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Distributed Multi-Target Tracking Via Generalized Multi-Bernoulli Random Finite Sets

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***Abstract*—In this paper, we address the problem of the distributed multi-target tracking with labelled set filters in the framework of generalized Covariance Intersection (GCI). Our analyses show that the label space mismatching phenomenon, which means the same realization drawn from label spaces of different sensors does not have the same implication, is quite common in practical scenarios and may bring serious problems. To get rid of the bad influence of label space mismatching phenomenon, firstly, we propose a robust strategy for distributed fusion with labelled set posteriors in which labelled set posteriors are transformed to their unlabelled versions firstly and the GCI fusion is performed with the unlabelled posteriors then. Secondly, we derive the unlabelled versions of common labelled set distributions in generalized labelled multi-Bernoulli (GLMB) family and show that they all belong to the same (unlabelled) random finite set (RFS) family, referred to as generalized multi-Bernoulli (GMB) family. Thirdly, we derive the explicit formula for GCI with GMB distributions, which enables the distributed fusion with GLMB filter family, including the GLMB, δ-GLMB, Mδ-GLMB and LMB filters. Simulation results for Gaussian mixture implementation have demonstrated the performance of the proposed distributed fusion algorithms in two challenging tracking scenarios.**

I. Introduction

Distributed multi-target tracking (DMTT) has become increasingly important due to its lower communication cost and stronger fault-tolerance abilities compared with centralized fusion. One of the remarkable problems of DMTT is that the estimates from different sensors exist unknown level of correlation. One solution is the optimal fusion [1], however, it is ruled out by its unacceptable cost of computing the common information between sensors. An alternative is to use suboptimal fusion technique, generalized Covariance Intersection (GCI) based on Exponential Mixture Densities (EMDs) proposed by Mahler [2]. The highlight of GCI is that it is capable to fuse both Gaussian and non-Gaussian formed multi-target distributions from different sensors with completely unknown correlation.

Based on the work in [2], Clark *et al*. proposed tractable formulations of GCI for special forms of multi-target distributions [3], i.e., Possion, independent identically distributed (i.i.d.) cluster and Bernoulli distributions. Using these derivations, a sequential Monte Carlo (SMC) realization of distributed fusion with probability hypothesis density (PHD) filter was presented in [4]. Meanwhile, [5] addressed the problem of DMMT with a Gaussian mixture cardinalized PHD (GM-CPHD) filter. The work of distributed detection and tracking with Bernoulli filter over a Doppler-shift sensor network has been completed in [6]. Compared with PHD/CPHD filters [7]–[11], the multi-Bernoulli (MB) filters [13]–[18] which are the extension of Bernoulli filters [12], are more useful in problems that require particle implementations or target individual existence probability. In our preliminary work, [19], we derived the explicit formula for GCI with MB distributions based on two step reasonable approximations, and proposed a robust distributed fusion with MB filter.

To summarize the filters mentioned above, on one hand, they are not multi-target trackers because target states are indistinguishable, and on the other hand, they are almost not the closed-form solution to the optimal Baysian filter even assuming a special observation model, such as standard observation model [7]. Recently, the notion of labelled random finite set (RFS) is introduced to address target trajectories and their uniqueness in [20], [22]–[24]. Accompanied with this notion, a class of labelled RFSs, generalized labelled multi-Bernoulli (GLMB) RFSs which are the conjugate priors for the standard multi-target likelihood function, provides a closed-form solution to the optimal Bayesian filter. Moreover, the relevant stronger results, δ-GLMB filter, which can be directly used to multi-target tracking, can not only produce trajectories formally but also outperform the PHD/CPHD and MB filters. Later, [22] and [27] respectively proposed two efficient approximations of δ-GLMB filter, i.e., labelled multi-Bernoulli (LMB) filter and Marginalized δ-GLMB (Mδ-GLMB) filter, where LMB filter approximates the δ-GLMB posterior as its first-order statistical moment matched MB distribution, while Mδ-GLMB filter preserves both first-order moment and cardinality distribution of the δ-GLMB posterior.

Due to the advantages of labelled set filters, it is really meaningful to investigate their generalization to the distributed environment. In [26], authors derived the closed-form solutions of GCI fusion with Mδ-GLMB and LMB posteriors which are, however, based on the assumption that the label spaces of each sensors are matching making it somewhat restrictive in

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real applications.

In this paper, we are devoted to study on the robust distributed fusion method for labelled set filters, especially for the GLMB filter family including GLMB, δ-GLMB Mδ-GLMB, and LMB filters. Our analyses show that the label spaces of each sensors are always mismatching in the sense that the same realization drawn from label spaces of different sensors does not have the same implication in practical scenarios, which is referred to as *"label space mismatching"* phenomenon. Moreover, when this phenomenon arises, it may be unreasonable in theory and a lack of robustness in practice if the fusion is still performed directly on the space augmented with target label. To solve this problem, two promising thoughts can be employed. One is that match the label spaces of different sensors first through some means and then perform fusion on labelled state space. Another is that get the unlabelled posteriors of different sensors first and perform fusion on unlabelled state space then. This paper mainly focuses on the latter, and the major contributions of our work are three-fold:

i) *We propose a robust fusing strategy for distributed fusion with labelled set posteriors which can get rid of the bad influence of "label space mismatching" phenomenon.* The strategy is to transform the labelled set posteriors to its unlabelled versions, and then perform GCI fusion with the unlabelled versions of posteriors.
ii) *We derive the mathematic representations of the unlabelled versions of common labelled set distributions in GLMB family, and prove that they all belong to the same (unlabelled) RFS family, named as generalized multi-Bernoulli (GMB) family.* These derivations are preconditions for the distributed fusion with labelled set posteriors in GLMB family, according to the proposed fusing strategy.
iii) *We derive the explicit formula for the GCI with GMB distributions through two-step approximation, which enables the distributed fusion with GLMB filer family.* The first step is to approximate the GMB distributions using its first-order statistical moment matched MB distributions [7], and the second is to derive the formula for EMD of approximated MB distributions using another reasonable approximation. The fused distribution which turns out to be another GMB distribution, can not only make the formula of universal GCI fusion tractable, but also enable the sequential fusion with sensor network which owns more than two sensors.

In numerical results, Gaussian mixture (GM) implementation of the proposed algorithm for point observation model verifies the robustness and effectiveness of the proposed fusing strategy for distribution fusion with labelled set filters and the derived formula for GCI with GMB distributions.

II. Background

A. Notation

In this paper, we inhere the convention that single-target states are denoted by the small letter "x", e.g., $x, \mathbf{x}$ and the multi-target states are denoted by capital letter "X", e.g., $X, \mathbf{X}$. To distinguish labelled states and distributions from the unlabelled ones, bold face letters are adopted for the labelled ones, e.g., $\mathbf{x}$, $\mathbf{X}$, $\boldsymbol{\pi}$. Observations generated by single-target states are denoted by the small letter "z", i.e., z, and the multi-target observations are denoted by capital letter "Z", i.e., Z. Moreover, blackboard bold letters represent spaces, e.g., the state space is represented by $\mathbb{X}$, the label space by $\mathbb{L}$, and the observation space by $\mathbb{Z}$. The collection of all finite sets of $\mathbb{X}$ is denoted by $\mathcal{F}(\mathbb{X})$ and $\mathcal{F}_n(\mathbb{X})$ denotes all finite subsets with n elements.

The labelled single target state $\mathbf{x}$ is constructed by augmenting a state $x \in \mathbb{X}$ with a label $\ell \in \mathbb{L}$. The labels are usually drawn form a discrete label space, $\mathbb{L} = \{\alpha_i : i \in \mathbb{N}\}$, where all α_i are distinct and the index space $\mathbb{N}$ is the set of positive integers.

The multi-target state X, the labelled multi-target state $\mathbf{X}$ and the multi-target observation Z are modelled by the finite set of single-target states, the finite set of labelled single-target states, and the finite set of observations generated by single-target states, respectively, i.e.,

$$\begin{aligned} X =& \{x_1, \cdots, x_n\} \subset \mathbb{X} \\ \mathbf{X} =& \{\mathbf{x}_1, \cdots, \mathbf{x}_n\} \subset \mathbb{X} \times \mathbb{L} \\ Z =& \{z_1, \cdots, z_m\} \subset \mathbb{Z} \end{aligned} \tag{1}$$

Also notice that the labelled multi-target state is an RFS on $\mathbb{X} \times \mathbb{L}$ with distinct labels. The set of labels of a labelled RFS $\mathbb{X}$ is given by $\mathcal{L}(\mathbf{X}) = \{\mathcal{L}(\mathbf{x}) : \mathbf{x} \in \mathbf{X}\}$, where $\mathcal{L} : \mathbb{X} \times \mathbb{L} \to \mathbb{L}$ is the projection defined by $\mathcal{L}((x, \ell)) = \ell$. The distinct label indicator

$$\triangle(\mathbf{X}) = \delta_{|\mathbf{X}|}(|\mathcal{L}(\mathbf{X})|) \tag{2}$$

is used to ensure distinct labels.

We use the multi-target exponential notation

$$h^X \triangleq \prod_{x \in X} h(x) \tag{3}$$

for real-valued function h, with $h^{\emptyset} = 1$ by convention.

To admit arbitrary arguments like sets, vectors and integers, the generalized Kronecker delta function is given by

$$\delta_Y(X) \triangleq \begin{cases} 1, \text{if} X = Y \\ 0, \text{otherwise} \end{cases} \tag{4}$$

and the inclusion function is given by

$$1_Y(X) \triangleq \begin{cases} 1, \text{if} X \subseteq Y \\ 0, \text{otherwise} \end{cases} \tag{5}$$

B. Multi-target Bayesian filter

Finite Set Statistics (FISST) proposed by Mahler, has provided a rigorous and elegant mathematical framework for the multi-target detection, tracking and classification problem in an unified Bayesian paradigm.

In the FISST framework, the optimal multi-target Bayesian filter [1] propagates RFS based posterior density $\pi_k(X_k|Z^{1:k})$

[1]Note that the Multi-target Bayesian filter in (6) and (7) is also appropriate for the labelled set posterior, and the labelled set integrals defined as [20] are involving.

conditioned on the sets of observations up to time k, $Z^{1:k}$, in time with the following recursion [7]:

$$\begin{aligned}&\pi_{k|k-1}(X_k|Z^{1:k-1})\\&=\int f_{k|k-1}(X_k|X_{k-1})\pi_{k-1}(X_{k-1}|Z^{1:k-1})\delta X_{k-1}\end{aligned} \tag{6}$$

$$\pi_k(X_k|Z^{1:k})=\frac{g_k(Z_k|X_k)\pi_{k|k-1}(X_k|Z^{1:k-1})}{\int g_k(Z_k|X_k)\pi_{k|k-1}(X_k|Z^{1:k-1})\delta X_k} \tag{7}$$

where $f_{k|k-1}(X_k|X_{k-1})$ is the multi-target Markov transition function and $g_k(Z_k|X_k)$ is the multi-target likelihood function of Z_k, and $\int \cdot \delta X$ denotes the set integral [7] defined by

$$\int f(X)\delta X = f(\emptyset)+\sum_{n=1}^{\infty}\int f(\{x_1,\cdots,x_n\})dx_1\cdots dx_n \tag{8}$$

C. GCI Fusion Rule

The GCI was proposed by Mahler specifically to extend FISST to distributed environments [9]. Consider two nodes 1 and 2 in the sensor network. At time k, each nodes maintain its own local posteriors $\pi_1(X|Z_1^{1:k})$ and $\pi_2(X|Z_2^{1:k})$ which are both the RFS based densities. Under the GCI [2] proposed by Mahler, the fused distribution is the geometric mean, or the exponential mixture of the local posteriors [2],

$$\pi_\omega(X|Z_1^{1:k},Z_2^{1:k})=\frac{\pi_1(X|Z_1^{1:k})^{\omega_1}\pi_2(X|Z_2^{1:k})^{\omega_2}}{\int \pi_1(X|Z_1^{1:k})^{\omega_1}\pi_2(X|Z_2^{1})^{\omega_2}\delta X} \tag{9}$$

where ω_1, ω_2 ($\omega_1+\omega_2=1$) are the parameters determining the relative fusion weight of each distributions. (9) is derived by following that the distribution that minimizes the weighted sum of its Kullback-Leibler divergence (KLD) with respect to a given set of distributions is an EMD, e.g.,

$$\pi_\omega = \arg\min_{\pi}(\omega_1 D(\pi\parallel\pi_1)+\omega_2 D(\pi\parallel\pi_2)) \tag{10}$$

where D denotes the KLD.

D. GLMB Family

The subsection provides a brief review of a series of common labelled set distributions, i.e., GLMB, δ-GLMB, Mδ-GLMB and LMB distributions, in GLMB family. The GLMB family is necessary for the results of this paper.

1) GLMB: A generalized labelled multi-Bernoulli (GLMB) RFS [21] is a labelled RFS with state space $\mathbb{X}$ and (discrete) label space $\mathbb{L}$ distributed according to

$$\boldsymbol{\pi}(\mathbf{X})=\triangle(\mathbf{X})\sum_{c\in\mathbb{C}}w^{(c)}(\mathcal{L}(\mathbf{X}))[p^{(c)}]^{\mathbf{X}} \tag{11}$$

where $\mathbb{C}$ is a discrete index set, $w^{(c)}(L)$ and $p^{(c)}$ satisfy

$$\begin{aligned}&\sum_{L\subseteq\mathbb{L}}\sum_{c\in\mathbb{C}}w^{(c)}(L)=1\\&\int p^{(c)}(x,\ell)dx=1\end{aligned} \tag{12}$$

[2]Note that GCI fusion rule in (9) is also appropriate for the labelled set posterior, and the labelled set integrals defined as [20] are involving.

2) δ-GLMB: A δ-generalized labelled multi-Bernoulli (δ-GLMB) RFS [21] is a special case of a GLMB with

$$\begin{aligned}&\mathbb{C}\triangleq\mathcal{F}(\mathbb{L})\times\Xi\\&w^{(c)}(L)=w^{(I,\xi)}(L)\triangleq w^{(I,\xi)}\delta_I(L)\\&p^{(c)}=p^{(I,\xi)}\triangleq p^{(\xi)}\end{aligned} \tag{13}$$

i.e. it is distributed according to

$$\boldsymbol{\pi}(\mathbf{X})=\triangle(\mathbf{X})\sum_{(I,\xi)\in\mathcal{F}(\mathbb{L})\times\Xi}\omega^{(I,\xi)}\delta_I(\mathcal{L}(\mathbf{X}))[p^{(\xi)}]^{\mathbf{X}} \tag{14}$$

where Ξ is a discrete space, each $p^{(\xi)}(\cdot,\ell)$ is a probability density, and each $\omega^{(\xi)}(I)$ is non-negative with

$$\sum_{(I,\xi)\in\mathcal{F}(\mathbb{L})\times\Xi}\omega^{(\xi)}(I)=1 \tag{15}$$

3) Mδ-GLMB: A Marginalized δ-GLMB (Mδ-GLMB) density $\boldsymbol{\pi}$ corresponding to the δ-GLMB density of form (14) is a probability density of form

$$\boldsymbol{\pi}(\mathbf{X})=\triangle(\mathbf{X})\sum_{I\in\mathcal{F}(\mathbb{L})}\delta_I(\mathcal{L}(\mathbf{X}))w^{(I)}\left[p^{(I)}\right]^{\mathbf{X}} \tag{16}$$

where

$$w^{(I)}=\sum_{\xi\in\Xi}w^{(I,\xi)} \tag{17}$$

$$p^{(I)}(x,\ell)=1_I(\ell)\frac{1}{w^{(I)}}\sum_{\xi\in\Xi}w^{(I,\xi)}p^{(\xi)}(x,\ell) \tag{18}$$

Notice that the Mδ-GLMB is also a special case of GLMB with

$$\begin{aligned}&\mathbb{C}\triangleq\mathcal{F}(\mathbb{L})\\&w^{(c)}(L)=w^{(I)}\delta_I(L)\\&p^{(c)}(x,\ell)=p^{(I)}(x,\ell)\end{aligned} \tag{19}$$

4) LMB: A labelled multi-Bernoulli (LMB) RFS $\mathbf{X}$ [24] with state space $\mathbb{X}$, label space $\mathbb{L}$ and (finite) parameter set $\{(r^\ell,p^\ell(x)):\ell\in\mathbb{L}\}$, is distributed according to

$$\boldsymbol{\pi}(\mathbf{X})=\triangle(\mathbf{X})w(\mathcal{L}(\mathbf{X}))p^{\mathbf{X}} \tag{20}$$

where

$$\begin{aligned}&w(L)=\prod_{i\in\mathbb{L}}(1-r^i)\prod_{\ell\in L}\frac{1_{\mathbb{L}}(\ell)r^\ell}{1-r^\ell}\\&p(x,\ell)=p^\ell(x)\end{aligned} \tag{21}$$

III. Distributed Fusion with Labelled Set Posteriors

Motivated by the advantages of labelled set filters, in this section, we are going to investigate their generalization to the distributed environment.

It is similar to the multi-target Bayesian filter that the universal GCI formula in (9) is also computationally intractable in general for the integral in (9) is not the integral in the conventional sense but rather a set-integral that integrates over all joint target-spaces, considering each cardinality (number of targets). Fortunately, when local posteriors are in convenient mathematical representations, such as possion processes, it is promising to obtain the tractable type of fused posteriors. In [26], authors derived the closed-form solution of GCI fusion with Mδ-GLMB and LMB posteriors, which can be used to the distributed fusion with Mδ-GLMB and LMB filters. However, the fusion method in [26] is based on the assumption that the label spaces of each sensors are matching, making it somehow restrictive in real applications.

A. "Label Space Mismatching" Phenomenon

Consider two nodes 1 and 2 in a distributed fusion network. Each nodes have received its own sensor information and maintain its own posteriors $\pi_1(\mathbf{X}|\mathbf{Z}_1^{1:k})$ defined on $\mathcal{F}(\mathbb{X}\times\mathbb{L}_1)$ and $\pi_2(\mathbf{X}|\mathbf{Z}_2^{1:k})$ defined on $\mathcal{F}(\mathbb{X}\times\mathbb{L}_2)$, where $\mathbb{L}_1$ and $\mathbb{L}_2$ denote the label spaces of sensors 1 and 2, respectively. The straightforward method of distributed fusion which is adopted in [27] is that assume $\mathbb{L}_\omega = \mathbb{L}_1 = \mathbb{L}_2$, substitute these two labelled posteriors to (9) directly, and perform GCI fusion on space $\mathbb{L}_\omega$. However, this method will be unreasonable in theory if "label space mismatching phenomenon" arises.

"Label space mismatching" phenomenon means that the same realization drawn from the different label spaces may not have the same implication and thus cannot represent the same trajectory. For instance, a realization α belongs to the label space $\mathbb{L}_1$ and $\mathbb{L}_2$ at the same time, however, $\alpha\in\mathbb{L}_1$ represents the different trajectory with $\alpha\in\mathbb{L}_2$.

"Label space mismatching" phenomenon is very common in practical scenarios. We analyse this from two aspects, two widely used track initiation approaches.

i) *Observation oriented birth process*: New-born targets are based on the observations not associated to the persisting targets. The observation sets provided by different sensors incorporate noisy observations of targets, stochastic miss-detections of targets, and stochastic clutters, as a result, it cannot guarantee the matching of label spaces from different sensors.

ii) *Known location oriented birth process*: New-born targets are based on the known locations about which different sensors have reached a consensus. The label space may be matching for the first few time steps, however, once some tracks deviate by the influence of clutters, or are truncated during to successive miss-detections, the matching of different sensors' tracks evolved from the same birth will be invalid.

In a word, for either birth processes, to ensure the matching of label spaces of each sensors, an ideal detecting environment, in which each sensor does not have miss-detections and clutters, and the estimate accuracy of each sensor is enough high, is required. However, it is not realistic in practice, thus "label space mismatching" is a common phenomenon.

When the "label space mismatching" phenomenon happens, if we still insist on performing fusion on space $\mathbb{X}\times\mathbb{L}_\omega$, the fusing performance will dramatically decrease and even collapse entirely. Suppose that the posteriors of sensors 1 and 2 are LMB distributions of form (20), i.e., $\boldsymbol{\pi}_s = \{(r_s^\ell, p_s^\ell(x))\}_{\ell\in\mathbb{L}_s}$ on space $\mathbb{X}\times\mathbb{L}_s$. Assume that $\mathbb{L}_1=\mathbb{L}_2=\{\alpha_1,\alpha_2\}$. The parameters for LMB posteriors of each sensor are shown as:

$$\begin{aligned} r_1^{\alpha_1} &= 0.92,\ p_1^{\alpha_1}(x) = \mathcal{N}(x;0,1)\\ r_1^{\alpha_2} &= 0.90,\ p_1^{\alpha_2}(x) = \mathcal{N}(x;10,1.3)\\ r_2^{\alpha_1} &= 0.95,\ p_2^{\alpha_1}(x) = \mathcal{N}(x;10,1.1)\\ r_2^{\alpha_2} &= 0.98,\ p_2^{\alpha_2}(x) = \mathcal{N}(x;0,1.5) \end{aligned} \tag{22}$$

Based on the solution of the GCI fusion with LMB posteriors derived in [27], the fused posterior is also a LMB distribution, $\pi_\omega = \{(r_\omega^\ell, p_\omega^\ell)\}_{\ell\in\mathbb{L}_\omega}$, where

$$\begin{aligned} r_\omega^{\alpha_1} &= 0.0010,\ p_\omega^{\alpha_1}(x) = \mathcal{N}(x;4.7619,1.0476)\\ r_\omega^{\alpha_2} &= 0.0028,\ p_\omega^{\alpha_2}(x) = \mathcal{N}(x;5.3571,1.3929) \end{aligned} \tag{23}$$

It seems that these two sensors share the same label space, however, these two label spaces are not matching. Actually, the label α_1 in $\mathbb{L}_1$ represents the same target as the label α_2 in $\mathbb{L}_2$. Hence, the fusion results (23) are completely insignificance with the existence probability tiny.

B. Robust Strategy for Distributed Fusion with Labelled set Posteriors

Now that the mismatching between label spaces of different sensors may arise such serious fusing problem as analysed in subsection III-A, in this subsection, we propose a novel strategy of distributed fusion with labelled set posteriors to get rid of the bad influence of "label space mismatching" phenomenon. The strategy is that the GCI fusion will not performed on space $\mathbb{X}\times\mathbb{L}_\omega$, but performed on space $\mathbb{X}$, the kinematic state space shared by all sensors. To carry out this strategy, the labelled set posteriors are transformed to the unlabelled versions firstly and the EMD of unlabelled posteriors are computed using (9).

IV. The Unlabelled Versions of GLMB Family

In the following, we primarily study on the distributed fusion with GLMB family posteriors, which are used in the GLMB filter family including GLMB δ-GLMB, Mδ-GLMB and LMB filters, according to the proposed strategy in Section III-B. This section provides the mathematics representations for the unlabelled versions of distributions in GLMB family, which also belong to the same (unlabelled) RFS family, named as generalized multi-Bernoulli (GMB) family. The relevant results are summarized in Propositions 1-4.

Definition 1. *A generalized multi-Bernoulli (GMB) RFS is an RFS with state space $\mathbb{X}$ distributed according to*

$$\pi(\{x_1,\cdots,x_n\}) = \sum_\sigma \sum_{(\mathcal{I},\phi)\in\mathcal{F}_n(\mathbb{I})\times\Phi} w^{(\mathcal{I},\phi)} \prod_{i=1}^n p^{(\phi),\mathcal{I}^v(i)}(x_{\sigma(i)}) \tag{24}$$

where the summation $\sum_\sigma$ is taken over all permutations on the numbers $1,\cdots,n$, Φ is a discrete space, $\mathbb{I}$ is the index set *of densities, $\mathcal{I}^v \in \mathbb{I}^{|\mathcal{I}|}$ is a vector constructed by sorting the elements of the set $\mathcal{I}$, $w^{(\mathcal{I},\phi)}$ and $p^{(\phi),\imath}(x)$ satisfy*

$$\begin{aligned}\sum_{\mathcal{I}\in\mathcal{F}(\mathbb{I})}\sum_{\phi\in\Phi} w^{(\mathcal{I},\phi)} = 1\\ \int p^{(\phi),\imath}(x)dx = 1, \imath \in \mathbb{I}\end{aligned} \quad (25)$$

A GMB distribution is constructed by a set of hypotheses, $\{(\mathcal{I},\phi) : (\mathcal{I},\phi) \in \mathcal{F}(\mathbb{I}) \times \Phi\}$. We define a set of densities for each $\phi \in \Phi$ as:

$$\mathfrak{D}^{(\phi)} = \{p^{(\phi),\imath}(x)\}_{\imath\in\mathbb{I}} \quad (26)$$

Under each hypothesis, the corresponding weight is $w^{(\mathcal{I},\phi)}$ and the corresponding density set is $\mathfrak{D}^{(\phi)}$. Thus a GM-B is completely characterized by the set of parameters $\{w^{(\mathcal{I},\phi)}, \mathfrak{D}^{(\phi)} : (\mathcal{I},\phi) \in \mathcal{F}(\mathbb{I}) \times \Phi\}$. Notice that the number of $\omega_(\mathcal{I},\phi)$ and $\mathfrak{D}^{(\phi)}$ which need to store and compute is $|\mathcal{F}(\mathcal{I}) \times \Phi|$ and $|\Phi|$, respectively.

The unlabelled version of a labelled RFS on $\mathbb{X}\times\mathbb{L}$ is given by $\mathcal{K}(X) = \{\mathcal{K}(\mathbf{x}) : \mathbf{x} \in \mathbf{X}\}$, where $\mathcal{K} : \mathbb{X}\times\mathbb{L} \to \mathbb{X}$ is the projection defined by $\mathcal{K}((x,\ell)) = x$.

Lamma 1. *If a labelled RFS $\mathbf{X}$ is distributed according to $\boldsymbol{\pi}$, then $X = \mathcal{K}(\mathbf{X})$ is distributed according the marginal [20]*

$$\pi(\{x_1,\cdots,x_n\}) = \sum_{(\ell_1,\ldots,\ell_n)\in\mathbb{L}^n} \boldsymbol{\pi}(\{(x_1,\ell_1),\cdots,(x_n,\ell_n)\}) \quad (27)$$

Proposition 1. *If a labelled RFS $\mathbf{X}$ on $\mathbb{X}\times\mathbb{L}$ is a GLMB RFS distributed according to (11), then $X = \mathcal{K}(\mathbf{X})$ is distributed as*

$$\pi(\{x_1,\ldots,x_n\}) = \sum_{\sigma}\sum_{(I,c)\in\mathcal{F}_n(\mathbb{L})\times\mathbb{C}} w^{(I,c)}\prod_{i=1}^{n} p^{(c),I^v(i)}(x_{\sigma(i)}) \quad (28)$$

where

$$\begin{aligned}w^{(I,c)} &\triangleq w^{(c)}(I), I \in \mathcal{F}(\mathbb{L})\\ p^{(c),\ell}(x) &\triangleq p^{(c)}(x,\ell), \ell \in \mathbb{L}\end{aligned} \quad (29)$$

Proposition 1 explicitly describes the relationship between the parameters of GLMB and the parameters of its unlabelled version. It is obvious that the unlabelled version of GLMB is a GMB with $\mathbb{I} = \mathbb{L}$ and $\Phi = \mathbb{C}$.

Proposition 2. *If a labelled RFS $\mathbf{X}$ on $\mathbb{X}\times\mathbb{L}$ is a δ-GLMB RFS distributed according to (14), then $X = \mathcal{K}(\mathbf{X})$ is distributed as:*

$$\pi(\{x_1,\ldots,x_n\}) = \sum_{\sigma}\sum_{(I,\xi)\in\mathcal{F}_n(\mathbb{L})\times\Xi} \omega^{(I,\xi)}\prod_{i=1}^{n} p^{(\xi),I^v(i)}(x_{\sigma(i)}) \quad (30)$$

where

$$p^{(\xi),\ell}(x) \triangleq p^{(\xi)}(x,\ell), \ell \in \mathbb{L} \quad (31)$$

Proposition 2 explicitly describes the relationship between the parameters of δ-GLMB and the parameters of its unlabelled version. It is obvious that the unlabelled version of δ-GLMB is a GMB with $\mathbb{I} = \mathbb{L}$ and $\Phi = \Xi$.

Proposition 3. *If a labelled RFS $\mathbf{X}$ on $\mathbb{X}\times\mathbb{L}$ is a Mδ-GLMB RFS distributed according to (16), then $X = \mathcal{K}(\mathbf{X})$ is distributed as:*

$$\begin{aligned}&\pi(\{x_1,\ldots,x_n\})\\ &= \sum_{\sigma}\sum_{(I',I)\in\mathcal{F}(\mathbb{L})\times\mathcal{F}(\mathbb{L})} w^{(I)}\delta_I(I')\prod_{i=1}^{n} p^{(I),I^v(i)}(x_{\sigma(i)})\\ &= \sum_{\sigma}\sum_{I\in\mathcal{F}(\mathbb{L})} w^{(I)}\prod_{i=1}^{n} p^{(I),I^v(i)}(x_{\sigma(i)})\end{aligned} \quad (32)$$

where

$$p^{(I),\ell}(x) \triangleq p^{(I)}(x,\ell), \ell \in \mathbb{L} \quad (33)$$

Proposition 3 explicitly describes the relationship between the parameters of Mδ-GLMB and the parameters of its unlabelled version. It is obvious that the unlabelled version of δ-GLMB is a GMB with $\mathbb{I} = \mathbb{L}$ and $\Phi = \mathcal{F}(\mathbb{L})$.

Proposition 4. *If a labelled RFS $\mathbf{X}$ on $\mathbb{X}\times\mathbb{L}$ is a LMB RFS distributed according to (20), then $X = \mathcal{K}(\mathbf{X})$ is distributed as:*

$$\pi(\{x_1,\cdots,x_n\}) = \sum_{\sigma}\sum_{I\in\mathcal{F}_n(\mathbb{L})} w^{(I)}\prod_{i=1}^{n} p^{I^v(i)}(x_{\sigma(i)}) \quad (34)$$

where

$$\begin{aligned}w^{(I)} &\triangleq w(I), I \in \mathcal{F}(\mathbb{L})\\ p^{\ell}(x) &\triangleq p(x,\ell), \ell \in \mathbb{L}\end{aligned} \quad (35)$$

Proposition 4 explicitly describes the relationship between the parameters of LMB and the parameters of its unlabelled version. It is obvious that the unlabelled version of LMB is a GMB with $\mathbb{I} = \mathbb{L}$ and the discrete space Φ only has one point. Notice that (34) is a MB s with a set of parameters $\{r^\ell, p^\ell(x)\}_{\ell\in\mathbb{L}}$ in nature.

V. GCI with GMB Distributions

Once the labelled set posteriors in GLMB family are marginalized to the GMB distributions based on propositions 1-4, the task of distributed fusion based on GCI rule turns to computing the EMD of GMB distributions, according to the proposed fusing strategy. Actually, a manipulatable formula for EMD of RFS distributions has two implicit demands: one is that the formula should have a convenient mathematical representation making the computation of (9) tractable; the other is that the fused distribution should belong to the same family of local posteriors or its unlabelled versions, in order to enable the sequential fusion with senor network owing more two nodes. In this section, we are devoted to derive explicit formula for EMD of GMB distributions, in line with the idea to satisfy these two demands.

A. Two-step Approximation

Consider two GMB distributions π_s of form (24) , $s = 1, 2$. These are the unlabelled versions of posteriors output by two local filters in GLMB filter family in a network. Omitting the conditioning on the observations for convenience,

$$\pi_s(\{x_1, \cdots, x_2\}) = \sum_{\sigma} \sum_{(\mathcal{I},\phi)\in\mathcal{F}_n(\mathbb{I}_s)\times\Phi_s} \omega_s^{(\mathcal{I},\phi)} \prod_{i=1}^{n} p_s^{(\phi),\mathcal{I}^v(i)}(x_{\sigma(i)}), s = 1, 2 \tag{36}$$

If we directly substitute (36) into (9), we are faced with a tough problem of simplifying the term,

$$(\pi_s)^{\omega_s} = \left(\sum_{\sigma} \sum_{(\mathcal{I},\phi)\in\mathcal{F}_n(\mathbb{I}_s)\times\Phi_s} \omega_s^{(\mathcal{I},\phi)} \prod_{i=1}^{n} p_s^{(\phi),\mathcal{I}^v(i)}(x_{\sigma(i)})\right)^{\omega_s} \tag{37}$$

It is really intractable to compute (37). Hence, in this paper, we try to change the train of thought. We will not substitute GMB distributions into (9) directly, but approximate it using a more tractable expression, i.e., the first moment matched MB distribution in the first place. Then we derive the explicit formula for EMD of the first moment matched MB distributions using a reasonable approximation. As the fused distribution is another GMB distribution, it can enable the distributed fusion with GLMB filter family even with more than two sensors in the sensor network. The two-step approximation in the derivation of EMD of GMB distributions is summarized in Propositions 5 and 6.

Proposition 5. *The MB distribution that matches exactly the first-order moment of GMB distribution $\pi_s(X)$ in (36), is $\widetilde{\pi}_s(X) = \{(\widetilde{r}_s^\imath, \widetilde{p}_s^\imath(x)\}_{\imath\in\mathbb{I}_s}$, where*

$$\begin{aligned} \widetilde{r}_s^\imath &= \sum_{\mathcal{I}\in\mathcal{F}(\mathbb{I}_s)} \sum_{\phi\in\Phi_s} 1_{\mathcal{I}}(\imath) w_s^{(\phi,\mathcal{I})} \\ \widetilde{p}_s^\imath(x) &= \frac{1}{\widetilde{r}_s^\imath} \sum_{\mathcal{I}\in\mathcal{F}(\mathbb{I}_s)} \sum_{\phi\in\Phi_s} 1_{\mathcal{I}}(\imath) w_s^{(\phi,\mathcal{I})} p_s^{(\phi),\imath}(x) \end{aligned} \tag{38}$$

Without loss of generalization, assume that $|\mathbb{I}_1| \leq |\mathbb{I}_2|$. The following gives the definition of a fusion map.

Definition 2. *A fusion map (for the current time) is a function $\tau : \mathbb{I}_1 \to \mathbb{I}_2$ such that $\tau(i) = \tau(i^*)$ implies $i = i^*$, The set of all such fusion maps is called fusion map space denoted by $\mathcal{T}$. The subset of τ with domain $\mathcal{I}$ is denoted by $\mathcal{T}(\mathcal{I})$.*

Proposition 6. *The EMD of the two MB distributions in (38), $\pi_\omega(X)$ can be approximated as a GMB distribution of the form*

$$\pi_\omega(\{x_1, \ldots, x_n\}) = \sum_{\sigma} \sum_{(\mathcal{I},\tau)\in\mathcal{F}_n(\mathbb{I}_1)\times\mathcal{T}(\mathcal{I})} w_\omega^{(\mathcal{I},\tau)} \prod\nolimits_{i=1}^{n} p_\omega^{(\tau),\mathcal{I}^v(i)}(x_{\sigma(i)}) \tag{39}$$

where

$$\overline{w}_\omega^{(\mathcal{I},\tau)} = \prod\nolimits_{\imath\in\mathcal{I}} \int \left(\frac{\widetilde{r}_1^\imath \widetilde{p}_1^\imath(x)}{1-\widetilde{r}_1^\imath}\right)^{\omega_1} \left(\frac{\widetilde{r}_2^{\tau(\imath)} \widetilde{p}_2^{\tau(\imath)}(x)}{1-\widetilde{r}_2^{\tau(\imath)}}\right)^{\omega_2} dx \tag{40}$$

$$C = \sum_{(\mathcal{I},\tau)\in\mathcal{F}(\mathbb{I}_1)\times\mathcal{T}(\mathcal{I})} \overline{w}^{(\mathcal{I},\tau)} \tag{41}$$

$$w_\omega^{(\mathcal{I},\tau)} = \overline{w}_\omega^{(\mathcal{I},\tau)}/C \tag{42}$$

$$p_\omega^{(\tau),\imath}(x) = \frac{\widetilde{p}_1^\imath(x)^{\omega_1} \widetilde{p}_2^{\tau(\imath)}(x)^{\omega_2}}{\int \widetilde{p}_1^\imath(x)^{\omega_1} \widetilde{p}_2^{\tau(\imath)}(x)^{\omega_2} dx}, \ \imath \in \mathbb{I}_1 \tag{43}$$

Proof. Substituting (38) in the term $\widetilde{\pi}_s(X)^\omega$, we obtain

$$\widetilde{\pi}_s(\{x_1,\ldots,x_n\})^{\omega_s} = \left(\widetilde{Q}_s \sum_{\sigma} \sum_{\mathcal{I}\in\mathcal{F}_n(\mathbb{I}_s)} \prod_{i=1}^{n} \widetilde{\beta}_s^{\mathcal{I}^v(i)}(x_{\sigma(i)})\right)^{\omega_s} \tag{44}$$

where

$$\widetilde{Q}_s = \prod_{\imath\in\mathbb{I}_s} (1 - \widetilde{r}_s^\imath), \quad \widetilde{\beta}_s^\imath(x) = \frac{\widetilde{r}_s^\imath \widetilde{p}_s^\imath(x)}{1-\widetilde{r}_s^\imath} \tag{45}$$

Motivated by [5], [28], we use the approximation

$$\left(\sum_i d_i\right)^{\omega} \approx \sum_i d_i^{\omega} \tag{46}$$

and (44) can be written as

$$\widetilde{\pi}_s(\{\mathbf{x}_1,\ldots,\mathbf{x}_n\})^{\omega_s} \approx \widetilde{Q}_s^{\omega_s} \sum_{\sigma} \sum_{\mathcal{I}\in\mathcal{F}_n(\mathbb{I}_s)} \prod_{i=1}^{n} \widetilde{\beta}_s^{\mathcal{I}^v(i)}(x_{\sigma(i)})^{\omega_s} \tag{47}$$

Then substituting (47) into the numerator of (9) and utilizing Definition 2, we obtain

$$\begin{aligned} &\widetilde{\pi}_1(\{x_1, \cdots, x_2\})^{\omega_1} \widetilde{\pi}_2(\{x_1, \cdots, x_2\})^{\omega_2} / \left(\widetilde{Q}_1^{\omega_1} \widetilde{Q}_2^{\omega_2}\right) \\ &= \sum_{\sigma} \sum_{\mathcal{I}\in\mathcal{F}_n(\mathbb{I}_1)} \sum_{\tau\in\mathcal{T}(\mathcal{I})} \prod_{i=1}^{n} \widetilde{\beta}_1^{\mathcal{I}^v(i)}(x_i)^{\omega_1} \widetilde{\beta}_2^{\tau(\mathcal{I}^v(i))}(x_{\sigma(i)})^{\omega_2} \\ &= \sum_{\sigma} \sum_{\mathcal{I}\in\mathcal{F}_n(\mathbb{I}_1)} \sum_{\tau\in\mathcal{T}(\mathcal{I})} \overline{w}_\omega^{(\mathcal{I},\tau)} \prod_{i=1}^{n} p_\omega^{(\tau),\mathcal{I}^v(i)}(x_{\sigma(i)}) \end{aligned} \tag{48}$$

Thus the denominator of (9) can be computed as:

$$\begin{aligned} &\int \pi_1(\{x_1, \cdots, x_2\})^{\omega_1} \pi_2(\{x_1, \cdots, x_2\})^{\omega_2} / \left(\widetilde{Q}_1^{\omega_1} \widetilde{Q}_2^{\omega_2}\right) \delta X \\ &= \sum_{n=0}^{\infty} \sum_{\mathcal{I}\in\mathcal{F}_n(\mathbb{I}_1)} \sum_{\tau\in\mathcal{T}(\mathcal{I})} \overline{w}_\omega^{(\mathcal{I},\tau)} \\ &= \sum_{\mathcal{I}\in\mathcal{F}(\mathbb{I}_1)} \sum_{\tau\in\mathcal{T}(\mathcal{I})} \overline{w}_\omega^{(\mathcal{I},\tau)} \end{aligned} \tag{49}$$

Finally, substituting (48) and (49) into (9), we obtain the fused density as the form of (39).

□

Remark: The approximation in (47) seems reasonable whenever the cross-products in (44) are negligible; this, in turn, holds provided that the means $\int x \widetilde{p}_s^\imath(x) dx$ and $\int x \widetilde{p}_s^{\imath'}(x) dx$, $\imath \neq \imath'$, are well separated, as measured by the respective covariances. This condition can be met when different hypothesized tracks described by $\widetilde{p}_s^{\imath}(x)$ are not closely spaced by measuring its kinematic parameters such as position and velocity. Our simulation results in [19] have shown that the required minimum distance between each tracks is about 2∼3 resolution cells which can be easily satisfied in most scenarios, shown as our empirical data.

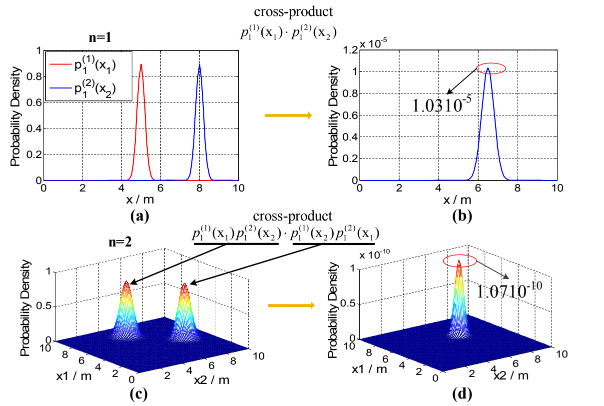


Fig. 1. (a) shows the $p_1^{(1)}(\mathbf{x}_1)$ and $p_1^{(2)}(\mathbf{x}_2)$; (c) shows the $p_1^{(1)}(\mathbf{x}_1)p_1^{(2)}(\mathbf{x}_2)$ and $p_1^{(1)}(\mathbf{x}_2)p_1^{(2)}(\mathbf{x}_1)$; (b) and (d) are the corresponding cross-products respectively.

For example, consider an MB distribution of form (38) with parameters meeting the condition mentioned above and denoted as $\pi_1 = \{r_1^{\imath}, p_1^{\imath}(x)\}_{\imath=1}^{2}$, where $r_1^1 = 0.8$ and $r_1^2 = 0.9$; $p_1^1(x) \sim \mathcal{N}(x, 5; 0.2)$ and $p_1^2(x) \sim \mathcal{N}(x, 7; 0.2)$ with $x \in \mathbb{R}$. The cross-products when $n = 1$ and $n = 2$ of (44) are shown in Fig.1 respectively. It can be seen that the maximums of cross-products are such low that they are negligible.

B. Summary and Discussions

In previous sections, we proposed a novel strategy for distributed fusion with labelled set posteriors. The labelled set posteriors are marginalized to its unlabelled versions firstly in order to avoid the problem arising from "label space mismatching" phenomenon. Propositions 1-4 give the mathematic representation of unlabelled versions of GLMB family and show that all these unlabelled distributions belong to the GMB family. Hence, the task of the distributed fusion with GLMB family posteriors turns to the GCI fusion with GMB posteriors. In this section, Propositions 5 and 6 provide the explicit formula for EMD of GMB posteriors and the fused posterior is also another GMB distribution. Thus the derived formula enables the distributed fusion with GLMB filter family over a sensor network owing more than two nodes. Fig. 2 shows the relationship between distributed fusion with GLMB filter family.

VI. Performance Assessment

In this section, the performance of the proposed GCI with GMB distributions (GCI-GMB) is examined in terms of the optimal sub-pattern assignment (OSPA) error [29]. We adopt

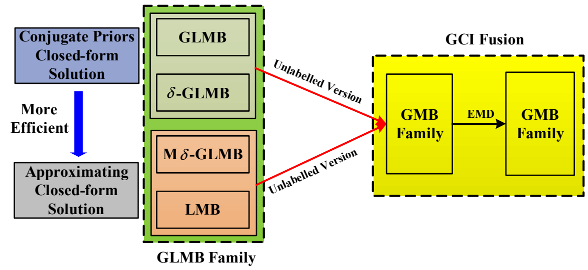


Fig. 2. The relationship between distributed fusion with GLMB filter family

GM implementations for the GCI-GMB fusion over distributed networks and the parameter ω in (9) is chosen as 0.5. All performance metrics are averaged over 200 Monte Carlo runs.

The following target and observation models are used. The target state variable is a vector of plannar position and velocity $x_k = [p_{x,k}, p_{y,k}, \dot{p}_{x,k}, \dot{p}_{y,k}]^T$. The single-target transition model is linear Gaussian specified by

$$\mathbf{F}_k = \begin{bmatrix} \mathbf{I}_2 & \Delta\mathbf{I}_2 \\ \mathbf{0}_2 & \mathbf{I}_2 \end{bmatrix}, \mathbf{Q}_k = \sigma_v^2 \begin{bmatrix} \frac{1}{4}\mathbf{I}_2 & \frac{1}{2}\Delta\mathbf{I}_2 \\ \frac{1}{3}\mathbf{0}_2 & \mathbf{I}_2 \end{bmatrix}$$

where $\mathbf{I}_n$ and $\mathbf{0}_n$ denote the $n \times n$ identity and zero matrices, $\Delta = 1$s is the sampling period, and $\sigma_\nu = 0.63m/s^2$ is the standard deviation of the process noise. The probability of target survival is $P_{S,k} = 0.98$; The probability of target detection in each sensor is independent of the probability of detection at all sensors and is $P_D = 0.95$. The single-target observation model is also a linear Gaussian with

$$\mathbf{H}_k = \begin{bmatrix} \mathbf{I}_2 & \mathbf{0}_2 \end{bmatrix}, \mathbf{R}_k = \sigma_\varepsilon^2 \mathbf{I}_2,$$

where $\sigma_\varepsilon = 1.4m$, is the standard deviation of the measurement noise. The number of clutter reports in each scan is Poisson distributed with $\lambda = 10$. Each clutter report is sampled uniformly over the whole surveillance region.

A. Scenario 1

To assess the robustness of the proposed GCI-GMB fusion algorithm, we compare it with the GCI with LMB filter (GCI-LMB) [26] under two assumptions:

A.1. the label spaces of birth processes for each local filters are matching, i.e.,

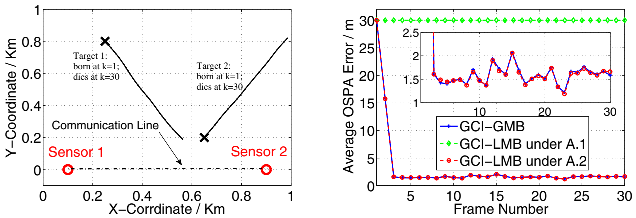
$$(r_{\Gamma,1}^1, p_{\Gamma,1}^1) \rightarrow (r, p) \rightarrow (r_{\Gamma,1}^2, p_{\Gamma,1}^2)$$
$$(r_{\Gamma,2}^1, p_{\Gamma,2}^1) \rightarrow (\overline{r}, \overline{p}) \rightarrow (r_{\Gamma,2}^2, p_{\Gamma,2}^2)$$

A.2. the label spaces of birth processes for each local filters are mismatching, i.e.,

$$\begin{array}{ccc} (r_{\Gamma,1}^2, p_{\Gamma,1}^2) \rightarrow (r, p) & & (r_{\Gamma,1}^2, p_{\Gamma,2}^2) \\ & \times & \\ (r_{\Gamma,1}^2, p_{\Gamma,2}^2) \rightarrow (\overline{r}, \overline{p}) & & (r_{\Gamma,2}^2, p_{\Gamma,2}^2) \end{array}$$

where $r = \overline{r} = 0.01$, $p(x) = \mathcal{N}(x; m, P)$, $\overline{p}(x) = \mathcal{N}(x; \overline{m}, \overline{P})$, $m = [250, 800, 12, -20]^T$, $\overline{m} = [650, 200, 10, 20]^T$, $P = \overline{P} = diag([800, 800, 25, 25]^T)^2$.

We consider a scenario involving two targets on a two dimensional surveillance region $[0, 1000]m \times [0, 1000]m$ shown in Fig. 3 (a). For both fusion algorithms, the local filters are chosen as the LMB filters.



(a) (b)

Fig. 3. (a) The scenario of distributed sensor network with two sensors tracking two targets, (b) the average OSPA errors under different assumptions.

Fig. 3 (b) shows the average OSPA errors for the GCI-GMB fusion and the GCI-LMB fusion under assumptions A.1 and A.2. Notice that GCI-GMB fusion is performed on the unlabelled state space and thus is independent of assumptions A.1 and A.2. It can be seen that the performance of GCI-GMB algorithm is almost at the same level with GCI-LMB algorithm under A.1, while GCI-LMB algorithm under A.2 complete collapse. According to the analysis in section III-A, in practical scenarios, "label space mismatching" phenomenon is very common. Thus, the performance of GCI-LMB will have considerable uncertainty and risk in practice. On the contrary, the proposed GCI-GMB algorithm is much more robust to accommodate the unknown relationship between each label spaces.

B. Scenario 2

In this scenario, the performance of the GCI-GMB fusion is compared to the GCI with PHD filter (GCI-PHD) proposed in [4]. A sensor network scenario involving four targets which appear at different time steps is considered as shown in Fig. 3. The local filters are chosen as PHD filter for GCI-PHD fusion, and the δ-GLMB filter for GCI-GMB fusion. In this

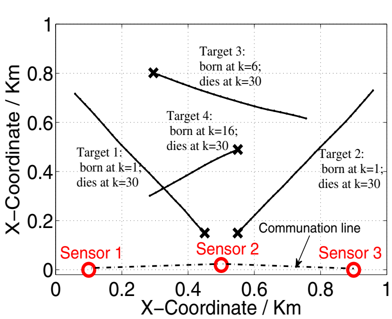
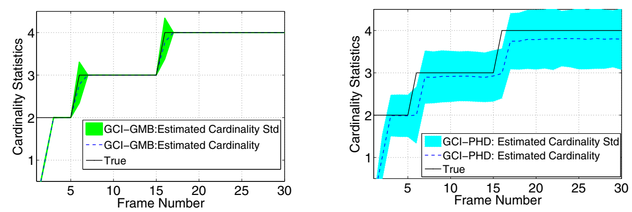


Fig. 4. The scenario of distributed sensor network with three sensors tracking four targets.

sensor network, each sensors have same quality and can only exchange posteriors with its neighbours. Specifically, both sensors 1 and 3 perform fusion of two posteriors from sensor 2 and the local filter, while sensor 2 performs fusion of three posteriors from sensor 1, sensor 3 and the local filter by sequentially applying the pairwise fusion (38) and (39) two times.

Figs. 5 (a) and (b) show the cardinality estimations of GCI-GMB fusion and GCI-PHD fusion for sensor 2, respectively. It can be seen that the cardinality estimation of GCI-GMB fusion is more accurate and exhibits much smaller covariance than GCI-PHD fusion.

The average OSPA errors shown in Fig. 6 illustrates the performance difference between the GCI-GMB and GCI-PHD fusion algorithms for sensor 2. It can be seen from Fig. 6 that



(a) (b)

Fig. 5. (a) Cardinality statistics for GCI-GMB, (b) Cardinality statistics for GCI-PHD.

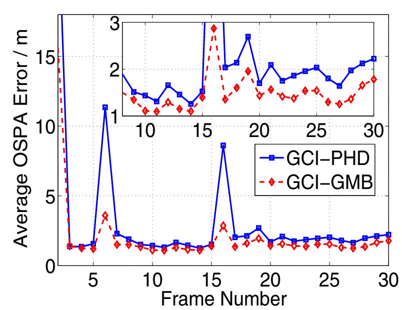


Fig. 6. Performance comparison, using average OSPA error, between GCI-GMB and GCI-PHD.

the proposed GCI-GMB fusion performs significantly better than GCI-PHD fusion over total time. The track initiations are much faster when new targets born at time step 6 and 16. Also, when the performances reach stable, the OSPA errors of GCI-GMB fusion is much more lower than GCI-PHD fusion algorithm. The results also demonstrate implicitly that the approximations used in the derivation of EMD of GMB distributions are reasonable.

Table I shows the performance comparisons between local filter, two and three sensors for GCI-GMB fusion in terms of averaged OSPA errors over 18th$\sim$30th frames and 200 Monte Carlo runs. The results demonstrate that more number of sensors contributing to the fusion, the smaller estimation errors are.

TABLE I
AVERAGE OSPA ERRORS *vs* NUMBER OF SENSORS

Number of sensor	One	Two	Three
OSPA Errors(m)	1.9416	1.5015	1.4124

VII. CONCLUSION

In this paper, we addressed the problem of the distributed multi-target tracking with labelled set filters based on generalized Covariance Intersection (GCI). Firstly, we proposed a robust strategy for distributed fusion with labelled set posteriors to get rid of the bad influence of "label space mismatching" phenomenon. Secondly, we derived the unlabelled versions of common labelled set distributions in generalized labelled multi-Bernoulli (GLMB) family and showed they all belong to the generalized multi-Bernoulli (GMB) family. Thirdly, we derived the explicit formula for the GCI with GMB distributions, which enables the distributed fusion with GLMB filter family, including the GLMB, δ-GLMB, Mδ-GLMB and LMB filters. Simulation results for Gaussian mixture implementation have demonstrated the robustness and effectiveness of GCI with GMB fusion algorithms in two challenging tracking scenarios.

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