzy Systems , 2014, 26 (2014) , 2391–2401

[18] A.S.A. Bakar, A. Gegov (2014). Ranking of fuzzy numbers based on centroid point and spread, Journal of Intelligent & Fuzzy Systems 27(2014), 1179–1186.

[19] Y.J. Wang (2015), Ranking triangle and trapezoidal fuzzy numbers based on the relative preference relation, Appl. Math, 2015, 39 (2015), 586-599.

[20] A. Deluca, S. Termini. (1972) A definition of nonprobalistic entropy in the setting of fuzzy sets theory. Information and Control, 1972 (20), pp. 301–302.

[21] G.C. Tang (1999).The Area Degree of Fuzziness in Fuzzy Scheduling, Journal of Shanghai Second Polytechnic University, 1999 (2), pp.49 – 56.

[22] Y.M. Wang, J.B. Yang, D. L. Xu, K. S. Chin (2006) On the centroids of fuzzy numbers. Fuzzy Sets and Systems, 2006(157), pp.919-92